Hors-série spécial 50 ans du BETA

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Notice introductive : **Sophie Béreau**

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Hors-série « 50 ans du BETA »
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Notice introductive

Il apparaît de façon cyclique dans l'histoire de notre discipline, des voix (souvent discordantes) pour affirmer la fin de la macroéconomie ou tout du moins, réclamer à grands cris son nécessaire *aggiornamento*[1] (voir Abraham-Frois, 2000, ou plus récemment De Grauwe, 2009, 2010 ; Romer, 2016 ; Stiglitz, 2018, et au-delà de nos « frontières » disciplinaires Bouchaud, 2008 ou Farmer et Foley, 2009 appelant à une profonde remise en cause de nos modèles et méthodes voire à une « révolution scientifique » de l'économie). D'autres, comme Reis (2018)[2] soulignent qu'en dépit des nombreuses critiques (la plupart légitimes) que l'on peut adresser aux macroéconomistes – et à leurs modèles d'équilibre général dynamique stochastique (ou DSGE) coupables de tous les maux (De Grauwe, 2010 ; Stiglitz, 2018 ; Storm, 2022), l'avenir du champ repose *in fine* sur la capacité de ces derniers à produire des travaux de recherche qui soient « stimulants, divers et enthousiasmants »[3]. Notre propos dans le cadre de ce dossier spécial consacré à la macroéconomie est d'illustrer dans quelle mesure les chercheurs du groupe MACRO du BETA ont contribué à entretenir/cultiver cette flamme, selon les grandes tendances internationales de notre discipline, tout en dessinant une trajectoire singulière en cohérence avec l'histoire et l'identité même de notre laboratoire.

Pour tenter d'apporter (modestement) quelques éléments de réponse à cette question, arrêtons-nous un instant sur la production scientifique en macroéconomie de ces dernières années et comparons les travaux réalisés au sein du BETA à ceux de nos collègues des départements d'économie et d'institutions de par le monde. A cet égard, l'étude bibliométrique de Glandon et al. (2021) permet de dessiner les grandes tendances de la discipline à partir de l'analyse d'articles publiés sur les 40 dernières années dans les revues généralistes et « *top-field* » les plus réputées. Les auteurs pointent la prédominance des approches théoriques micro fondées, avec une progressive montée en puissance des approches DSGE (qui représentent plus de 60 %[4] des contributions publiées en macroéconomie sur les deux dernières années de l'étude, au détriment des modèles d'équilibre partiel), dont une proportion croissante (mais néanmoins encore minoritaire) tente de répondre aux critiques évoquées plus haut, soit au travers de règles de comportements hétérogènes des agents (sans que cela n'implique de rupture forte avec l'hypothèse de rationalité ou de remise en cause du concept d'équilibre unique), soit par l'introduction de (frictions sur les) marchés financiers. Les auteurs pointent en outre une ouverture croissante vers la multidisciplinarité, qui s'entend ici comme des emprunts/approches croisées depuis et vers d'autres questions/domaines de la même discipline (codes J.E.L. à l'appui). Enfin, du côté des approches empiriques, ils constatent une montée en puissance des données micro ainsi que des méthodes économétriques associées, au détriment des approches traditionnelles relevant des séries temporelles macro-financières.

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[1] L'usage de ce terme est un petit clin d'œil à la première publication de notre série de DT spéciale 50 ans, consacrée à « L'aggiornamento des sciences économiques en France : le cas strasbourgeois au tournant des années 70 » de Rodolphe Dos Santos Ferreira, Ragip Ege et Sylvie Rivot, le premier étant à nouveau mis à l'honneur dans ce numéro spécial avec deux contributions marquantes, voir *infra*.
[2] Les contributions de Stiglitz (2018) et Reis (2018) font partie d'un numéro spécial de l'*Oxford Review of Economic Policy* consacré à la refondation de la macroéconomie des suites de la crise de 2008.
[3] Ou selon la formulation de l'auteur en anglais dans le texte : « *doing vibrant, varied and exciting work* ».
[4] 61% des papiers publiés selon le code J.E.L. « E : Macroeconomics and Monetary Economics » entre 2016 et 2018 relèvent de modélisations DSGE, ce chiffre atteignant 42% pour l'ensemble des publications du champ, tous codes J.E.L. confondus durant la même période.

Pour rendre compte de la façon la plus juste des spécificités propres aux recherches menées au sein de l'équipe MACRO au cours du temps, nous avons pris le parti de mettre à l'honneur non pas une mais plusieurs des contributions ayant émaillé l'histoire de notre groupe dans le cadre d'un dossier thématique[5]. Aussi, les articles sélectionnés témoignent-ils de la grande variété des recherches conduites. Cette variété tient aux objets d'étude : concurrence imparfaite et chômage involontaire, « esprits animaux » et fluctuations de court terme, impacts croisés de la transparence des banques centrales et des décisions de politiques budgétaires des gouvernements sur l'activité économique, ou encore liens entre (biodiversité), diffusion épidémique et économie. Elle tient aussi aux approches : modélisations en équilibre général vs. équilibre partiel, DSGE, modélisations empiriques. Sans oublier les supports de publication allant des revues *mainstream* aux revues multidisciplinaires en passant par les numéros spéciaux/livres. Tous les articles retenus sanctionnent le fruit de recherches exigeantes au long court, voire parfois, pour certains d'entre eux, de la capacité à répondre en temps réel mais néanmoins de façon rigoureuse, à des problématiques plus immédiates.

En écho aux évolutions évoquées par Glandon et al. (2021), une première caractéristique de la recherche en macroéconomie menée au BETA tient ainsi à l'exploration d'un grand nombre de questions à l'aide d'outils formels relevant de ce qu'il est commun d'appeler les « fondements micro de la macro ». Ici, la démarche, loin d'obéir à un *diktat* ou un dogme, se révèle des plus fécondes, puisqu'il s'agit de recourir à des résultats micro pour (i) illustrer/éclairer la logique à l'œuvre au niveau macro de façon pertinente (notamment lorsque cela permet de coller au plus près aux faits stylisés sans recourir à des contorsions de modélisation invraisemblables ou problématiques, e.g. un niveau de *mark-up* « dans les limbes » incohérent avec les faits stylisés, Dos Santos Ferreira et Dufourt, 2006) ; ou (ii) de réconcilier intuitions anciennes (e.g. Keynes et sa théorie du chômage involontaire, d'Aspremont, Dos Santos Ferreira et Gérard-Varret, 1990, le même Keynes et ses « esprits animaux », Dos Santos Ferreira et Dufourt, 2006) et formalisme rigoureux dans les normes admises de la discipline.

Dans cette veine, le premier article que nous mettons à l'honneur dans ce dossier est l'une des premières publications[6] issue du travail d'un trio « presque légendaire » pour reprendre la formulation de Jean Gabszewicz (Gabszewicz, 2001), j'ai nommé Claude d'Aspremont, Rodolphe Dos Santos Ferreira et Louis-André Gérard-Varret (d'Aspremont, Dos Santos Ferreira et Gérard-Varret, 1990). Tout a déjà été écrit sur leur relation d'amitié et leur complémentarité intellectuelle des plus fructueuses (plus de 20 publications toutes plus prestigieuses les unes que les autres, avec des contributions principalement théoriques contenant de nombreuses applications à des questions très variées dont certaines, sur lesquelles nous nous concentrons ici, relèvent de la macroéconomie)[7]. Comme rappelé par Pierre Dehez dans sa notice du numéro spécial consacré à un cadre d'analyse similaire développé par ces mêmes auteurs[8], le point de départ du papier consiste en une analyse de concurrence imparfaite

[5] Sans prétendre à l'exhaustivité, cette sélection (subjective, mais néanmoins concertée avec les principaux intéressés au gré d'allers-retours entre le comité éditorial et les différentes équipes de travail au sein du groupe) se veut la plus représentative des travaux menés au sein du groupe, divers tant par leurs objets que leurs méthodes.

[6] Si mes informations sont exactes, la première publication des mêmes auteurs (hors documents de travail du CORE dont on trouve trace ici) portait déjà sur le lien entre concurrence oligopolistique et chômage, publiée dans l'*Oxford Economic Papers* en 1989 (d'Aspremont, Dos Santos Ferreira et Gérard-Varret, 1989) et dont une version libre de droits est disponible ici : https://www.di.ens.fr/~aspremon/Claude/PDFs/dAsp89a.pdf

[7] Une lettre de l'EHESS (http://lettre.ehess.fr/index.php?2311) retrace l'histoire de leur collaboration ainsi que des laboratoires qu'ils ont contribué à façonner.

[8] Voir https://beta-economics.fr/uploads/2022/08/WP-HS2022-14.pdf.

en équilibre général selon l' « approche objective » de Nikaido (1975) (par opposition à l'approche « subjective » de Negishi, 1961) où les producteurs sont supposés connaître la « vraie » courbe de demande à laquelle ils sont assujettis. Plus spécifiquement ici, partant d'une structure oligopolistique sur le marché des biens, les auteurs démontrent la possibilité théorique du chômage involontaire - ce qui revient formellement à prouver l'existence de configurations de marché où aucune valeur strictement positive de salaire ne permet de garantir l'équilibre sur le marché du travail - l'ampleur de ce déséquilibre dépendant d'un certain nombre de choix de modélisation dont les auteurs discutent la pertinence et les limites de façon détaillée et argumentée.

Un peu moins de deux décennies plus tard mais néanmoins dans une même lignée, la contribution de Rodolphe Dos Santos Ferreira et Frédéric Dufourt (Dos Santos Ferreira et Dufourt, 2006) s'intéresse elle aux caractéristiques d'équilibre de libre entrée des firmes sur le marché en concurrence imparfaite et en particulier, à la possible indétermination de leur statut (active vs. passive), justifiant le recours dans un second temps à une échelle macro, à la formalisation de règles de comportements dépendant d'« esprits animaux » dont ils étudient ensuite l'influence sur les cycles réels. Plus précisément, les auteurs raisonnent d'abord à l'échelle micro. Partant de l'observation qu'en présence de marchés contestables, l'équilibre n'est atteignable qu'en présence de profits nuls des firmes, condition à partir de laquelle il est alors possible d'identifier le nombre optimal de firmes, les auteurs explorent le cadre de concurrence imparfaite « à la Cournot »[9]. Ils montrent que des niveaux de profits strictement positifs (et potentiellement non négligeables) peuvent être atteints par les firmes à l'équilibre, rendant dès lors impossible l'identification de leur nombre optimal. Ce résultat actant l'indétermination du statut des firmes à l'équilibre est ensuite mis à profit dans un modèle d'équilibre général dynamique (DGE) où les firmes sont supposées choisir d'entrer ou non sur le marché au gré d'« esprits animaux » chers à Keynes (modélisées ici comme des vagues d'optimisme ou de pessimisme) jouant sur leurs anticipations des comportements de leurs concurrents. Les auteurs étudient ensuite comment un tel mécanisme, associé à des chocs externes sur les conjectures des producteurs (et les comportements de marge qui en découlent), permettent de rendre compte des fluctuations observées de l'économie américaine, résolvant au passage quelques incohérences rencontrées précédemment dans cette littérature. Ces deux travaux sont un régal pour le lecteur : reposant sur des modélisations relativement simples, limpides et élégantes, présentées de façon très didactique ils constituent (presque !) une lecture de chevet et donnent du grain à moudre au *mantra* de notre discipline - qui ne reconnaît comme valides que des résultats micro fondés, en permettant de ré-explorer des intuitions keynésiennes dans un cadre analytique rigoureux.

Au-delà des thèmes traditionnels évoqués plus haut, les chercheurs du groupe MACRO se sont également intéressés à des questions variées de macroéconomie ouverte et de macroéconomie européenne. Ainsi, la contribution de Romain Restout, co-écrite avec Olivier Cardi et Peter Claeys (Cardi, Restout et Claeys, 2020), articule explorations empiriques *via* une série d'estimations VAR en panel sur un ensemble de pays de l'OCDE avec un modèle d'équilibre général dynamique (DGE) étendu au cas d'une petite économie ouverte à deux secteurs en supposant, outre l'imparfaite mobilité du travail, que le secteur de biens échangeables est largement subventionné. Ainsi, ils documentent et expliquent l'impact potentiellement différentié de chocs de politiques budgétaires dans les secteurs des biens échangeables vs. non

[9] Il est à noter que ce résultat n'est pas propre à ce seul cadre d'analyse. Comme le montrent les mêmes auteurs dans un travail antérieur, de nombreux *setups* permettent d'aboutir à l'indétermination du nombre de firmes à l'équilibre, voir Dos Santos Ferreira et Dufour (2007).

échangeables. Les auteurs montrent que les chocs de dépenses publiques tendent à générer une réallocation du travail du secteur des biens échangeables vers le secteur des biens non échangeables, particulièrement dans les pays où les coûts associés sont les plus faibles. Ces chocs bénéficient ainsi de façon plus marquée au secteur de biens non échangeables, ce qui corrobore et explique les faits empiriques mis en lumière dans la littérature récente sur cette question. L'approche originale développée dans le papier ainsi que l'aller-retour entre données et théorie selon le triptyque *data-theory-data*, s'avère particulièrement éclairant et convaincant, inspirant depuis un certain nombre de travaux publiés dans les meilleures revues du domaine, e.g. Bouakez, Rachedi et Santoro (2023), Cox, Müller, Pastor et Schoenle (2020) ou encore Proebsting (2022).

L'article de Meixing Dai et Moïse Sidiropoulos (Dai et Sidiroppoulos, 2011), deux des piliers de l'équipe de macroéconomie européenne rattachée à l'aile strasbourgeoise du BETA est assez emblématique des questions auxquelles le groupe s'intéresse depuis maintenant 25 ans[10]. Ce groupe qui compte à son actif de nombreuses contributions significatives au débat public notamment sur les questions de l'articulation des différents outils de politiques économiques et de coordination des états membres face aux défis rencontrés au sein des pays de l'Union, a vu au fil des ans ses travaux sanctionnées par deux chaires Jean Monnet, un prix Nobel de la paix (!)[11], et plus récemment, la nomination d'Amélie Barbier-Gauchard à l'IUF en tant que membre junior. Dans ce papier, les auteurs discutent de l'impact croisé de la transparence des banques centrales et des caractéristiques de leur politique budgétaire, sur la performance et la volatilité macroéconomiques. Habituellement discutée – et prônée – pour une implémentation efficace de la politique monétaire, la transparence des banques centrales (BC) a fait l'objet de nombreuses études dans la littérature[12] comme rappelé par les auteurs en préambule de leur article. Les effets d'une telle démarche lorsque des outils budgétaires sont mis en œuvre par les gouvernements sont en revanche beaucoup moins clairs, voire plaident pour plus d'opacité. Alors que les effets distorsifs des différentes formes de taxation sont connus et abondamment discutés dans la littérature[13], les auteurs rappellent la nécessité de tenir compte des effets positifs de l'investissement public sur la productivité du travail et, partant, sur le niveau de production de long terme, les seconds contrebalançant potentiellement les premiers. Aussi, afin de faire la lumière sur l'impact net des différents outils (monétaire, budgétaire, fiscal) à disposition des gouvernements selon le niveau de transparence de leurs politiques, les auteurs proposent une modélisation à deux périodes et trois agents : (i) une firme représentative, (ii) un gouvernement soucieux de stabiliser à la fois l'inflation et l'*output gap*, qui dispose pour

[10] La création de l'Observatoire de Politiques Economiques Européennes (OPEE) par Michel Dévouy et Moïse Sidiropoulos date de 1998, suivie de l'équipe ERMEES au sein du groupe MACRO (ex-axe « Macroéconomie et Politiques Publiques ») en 2013 par Amélie Barbier-Gauchard, Francesco de Palma et Moïse Sidiropoulos.

[11] On se rapportera avec intérêt à la présentation de Moïse Sidiropoulos réalisée à l'occasion de la conférence anniversaire du BETA pour comprendre pourquoi le BETA (et l'équipe de macroéconomie européenne en particulier) peut être considéré comme le juste récipiendaire du Prix Nobel de la Paix décerné « à l'Europe » en 2012.

[12] Une référence souvent oubliée sur le sujet et pourtant lumineuse est celle de Bernhard Winkler (Winkler, 2000), qui décrit de façon subtile et documentée les différentes acceptions de la transparence – en distinguant notamment le « degré d'ouverture » (« *openness* ») de la banque centrale, soit la quantité et la précision des informations qu'elle délivre auprès du public (souvent l'unique dimension considérée dans la littérature sur le sujet), de la « clarté » (« *clarity* ») de son discours, qui, pour être compris, doit être basé sur un corpus commun (« *common understanding* »), partagé avec le public, dans un soucis d'honnêteté (« *honesty* »), conférant ainsi audit public la capacité de traiter l'information de façon non ambigüe. En se référant à la littérature, il invoque différents exemples pour discuter des effets attendus - potentiellement antagonistes - de ces différentes composantes sur l'efficacité des politiques monétaires.

[13] Voir Nicodème (2008) pour un tour d'horizon des effets économiques de la taxation des revenus des entreprises et plus récemment Martin et Trannoy (2019) sur l'imposition de la production en France.

cela de deux outils, l'outil fiscal qu'il contrôle et l'outil monétaire qu'il délègue à (iii) une banque centrale dont le niveau de transparence est capturé par une composante stochastique affectant la prédictibilité, par les agents public et privé, des poids alloués aux cibles d'inflation et d'*output gap* dans la fonction de perte. Le timing du jeu est alors le suivant : la firme représentative forme ses anticipations d'inflation, puis le gouvernement fixe le taux d'imposition ainsi que le niveau investissement public, et enfin la BC choisit le taux d'inflation. Formellement, les auteurs supposent un jeu séquentiel où le secteur privé joue un jeu de Nash contre la BC alors que le gouvernement (leader) joue un jeu de Stackelberg contre la BC (suiveur), résolu par induction à rebours. Différentes configurations sont envisagées (avec ou sans investissement public) et montrent l'importance des interactions entre les différents effets envisagés : sans investissement public, plus d'opacité impacte positivement la performance macroéconomique via une réduction du taux de taxation et donc des distorsions, alors qu'en présence d'investissement public l'absence de transparence de la BC n'apparaît pas avoir d'effet, que ce soit sur le niveau ou sur la volatilité (stabilité) macroéconomique. Ces questions demeurent essentielles à l'heure où de nombreuses sources d'instabilités menacent l'économie mondiale, comme en témoigne, entre autres, l'intervention récente de Philip Lane, s'exprimant en tant que membre du directoire de la BCE dans le cadre d'un panel dédié à la question de la gouvernance budgétaire en Europe[14].

Enfin, en écho aux tentatives d'ouverture évoquées par Glandon et al. (2021), une troisième voie explorée au sein de notre groupe tient à l'hybridation de modèles. Tout en ne cédant rien sur la rigueur de l'approche formelle, l'objectif affiché est d'explorer des questions antérieurement tenues en dehors des frontières de notre champ disciplinaire, en combinant des éléments de langage et de formalisme issus de nos modélisations standards, i.e. le modèle de Ramsey pour l'un (Bosi et Desmarchelier, 2020), un modèle DSGE pour l'autre (Acurio Vásconez, Damette et Shanafelt, 2021), à ceux de sciences connexes, en l'occurrence l'écologie et l'épidémiologie. Dans les deux articles que nous avons retenus dans ce dossier, les auteurs introduisent des éléments théoriques issus de la même classe de modèles de diffusion épidémiologique à compartiments où la population est divisée en différents groupes (les « compartiments ») d'individus selon leur exposition/statut vis-à-vis de l'agent pathogène (e.g. virus, bactérie, prion, etc.) considéré dans l'étude. Dans les configurations les plus simples on distingue les individus susceptibles de contracter la maladie (labellisés « S » pour « *susceptible* »), les individus infectés dès lors supposés contagieux (labellisés « I » pour « *infected* »), et éventuellement, lorsqu'une forme d'immunité peut être acquise à l'issue de l'infection, les individus guéris (labellisés « R » pour « *recovered* »). Puis, des règles formalisées sous formes d'équations différentielles spécifient ensuite la proportion d'individus passant d'un compartiment à l'autre dans chacun des cas, l'acronyme utilisé pour le modèle précisant l'ordre de ces différentes règles. Ainsi, un individu initialement sain (S) peut devenir infecté (I). La troisième étape diffère selon le type de maladie envisagée : il peut se remettre avec immunisation (R) dans la configuration de type SIR comme celle introduite par Acurio Vásconez, Damette et Shanafelt (2021), adéquate pour répliquer le mécanisme de diffusion d'un virus comme le SARS-CoV-2 ; dans l'article de Bosi et Desmarchelier (2020) la modélisation renvoyant à des maladies endémiques, les individus redeviennent sains après les phases d'exposition puis d'infection, mais susceptibles de contracter à nouveau la maladie (configuration SIS).

[14] Voir https://www.ecb.europa.eu/press/key/date/2021/html/ecb.sp211112~739d3447ab.en.html.

Plus précisément, dans leur article David Desmarchelier et Stefano Bosi (Bosi et Desmarchelier, 2021) s'intéressent aux conséquences macroéconomiques des effets croisés de la pollution sur la perte de biodiversité et partant, sur la prévalence chez l'homme des zoonoses - ces maladies infectieuses transmissibles de l'animal vers l'homme – qui, du fait de la raréfaction des hôtes dits secondaires, tend à s'accroître, un effet connu dans la littérature éco-épidémiologique comme « effet de dilution »[15]. Les auteurs incorporent une composante « SIS » dans un modèle de Ramsey dans lequel le processus de production génère des externalités négatives sous forme de pollution néfaste à la biodiversité. Celle-ci est modélisée comme une ressource renouvelable dont la dynamique est affectée négativement par le niveau de pollution contre laquelle lutte le gouvernement au moyen d'une taxe Pigouvienne. Dans ce cadre, les auteurs discutent de façon très convaincante et documentée les effets attendus de la biodiversité sur les différentes composantes du modèle, puis étudient, selon l'intensité de l'effet de dilution, les caractéristiques de court et de long terme du modèle : les états stationnaires possibles selon deux régimes (avec ou sans maladie endémique) ainsi que les trajectoires potentielles vers ces points d'équilibre, ouvrant la voie dans certaines configurations à des bifurcations de Hopf où la biodiversité alterne entre niveaux haut et bas, en conformité avec la littérature documentant les extinctions de masse successives ayant ponctué l'histoire de notre planète. Un certain nombre de résultats clé émergent, parmi lesquels la possibilité que l'action humaine (i.e. le processus de production, polluant, à risque pour la survie des espèces) cause une extinction de masse lorsque l'effet de dilution est faible et en l'absence de maladie endémique à l'état stationnaire susceptible de constituer une force de rappel vers un niveau de production (et de pollution) moindres. A l'inverse, lorsque l'effet de dilution est supposé fort, il permet de préserver la biodiversité à long terme et évite au système économique de sombrer dans le « paradoxe vert »[16], soit l'existence d'une relation positive entre « taxe verte » et niveau de pollution à l'état stationnaire. Tous ces résultats foisonnants témoignent du caractère extrêmement riche et complexe des interactions entre biodiversité, chaîne épidémiologique et dynamique économique, et appellent sans nul doute à de plus amples explorations dans le futur.

Dans une même veine mais au moyen d'outils différents, la contribution de Verónica Acurio Vásconez, Olivier Damette et David Shanafelt (Acurio Vásconez, Damette et Shanafelt, 2021), a quant à elle pour ambition de documenter l'impact économique d'une pandémie, à l'instar de celle de la COVID-19. Plus spécifiquement, l'objectif affiché est d'étudier la capacité de politiques monétaires non conventionnelles, dites d'« assouplissement quantitatif », à diluer les effets délétères sur l'économie réelle attendus dans un tel contexte, soit une perte de 20 % de PIB selon le scénario de base du modèle. Les auteurs conceptualisent alors un dispositif d'« *epi loans* », s'assimilant à un accroissement exogène du montant des créances détenues par la banque centrale qui profite directement aux producteurs en besoin de capital physique sans pour autant constituer de l'argent « gratuit »[17]. Ce dispositif original s'écarte d'un côté d'une

[15] Voir notamment Civitello et al. (2015) qui dans le cadre d'une méta-analyse documentent les preuves empiriques en faveur de l'hypothèse de dilution, arguant que le déclin de la biodiversité induit par l'homme – du fait de son impact délétère sur la vie parasitaire et herbivore, pourrait augmenter la prévalence des maladies humaines et fauniques, et diminuer la production agricole et forestière.

[16] Ce terme a été introduit par Sinn en 2008 (Sinn, 2008) dans le contexte d'un modèle d'extraction de ressource optimale « à la » Hotelling.

[17] En pratique, les mesures d'assouplissement quantitatif mises en place par la BCE pour endiguer la crise des suites de la pandémie inclurent, outre l'extension du programme d'achat d'actifs en vigueur des suites de la crise de la dette de 2010, un vaste plan spécifique dans la même veine appelé PEPP pour « *Pandemic emergency purchase programme* » à destination des acteurs publics et privés à hauteur de 1850 milliards d'euros. Pour plus de détails, voir notamment ici et Blot (2021) pour une analyse détaillée des outils de politique monétaire mobilisées par le BCE des suites de la pandémie.

politique de Quantitative Easing traditionnelle, puisqu'il ne passe pas par l'intermédiaire des banques privées et n'a donc pas pour objectif de stimuler le crédit au travers d'un ajustement à la baisse des taux d'intérêt alors que ceux-ci avoisinent déjà des niveaux historiquement bas. Mais il ne correspond pas non plus tout-à-fait à de la « monnaie hélicoptère » conceptualisée par Milton Friedman puisque l'injection de liquidités directement au sein de l'économie réelle est couteuse pour les producteurs. Les auteurs, à l'instar des approches les plus récentes de notre discipline, partent d'une modélisation DSGE dans laquelle sont ajoutés des marchés financiers à la manière de Gertler et Karadi (2011), mais, et c'est l'innovation majeure de leur contribution, où l'offre de travail des seuls agents sains dépend directement de la proportion de ces derniers dans la population, gouvernée par l'évolution exogène de la composante « SIR » introduite dans le modèle comme un choc permanent de nature déterministe. Leurs résultats illustrent l'influence prédominante du taux de rémission sur l'activité économique – plus encore que celle du taux d'infection. Aussi le taux de reproduction d'une épidémie, le fameux R_0 largement suivi et commenté dans les médias, qui s'assimile au ratio du second sur le premier et mesure donc le nombre de nouveaux cas qu'une seule personne infectée et contagieuse va générer en moyenne dans une population sans aucune immunité (i.e. lorsque tous les individus sont supposés être de type "S"), s'il mesure bien la contagiosité d'une épidémie, ne prédit pas pour autant les pertes en bien-être encourues par l'économie réelle. Pour y faire face, et après avoir testé différents outils de politique monétaire non conventionnelle, les auteurs montrent que seuls les « *epi loans* », permettent de réduire les effets récessifs de la crise en évitant l'écueil inflationniste, offrant une avenue de recherche des plus intéressantes pour la suite.

Au vu de ces éléments, le groupe MACRO du BETA apparaît (1) en équilibre sur ses deux « pieds » en faisant la part belle à (i) la théorie d'une part, micro fondée avec conscience, parcimonie et élégance, tout en proposant des travaux (ii) à vocation appliquée d'autre part dans le but d'alimenter le débat public, notamment sur des questions environnementales et/ou de politique européenne. A cet égard, il marque ainsi son (2) ancrage dans un territoire, celui de la région Grand-Est où Strasbourg, « capitale de l'Europe » - et de ses institutions -, fournit un terrain d'exploration riche et sans cesse renouvelé, et finalement, se montre (3) à la pointe de l'hybridation, avec un goût marqué pour des approches audacieuses visant à dépasser les frontières de notre seule discipline, notamment dans le cadre de collaborations fécondes avec des chercheurs d'INRAe. Fort de ces trois axes, comment envisager l'avenir de la macroéconomie (au sein du BETA) ? Gaffard (2013) dans une rétrospective historique décrit comment les faits économiques ont de tous temps façonné les théories macroéconomiques, avec la résurgence d'idées et de concepts « anciens » au gré des époques et surtout des crises successives (de la Grande dépression à la crise de la dette souveraine européenne)[18]. A l'aune des défis environnementaux et sociétaux majeurs de notre époque, quelles sont les possibles directions pour le futur de notre discipline ? En quoi les recherches menées au sein du groupe MACRO dont nous avons rendu compte (de façon bien imparfaite ici) sont-ils les témoins (voire parfois les précurseurs ?) de la (nécessaire) mue opérée par notre champ tant dans ses objets d'études que ses méthodes, pour rendre compte de ces enjeux ? Cette question n'appelle évidemment pas de réponse définitive, si ce n'est l'intuition (tout à fait personnelle et

[18] Un ouvrage récent d'Assous et Carret (2022) propose une relecture tout à fait passionnante de l'histoire de la macroéconomie avec comme point focal la question de l'instabilité des systèmes économiques au centre des préoccupations d'un ensemble d'économètres/économistes quantitatifs de ce qui allait devenir l'*Econometric Society* du début des années 20 à la fin des années 50, et au détriment de la figure tutélaire de John Maynard Keynes qui bien que demeurant un contributeur important, apparaît ici comme secondaire. Un blog associé à l'ouvrage propose un ensemble de billets ainsi que des simulations stylisées à partir des différents cadres théoriques étudiés : https://www.economic-instability.com/

éminemment subjective) que les approches hybrides dont nous nous sommes faits l'écho apportent des éléments de réponse particulièrement bienvenus pour faire avancer la science et bousculer les limites de ce qu'il est possible et raisonnable de faire ou dire sur la base d'une modélisation macroéconomique. Hybridation des méthodes mais aussi hybridation des équipes, où chercheurs strasbourgeois et nancéiens collaborent main dans la main autour d'autres projets tout aussi « stimulants, divers et enthousiasmants » à l'instar, par exemple, du travail en cours de Desmarchelier et al. (2021) qui étendent l'article de Bosi et Desmarchelier (2021) évoqué dans ce dossier à la question de l'impact de mesures de *lock-down* sur le système économique, ou encore du groupe de travail[19] sur la modélisation en macroéconomie comportementale visant à mesurer l'impact des chocs climatiques sur la stabilité macro-financière en s'attaquant notamment à l'une des pierres angulaires des critiques portées aux approches relevant de la macroéconomie standard : la prise en compte de la nature complexe du système économique (Bouchaud, 2008 ; Leijonhufvud, 2009) au travers de la modélisation des interactions individuelles et des effets de composition dans la dynamique des agrégats macro, autorisant la possibilité d'équilibres multiples (voire de dynamiques chaotiques). *Last but not least,* le recrutement récent de nombreux (micro)économètres au sein de notre laboratoire ainsi que les contacts d'un certain nombre d'entre nous avec des collègues spécialistes de l'analyse de données de grande dimension et des algorithmes d'apprentissage statistique laissent augurer des collaborations fructueuses entre équipes (au sein du BETA et en dehors !) pour des explorations empiriques futures.

Sophie Béreau[20], Nancy le 24 janvier 2023

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[19] Le projet MiraCLE regroupe des chercheurs du groupe MACRO nancéiens et strasbourgeois et vise à (1) comparer différents modèles macroéconomiques allant de DSGE aux modèles multi-agents macroéconomiques en passant par des modélisations à agents hétérogènes, avec un étalonnage similaire chaque fois que cela est possible et (2) proposer une nouvelle approche hybride dans la veine des travaux évoqués dans Hommes (2021) avec pour ambition de rendre compte des liens complexes entre les sphères du climat, de la finance et de l'économie réelle.

[20] Je remercie les auteurs des contributions figurant dans ce dossier spécial ainsi qu'Agnès Gramain et Magali Jaoul Grammare pour leurs relectures attentives de versions préliminaires de ce texte.

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Macroepidemics and unconventional monetary policy: Coupling macroeconomics and epidemiology in a financial DSGE-SIR framework

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Abstract

Despite the fact that the current COVID-19 pandemic was neither the first nor the last disease to threaten a pandemic, only recently have studies incorporated epidemiology into macroeconomic theory. In our paper, we use a dynamic stochastic general equilibrium (DSGE) model with a financial sector to study the economic impacts of epidemics and the potential for unconventional monetary policy to remedy those effects. By coupling a macroeconomic model to a traditional epidemiological model, we are able to evaluate the pathways by which an epidemic affects a national economy. We find that no unconventional monetary policy can completely remove the negative effects of an epidemic crisis, save perhaps an exogenous increase in the shares of claims coming from the Central Bank ("*epi loans*"). To the best of our knowledge, our paper is the first to incorporate disease dynamics into a DSGE-SIR model with a financial sector and examine the effects of unconventional monetary policy.

JEL codes: D58, E32, E52

Keywords: New-Keynesian model, DSGE, COVID-19, epidemiology

Running header: Macroepidemics and unconventional monetary policy

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§All remaining errors are ours.

1 Introduction

The economic effects of the COVID-19 pandemic are unprecedented, far-reaching, and extend to virtually every member of the global market. Global growth was projected at minus 4.9 percent in 2020, and at 6 percent to 7.6 percent depending on the emergence of a second wave (IMF (2020)). COVID-19 was not the first emerging zoonotic or epizoonic disease to threaten a pandemic (Boissay and Rungcharoenkitkul (2020), LePan (2020)), nor will it be the last (Daszak et al. (2001), Jones et al. (2008), Wu et al. (2017)).

Prior to the COVID-19 pandemic, few studies incorporated epidemiology into macroeconomic theory, though this was not the case in microeconomics (see Horan and Wolf (2005), Horan and Fenichel (2007), Fenichel et al. (2011), Lenhart and Workman (2007), Morin et al. (2014), and Morin et al. (2015) for examples). Recent studies have examined the potential economic impacts of pandemics on a macroeconomic scale using Susceptible-Infected-Recovered (SIR) epidemiological models in the line with the macro model developed by Eichenbaum et al. (2020b). However, the role of financial intermediaries in coupled epidemic-economic frameworks has yet to be studied. In addition, previous papers have not focused on the effect of economic remedies - in the form of monetary policies - to reduce the economic burden of epidemics.

In this paper, we use a dynamic stochastic general equilibrium (DSGE) model as in Smets and Wouters (2007), but with a financial sector as in Gertler and Karadi (2011) (GK hereafter), to study the economic effects of an epidemic and the ability of monetary policy to remedy the crisis. Thus, our model is a financial DSGE-SIR model. To the best of our knowledge, we are the first to incorporate SIR dynamics into a DSGE model with a financial sector. Using the GK framework enables us to account for the financial sector of the economy and to assess the efficiency of unconventional monetary policy to combat the economic burdens of an epidemic. It enables us to investigate different recovery paths of the economy following shocks to the system, including an epidemic crisis. For instance, the GK model was used to extensively examine the effects of unconventional monetary policy on macroeconomic outputs following the subprime crisis (Gertler and Karadi (2011), Dedola et al. (2013), Gelain and Ilbas (2017)). Gertler and Karadi (2011) showed that when there is a financial crisis (understood as a negative shock in the quality of capital), the stronger the reaction by the Central Bank, and the smaller the total losses in GDP. In comparison to a simpler model without financial frictions *à la Smet-Wouters*, our financial DSGE-SIR model enables us to study macro-financial feedback loops.

We evaluate the effects of unconventional monetary policy, in particular a form of quantitative easing (QE) or "*epi loans*" policy. We model "*epi loans*" as a Central Bank liquidity injection into the real sector in the form of claims that do not pass-through private banks, similar to those that followed the sub-prime crisis in Europe. This measure can be understood as a light form of "helicopter money" (Friedman (1969)), in the sense that the injected liquidity goes directly to the real sector without direct involvement of fiscal authorities or private banks. However, contrary to"helicopter money", our "*epi loans*" policy must be repaid, thus changing the Central Bank balance sheet by increasing its assets. Further, while "helicopter money" may be highly inflationist, there is no proof that QE policies are, at least not in developed countries (Qianying et al. (2016), Albertazzi et al. (2018), Baumeister and Benati (2013)). In this regard, the Central Bank behaves as last resort lender for the economy.

Our model incorporates six different agents: households, financial intermediates, non-financial goods producers, capital producers, retailers and a government. It also considers the existence of a Central Bank that conducts conventional and unconventional monetary policy. From a methodological point of view, this study goes further than Smets and Wouters (2007) and Gertler and Karadi (2011) by coupling the labor sector to an epidemiological SIR model rather than assuming that each household chooses the quantity of hours it wants to work in each period. We suppose that labor supply is given by the quantity of people in good health, and is exogenously driven by the SIR model. In addition, we suppose that the government may dispense unemployment benefits to those who can no longer work due to illness.

In general, we find significant GDP losses due to an epidemic shock, with the effect on the labor market echoing throughout the economy. We observe declines in household consumption, non-financial intermediary capital, and capital producer investment following the trajectories of labor and production, and financial intermediaries experiencing declines in the quantity and composition of expected discounted terminal wealth. The Central Bank increases its share of total credits that it finances to compensate for losses in investment and production. What is particularly interesting is that it is feasible to have a severe epidemic that does not result in a large economic loss, provided that the recovery rate is sufficiently high to allow workers to quickly return to the labor force. The nature of the epidemic thus has a strong impact on the macroeconomic response.

In terms of monetary policy, we find that no unconventional monetary policy can completely remove the negative economic effects of the crisis, besides perhaps an exogenous increase in the share of claims coming from the Central Bank. Our "*epi loans*" policy is a form of QE policy related to Friedman (1969) "helicopter money", in that the Central Bank takes savings from households and issues it as claims to be used to buy physical capital rather than re-financing private banks. The injected liquidity goes directly to the real sector.

Our framework is not directly targeted towards COVID-19, but instead models a representative epidemic. That being said, it can be tailored to any combination of epidemiological models or economic parameters, making it possible to calibrate the model to a specific disease or country. While we believe that our model is relevant to the current pandemic, we hope that its contribution extends to epidemics more generally.

The paper is structured as follows. Section 2 presents related literature. The model is presented in Section 3, whereas Section 4 describes the elements of the calibration and model simulation. Section 5 analyzes the response of the economy to the epidemic shock and investigates the effect of monetary policy. Finally, Section 6 concludes.

2 Related Literature

Since the beginning of the COVID-19 pandemic, there has been an explosion of literature investigating the macroeconomics of pandemics. In this section, we briefly survey the literature, presenting the main methodological choices and key results, and explain in more detail how we depart from those studies. We categorize the literature into two thematics: the economic impacts of a pandemic and the effects of policy response.

2.1 Economic impacts of a pandemic

A first line of literature outlines the channels through which the pandemic shock affects the economy. Carlsson-Szlezak et al. (2020a), Carlsson-Szlezak et al. (2020b), and Brodeur et al. (2020), identified three broad patterns that have emerged from the current pandemic. The first is a direct impact generated by a reduced consumption of goods and services (a demand shock), which is exacerbated by social distancing and pessimistic expectations in the short-run. The second is an indirect impact based on financial market shocks and their effects on the real side of the economy. Household wealth will likely fall (wealth effects) as precautionary savings increase (due to uncertainty), leading to declines in new consumption spending. The third set of effects consist of supply-side disruptions. Declines in production due to containment and mitigation policies negatively impact supply chains, labor demand, and global employment and, as a consequence, unemployment and GDP losses strongly increase. In addition, a negative supply shock can trigger a demand shortage that leads to a contraction in output and employment larger than the supply shock itself (Guerrieri et al. (2020)). The existence of "wait-and-see" attitudes adopted by economic agents (described by Baldwin and DiMauro (2020)) are likely to reinforce the previous effects by generating additional uncertainty. All in all, different types of recovery geometry - "V-shaped", "U-shaped", "WU-shaped", or "L-shaped"- are possible depending on the persistence of shocks and government interventions.

The basis for these findings are predominantly theoretical in nature, and can be seen as hypotheses to be tested and re-evaluated. Therefore, economists have empirically assessed the economic impacts of the pandemic, as well as delved deeper into their theoretical foundations. We divide them into three sub-groups based on their methodology.

Our first sub-group quantitatively assesses the potential response of the economy to a pandemic crisis, mostly from a macroeconometric perspective. Ludvigson et al. (2020) assessed the macroeconomic impact of COVID-19 in the United States from historical data using a vector auto-regression VAR model. They quantified the potential response of the economy by comparing the current pandemic shock to a series of large disaster shocks in US time series data. Using the costly disaster index, they found that a 60 standard deviations shock from the mean can generate a 12.75 percent drop in industrial production. Chudik et al. (2020) developed a threshold-augmented dynamic multi-country model (TGVAR) to estimate the global as well as country-specific macroeconomic effects of the identified COVID-19 shock. They showed that the most-developed economies will likely experience deeper, longer-lasting effects. For example, they found evidence of long-term, carry-over effects for countries like the United States and the United Kingdom, but not for developing Asian countries. Milani (2021) used a standard GVAR to investigate the importance of interconnections between countries. He found that the unemployment responses varied widely across countries after a health shock. Bonadio et al. (2020) developed a quantitative framework to simulate a negative global labor shock and examine the role of global supply chains in explaining the intensity of the real GDP downturn due to the COVID-19 shock. They found that "re-nationalization" of global supply chains would not make countries more resilient to pandemic-induced contractions in labor supply. Baqaee and Farhi (2020) stressed the role of non-linearities associated with complementarities in consumption and production in response to the COVID-19 shock using a multi-sector, neoclassical model.

Another set of studies relies on static or dynamic computable general equilibrium models, focusing on international spillovers and sectoral effects. A family of Computable General Equilibrium (CGE) were developed to study the macroeconomic impacts of pandemics on a global scale and trade. In particular, the popular CGE G-Cubed (Mckibbin and Fernando (2020)) and ENVISAGE (Maliszewska et al. (2020)) models have been extended to account for COVID-19. Both extensions focused on the importance of spillover effects in a globalized economy when assessing the GDP and macroeconomic losses. Mihailov (2020) implemented potential economics responses within a standard Galí-Smets-Wouters DSGE model (Galí et al. (2011)) calibrated to US, France, Germany, Italy and Spain. In all cases, the negative effects are quite damaging and last between one and two years on average. However, these papers treat epidemics as completely exogenous shocks without the integration of epidemic dynamics. Our work extends this literature by explicitly incorporating an epidemiological model into a macroeconomic framework, taking into account the dynamics of the economic patterns, incorporating a financial sector, and exploring the role of financial intermediaries and the use of unconventional monetary policies. The introduction of financial market disruptions, as in GK, allow us to analyze the effects of unconventional monetary policies.

Our work is more akin to the works of Bodenstein et al. (2020), Eichenbaum et al. (2020a,b,c), Angelini et al. (2020) or Krueger et al. (2020). These studies develop more-or-less simple macroeconomic neoclassical models, in which agents consume goods and work, combined with disease models that are standard in the epidemiology literature. However, they treat the labor market in a markedly different way than us. To be more specific, in those models agents choose the number of hours to work, with household consumption and labor changing the number of susceptible and infected individuals. The more a person consumes or works, the more s/he is in contact with others and the probability of infection is higher. Supply hours decrease not because people of getting sick, but because infected individuals are less productive (lower revenue) (Eichenbaum et al. (2020b)) and individuals know that if they work, they have a higher risk of infection. We do not follow this assumption, choosing to assume that sick individuals cannot or are not allowed to work. We believe that this assumption is reasonable, does not impact our results, and avoids introducing addition assumptions (such as homogenous mixing) into the model. Further, to the best of our knowledge, our paper is the first to directly consider the financial sector in this framework.

From a methodological point of view, our model is closest to Bodenstein et al. (2020), whom enlarge a ECB-BASE model with the dynamics of a SIR model with two distinct population groups. They embed a canonical epidemiology model (SIR) in a Real Business Cycle (RBC) type model. In contrast, we mix a financial DSGE *à la* GK and a SIR model and as a consequence, our model enables us to study the interplay between the real economy and the financial sector.

2.2 Economic Policies

A key challenge for policy makers is to identify suitable policies to mitigate the adverse economic effects of epidemics. Kaplan et al. (2020) demonstrated that the role of the government is not just to balance lives and livelihood (health versus economic output), but also over who should bear the burden of the economic crisis. This should be taken into account when investigating the optimality of lockdown and fiscal policies. Krueger et al. (2020) extended the Eichenbaum et al. (2020a,b,c) studies to analyze the "Swedish case". They found that a no government intervention with flexible resource allocation can lead to a substantial mitigation of economic and human costs of the COVID-19 crisis. Other papers have stressed the need for government intervention, particularly economic policies. Elenev et al. (2020) focused on the interrelationships between corporate and financial sectors and real macro-economy output. They found evidence that a no-intervention policy generates a negative feedback loop between corporate default and weakness in the financial intermediary sector and creates a macroeconomic disaster. They studied the role of corporate credit policies to mitigate this situation, and suggested the implementation of conventional or unconventional monetary policies, which we explicitly consider here. Faria-e Castro (2020) analyzed different types of discretionary fiscal policies to smooth household incomes in a simple DSGE model. Conditional and unconditional transfers to households were effective mitigation policies, with expansion of unemployment insurance as the best targeted measure.

In a theoretical model with multiple equilibria, Céspedes et al. (2020) demonstrated that traditional expansionary fiscal policy had no beneficial effects, while conventional monetary policy had a limited effect when the discount rate was low. Unconventional policies, including helicopter drops of liquid assets, equity injections and loan guarantees, were able to keep the economy at a higher equilibrium in terms of productivity and unemployment. In a similar fashion, Sharma et al. (2020) developed a so-called "Mark-0 Agent-Based Model" based on the model by Gualdi et al. (2015). They simulated several policies including giving easy credit to firms and "helicopter money", i.e. injecting new money into households savings. Here, we analyze similar policy questions but, in contrast to Sharma et al. (2020), we build a DSGE-SIR framework with microeconomic foundations. Kiley (2020) added exogenous shocks to a GK framework to mimic the COVID-19 recession. He found that the use of extraordinary policy actions, such as a QE program of government bonds, may support recovery. We also depart from the GK model, but contrary to Kiley (2020) we explicitly incorporate epidemic dynamics. Our main value added is that our model enables us to take into account interactions between an epidemic and the economy, as well as the financial and real economic sectors, and to study the potential for monetary policy (specifically unconventional monetary policy) to mitigate the effects of an epidemic.

3 The Model

In this paper, we construct a so-called financial DSGE model like the one developed in Gertler and Karadi (2011). However, in contrast to the usual financial DSGE models, we enlarge our model with a SIR block (see Atkeson (2020)).

Our DSGE model is a neo-keynesian micro-founded aggregate representation of a national economy, in which we assume that there is an infinite number of economic agents divided into households, financial intermediates, non-financial goods producers, capital producers, and retailers, which individually chooses quantities of goods, production factors, bonds and eventually prices in order to maximize their own well-being (e.g. preferences for households and profits for bankers, capital producers, non-financial firms, and retailers). The model also includes a government and a Central Bank that conducts conventional and unconventional monetary policy.

We couple the DSGE model to a classic epidemiological model of an epidemic

(F.Brauer and Castillo-Chavez (1994, 2012), Hethcote (2000)) and suppose that labor supply is directly tied to the proportion of healthy individuals. For the sake of simplicity, we do not impose stochastic shocks to the economy, and take the trajectory of labor supply, which is affected by the disease, as a deterministic, exogenous shock to the economy. In this way we isolate the effects of the epidemic on the model economy.

In this section, we first describe the epidemiological model and how it relates to households and labor supply. We then describe how households behave, the structure of financial, non-financial and capital producers, and retailers. Finally, we explain how the government intervenes in the economy and monetary policies conducted by the Central Bank. Variables, definitions, and parameters are summarized in Figures 1 and 2 and Tables 1 to 3. For details on the full derivation of the model, see the Appendix.

3.1 Epidemiological Model

In order to model the spread of an epidemic, we use a Susceptible-Infected-Recovered (SIR) model as in F.Brauer and Castillo-Chavez (1994, 2012), Hethcote (2000), and Lenhart and Workman (2007). The SIR model is a type of compartmental epidemiological model in which the total population, N_t, is divided into three classes or types of individuals: susceptible individuals, S_t, who can incur the disease but are not yet infected; infected individuals, $\widetilde{I}_t$, who have the disease and can spread it to susceptible individuals; and recovered individuals, $\widetilde{R}_t$, who have contracted the disease but have recovered and are immune to future infections (Figure 2). For simplicity, we assume a constant population size, abstracting from natural births and deaths[1], and

[1]The validity of this assumption depends on the timescale of the analysis and the nature of the disease in question. Take for example, a single, localized epidemic and a population such that the disease could reasonably circulate throughout the entire population. For diseases like the cold, flu, or measles, an epidemic may last weeks or months and accounting for births and deaths would not be appropriate; for diseases lasting years or a lifetime (AIDS/HIV, hepatitis C, or tuberculosis), including births and deaths is more reasonable (Hethcote (2000)).

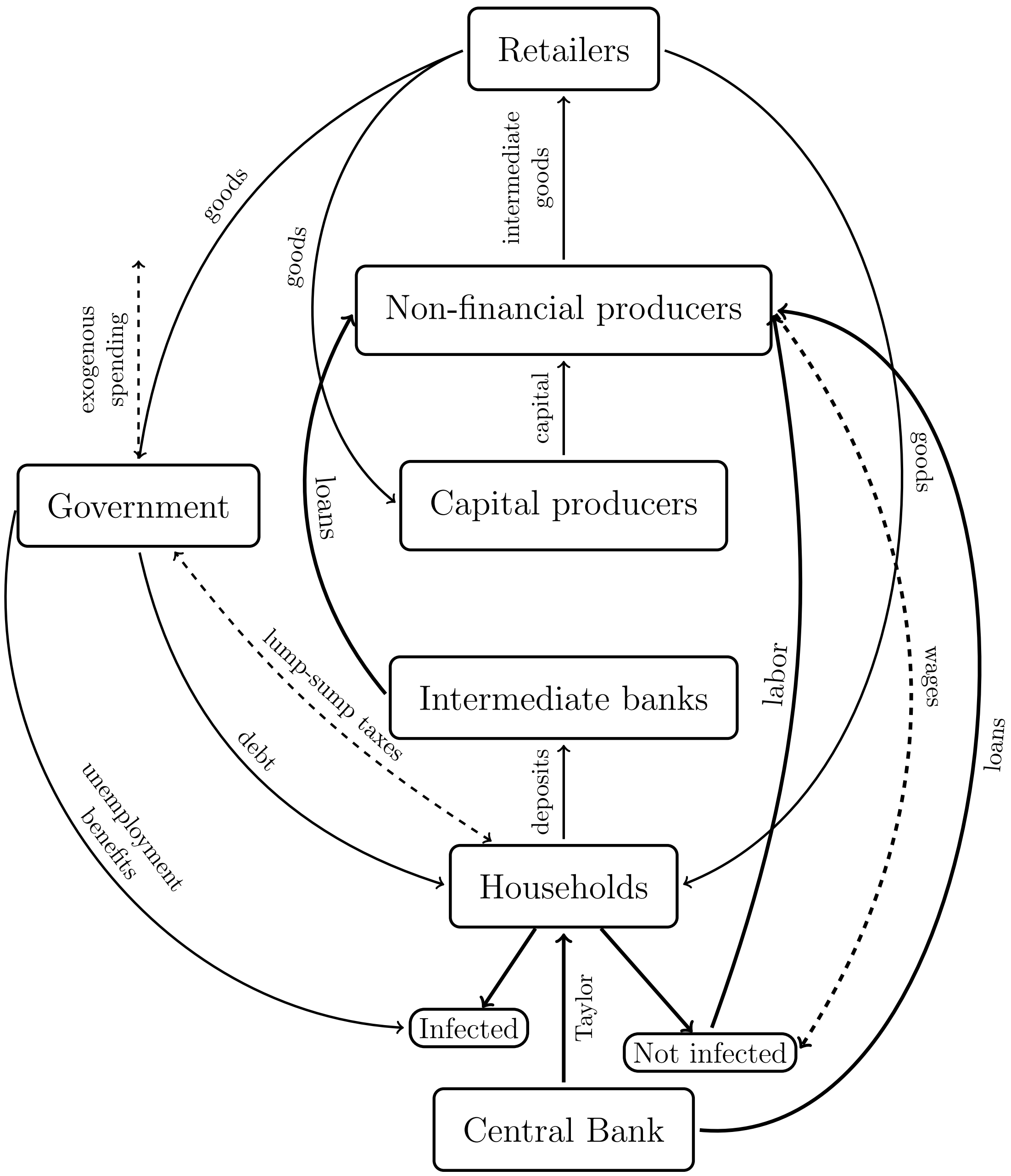

Figure 1: Economic Model Schema

normalize N_t to 1. Then S_t, $\widetilde{I}_t$ and $\widetilde{R}_t$ can be interpreted as shares or proportions of individuals of each class in the general population.

We can write the dynamics of the epidemic over time as:

$$S_{t+1} - S_t = -\alpha_v S_t \widetilde{I}_t \tag{1}$$

$$\widetilde{I}_{t+1} - \widetilde{I}_t = \alpha_v S_t \widetilde{I}_t - \gamma_v I_t \tag{2}$$

$$\widetilde{R}_{t+1} - \widetilde{R}_t = \gamma_v \widetilde{I}_t \tag{3}$$

where $1 = S_t + \widetilde{I}_t + \widetilde{R}_t$. The difference equations in (1)-(3) are equivalent to a system of ordinary differential equations solved via a Euler approximation. Susceptible and infected individuals make contact and transmit the disease with a constant probability α_v, and infected individuals recover at a rate γ_v. We assume that after recovery, individuals are immune from future infection.

Figure 2: SIR Schema

The model assumes a closed population (no immigration or emigration) with a constant population size (no births or deaths) and a well-mixed population. That is, each individual in the population has an equal probability of interacting with every other individual. Extensions of the basic SIR model relax these assumptions to take into account multiple populations of individuals (Bichara et al. (2015)), endemic disease (Hethcote (2000)), heterogeneous mixing (Morin et al. (2014), Morin et al. (2015), Toxvaerd (2020)), age structure (Hethcote (2000)), other classes of individuals such as exposed or asymptomatic, vaccinated or hospitalized (Chowell et al. (2003), Hethcote (2000), Lenhart and Workman (2007)), and management strategies such as treatment and vaccination (Hethcote (2000), Lenhart and Workman (2007), Toxvaerd and Rowthorn (2020)). However, relaxing our basic assumptions greatly complicates the analysis and is left for future work.

The epidemic affects the economy via the labor supply. Following Bodenstein et al. (2020), we assume that in absence of disease, labor supply L_t is equal to the total working force, $L_t = N_t$. However, as the epidemic spreads in the general population, we assume that infected individuals stay home and do not work, then the labor force is reduce by the quantity of infected people I_t. Thus, in each period, labor supply is given as $L_t = N_t$ - $\widetilde{I}_t$.

3.2 Households

We assume a continuum of perfectly competitive households in the economy indexed by $j \in [0, 1]$. Susceptible, infected, and recovered individuals are assumed to be evenly distributed among households. Each household consumes domestic goods, and, if healthy, supplies identical labor services to the non-financial production sector. Households pay/receive lump sum taxes, collect profits from all firms, have the option to lend funds to competitive financial intermediates or buy government bonds and, when infected, receive unemployment benefits.

At each time period t, a typical household j chooses consumption C_t to maximize the following lifetime expected utility function:

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \beta^k U \left(C_{t+k}(j) \right) \right] \tag{4}$$

where $U(C_t(j))$ is the net utility of household consumption of non-financial goods and $\beta \in (0, 1)$ is the discount factor.

We allow for internal habit formation in consumption as in Christiano et al.

(2005). Thus, the instantaneous utility at time t is given by:

$$U(C_t(j)) = (log(C_t(j) - hC_{t-1}(j))) \tag{5}$$

where $h \in [0, 1)$ represents the internal habit formation parameter. The latter governs how household preferences for past consumption affects utility over time. A high value of h means that past consumption is important, so as to maintain the current level of utility, the household must consume at least the same quantity as the last time period. A low value of h implies that households only care about present consumption. Note that we do not introduce a trade-off between consumption and labor since labor supply is determined by the epidemic. With this formulation, we implicitly assume that all those who can work are willing to do it.

Within each household there may be a portion of infected people, whom do not work but receive unemployment compensation b_t. The remaining individuals - susceptible and/or recovered - may be divided in two groups: workers and bankers. Workers do so for non-financial intermediate firms and receive a real salary W_t in exchange for the total amount of labor provided L_t. Bankers manage financial intermediaries and gain earnings. We assume that each member of the household gives their respective revenues to the household and that there is perfect consumption insurance. That is, consumption is equally distributed within households regardless if everyone in them is able to work.

Each household consumes final goods produced by retailers at price P_t and invests/deposits an amount B_t in government bonds and intermediary deposits. We assume that investing in government bonds and depositing into intermediate banks are equivalent and perfectly substitutable, as both are risk-less and pay the same rate. Each are one-period real bonds, which pay a gross real rate of return R_t such that $R_{t+1} := \frac{1+i_t}{\Pi_{t+1}}$, where i_t is the nominal interest rate fixed by the Central Bank and $\Pi_{t+1} := \frac{P_{t+1}}{P_t}$ represents price inflation.

Share holders of retailers, capital firms, financial and non-financial firms receive real profits. We assume that each household owns an equal share of all firms and receives an aliquot share $D_t(j)$ of aggregate profits D_t, i.e. the sum of dividends of all retailers $D_{r,t}$, intermediate private banks $D_{b,t}$, intermediate non-financial firms $D_{m,t}$, and capital producers $D_{k,t}$. Thus $\int_0^1 D_t(j) = D_t := \int_0^1 (D_{r,t}(i) + D_{b,t}(i) + D_{m,t}(i) + D_{k,t}(i))di$ where i indexes an individual firm in each sector. Households pay/receive T_t lump-sum transfers.

For the sake of tractability, all households are identical and choose consumption and investment in the same manner. Then dropping the j subscript, we may write the real budget constraint for each household as:

$$C_t + B_{t+1} \leq b_t (1 - L_t) + W_t L_t + R_t B_t + T_t + D_t \tag{6}$$

Each household solves (4) under the budget constraint (6). The solution of this maximization problem gives us the following Euler equation that describes the evolution of consumption along an optimal path[2]:

$$1 = \beta \mathbb{E}_t \left[\frac{\lambda_{c,t+1}}{\lambda_{c,t}} R_{t+1} \right] \tag{7}$$

where $\lambda_{c,t}$ represents the marginal lifetime discounted utility function at t. Equation (7) says that, at the optimum, each consumer is indifferent to consuming one more unit today and saving that unit (by buying bonds) to consume in the future.

Assuming internal habit formation yields:

$$\lambda_{c,t} = \frac{1}{C_t - hC_{t-1}} - \beta h \mathbb{E}_t \left[\frac{1}{C_{t+1} - hC_t} \right] \tag{8}$$

Thus we define the stochastic real discount factor for the entire economy from

[2]Cf. Appendix for derivation.

period t to $t+i$ as:

$$\Lambda_{t,t+i} := \beta^i \frac{\lambda_{c,t+i}}{\lambda_{c,t}} \tag{9}$$

3.3 Financial Intermediates

For the time being we present the financial intermediate's problem assuming that the Central Bank does not apply unconventional monetary policy, i.e. it does not directly lend to non financial firms. We will relax this hypothesis in the next section.

We assume an infinite continuum of financial intermediates indexed by j. Each intermediate recovers a quantity $B_{t+1}(j)$ of deposits from households, which pays a gross interest rate R_{t+1}, and issues a quantity $Z_t(j)$ of financial claims to non-financial producers at a real price of Q_t per claim[3]. Denote $\Omega_t(j)$ as the net worth of banker j in period t such that:

$$\Omega_t(j) = Q_t Z_t(j) - B_{t+1}(j) \tag{10}$$

Given that assets acquired by bankers earn a rate of return $R_{k,t+1}$ on claims, then bankers' wealth at period $t+1$ is:

$$\Omega_{t+1}(j) = R_{k,t+1} Q_t Z_t(j) - R_{t+1} B_{t+1}(j) \tag{11}$$

And using equation (10) yields:

$$\Omega_{t+1}(j) = (R_{k,t+1} - R_{t+1}) Q_t Z_t(j) + R_{t+1} \Omega_t(j) \tag{12}$$

Note the difference in subscripts between the banker rate of return ($R_{k,t+1}$) and the

[3] In reality, the Central Bank also sells claims. Therefore, we should differentiate private claims $Z_{p,t}$ from government claims $Z_{g,t}$. However, for the sake of presentation, we abstract from this distinction in this section.

gross interest rate (R_{t+1}).

We assume that bankers cannot default on their loans. Then a banker j operates if and only if the following condition holds:

$$\mathbb{E}_t \Lambda_{t,t+1+i} \left(R_{k,t+1+i} - R_{t+1+i}\right) \geq 0, \quad i \geq 0 \tag{13}$$

where $\Lambda_{t,t+1+i}$ is defined as in (9). In other words, if a banker must borrow more than its income, then it will not remain a banker.

In each period t, a fraction f of household members are bankers; the remaining proportion are workers. We assume that a fraction θ of bankers in the current period remain bankers in the next time period. That is, $(1-\theta)f$ bankers become workers and a similar number of workers become bankers[4].

Accordingly, each banker has the following expected discounted terminal wealth:

$$\begin{aligned} V_t(j) &= \sum_{i=0}^{\infty} (1-\theta)\theta^i \Lambda_{t,t+1+i} \Omega_{t+1+i}(j) \qquad (14) \\ &= \sum_{i=0}^{\infty} (1-\theta)\theta^i \Lambda_{t,t+1+i} \left((R_{k,t+1+i} - R_{t+1+i}) Q_{t+i} Z_{t+i}(j) + R_{t+1+i} \Omega_{t+i}(j)\right) \end{aligned}$$

Under condition (13), bankers may want to increase their assets indefinitely by borrowing more and more funds from households. Furthermore, a banker can decide to divert funds, i.e. transfer a fraction or even the totality of assets to its own household for personal gain. Creditors are aware of this possibility as they know that there may be a fraction λ of funds that will never be recovered. However, they can impose a borrowing constraint to ensure that bankers do not divert all funds. Therefore, households are willing to supply funds to a bank only if the banker's expected discounted terminal wealth $V_t(j)$ is at least as large as the banker's gain

[4] As explained in Gertler and Karadi (2011), this assertion implies that the average "survival time" for a banker at any period is $\frac{1}{1-\theta}$. This insures that bankers cannot fund all investments from their own capital and that the relative proportion of each type of household remains constant over time.

form diverting funds $\lambda Q_t Z_t(j)$[5]:

$$V_t(j) \geq \lambda Q_t Z_t(j) \tag{15}$$

where in each period t, banker j chooses $Z_t(j)$ in order to maximize (14) subject to constraint (15).

The leverage ratio is the value of total loans of a banker to non-financial producers divided by the net worth of that banker. It is a measure of the proportion of worth that a banker lends. Define $\phi_t(j)$ as the leverage ratio of banker j as:

$$\phi_t(j) := \frac{Q_t Z_t(j)}{\Omega_t(j)} \tag{16}$$

Note that the leverage ratio can be greater than one (e.g. bankers can lend more than they have), depending on interest rates.

As in Gertler and Karadi (2011), suppose that the solution of this problem has the following form:

$$V_t(j) = \nu_t Q_t Z_t(j) + \eta_t \Omega_t(j) \tag{17}$$

where ν represents the expected discounted marginal value that the banker gains by expanding claims, and η represents the expected marginal value of an extra unit of wealth. Equation (17) forms the initial guess of the solution, which is required in order to solve the problem. See the Appendix for details.

If constraint (15) is binding, then we arrive at an interior solution with:

$$\nu_t = \mathbb{E}_t \Lambda_{t,t+1} \Gamma_{t+1} \left(R_{k,t+1} - R_{t+1} \right), \quad \eta_t = \mathbb{E}_t \Lambda_{t,t+1} \Gamma_{t+1} R_{t+1} \tag{18}$$

$$\Gamma_{t+1} = 1 - \theta + \theta \left(\nu_{t+1} \phi_{t+1}(j) + \eta_{t+1} \right), \quad \phi_t(j) = \frac{\eta_t}{\lambda - \nu_t} \tag{19}$$

[5] See Gertler and Karadi (2011) for an extensive explanation of this condition.

If constraint (15) does not bind, then our solution is a corner with:

$$\nu_t = 0, \quad \eta_t = 1, \quad \Gamma_t = 1, \quad \phi_t(j) \text{ is undetermined} \tag{20}$$

As long as $0 < \nu_t < \lambda$, the incentive constraint holds and the banker will increase its assets. In contrast, when $\nu_t > \lambda$, the incentive constraint is not binding and the expected discounted value of the banker always exceeds gains from diverting funds.

Aggregating the wealth of all existing bankers, we have[6]:

$$\Omega_{t+1} = \left(\left(R_{k,t+1} - R_{t+1}\right)\phi_t + R_{t+1}\right)\Omega_t \tag{21}$$

Recall that, at each date t, not all bankers remain bankers to the next time period, and a portion of households become new bankers. We assume that bankers who exit give their earnings to their own household and the household gives the new banker startup funds, equal to a fraction $\frac{\epsilon}{1-\theta}$ of the value of assets that existing bankers had earned in their last operating period.

Accordingly, the total net worth of all bankers is the sum of the existing bankers and new bankers such that:

$$\Omega_t = \Omega_{e,t} + \Omega_{n,t} \tag{22}$$

Given that the probability of a banker at time t remaining a banker at time $t+1$ is equal to θ, then we may re-write (22) as:

$$\Omega_t = \theta\left(\left(R_{k,t} - R_t\right)\phi_{t-1} + R_t\right)\Omega_{t-1} + \epsilon Q_t Z_{t-1} \tag{23}$$

[6]Since all bankers are created equal and they choose the same quantity of claims, then their choice of $Z_t(j)$ will not depend upon j, neither deposits $B_t(j)$. Then ϕ_t is independent of j.

3.4 Central Bank and Public Loans

Until now, we have assumed that only private banks receive deposits from households (B_t) and lend funds to intermediate producers (Z_t). Here, we relax this assumption to consider a Central Bank which conducts unconventional monetary policy, managing the epidemic by issuing of bonds and lending money to non-financial firms.

As explained in Gertler and Kiyotaki (2010), there are many ways in which the Central Bank may behave. Since our objective is to study how the public authority may fight an epidemic crisis using public loans, we assume that the Central Bank issues government bonds $B_{g,t}$ to consumers at gross interest rate R_t and - using that income with respect to its budget constraint - issues financial claims $Z_{g,t}$ to intermediate non-financial producers at price Q_t, for which the government earns a stochastic rate of return $R_{k,t+1}$.

Let $Q_tZ_{p,t}$ be the value of assets coming from private banks, $Q_tZ_{g,t}$ the value of assets coming from the Central Bank, and Q_tZ_t the total value of intermediate assets (i.e. the sum of assets from private and Central banks). Note that in the eyes of borrowers and lenders in our model, private deposits/claims and government bonds/claims are equivalent in the sense that they have the same price and interest rates.

The Central Bank has both an advantage and a disadvantage with respect to private lenders. We assume that government assets come with an efficiency cost of τ per claim[7], but that, assuming the government can always honor its debts, there are no limitations in the number of bonds it can supply[8]. Therefore, it is not subject to an incentive constraint. As a consequence the Central Bank may also

[7] As explained in Gertler and Karadi (2011) and Gertler and Kiyotaki (2010), the government faces additional costs of evaluating and monitoring borrowers that privates banks do not have. This is because private banks possess specific knowledge of the market not readily available to the Central Bank.

[8] By abstracting from solvency problems, we are assuming that the government can always print money to pay its debts. In reality, solvency problems can emerge and be aggravated by sovereign debt and credit-rating agencies. We leave this for future work.

issue government debt to financial intermediates without constraint. Private banks fund government bonds by issuing households deposits at the same rate as they lend them from the Central Bank. Thus, only private assets financed with private banks face the incentive constraint.

Suppose that in each period the Central Bank lends a fraction ψ_t of total credit. Then, using equation (16), we write the total value of intermediate assets as:

$$Q_t Z_t = \phi_t \Omega_t + \psi_t Q_t Z_t = \Phi_t \Omega_t \tag{24}$$

where $\Phi_t := \frac{\phi_t}{1-\psi_t}$ is the leverage ratio for total intermediate funds (public and private). The choice of ψ_t will be explained in Section 3.8.

3.5 Intermediate Non-Financial Firms

Let there exist a continuum of perfectly competitive, homogenous intermediate goods producers that produce a differentiated non-financial good that is sold at real price $P_{m,t}$[9]. Each of them uses two inputs: labor L and capital K.

Following Gertler and Karadi (2011) we assume that at the end of period t, each intermediate producer acquires a quantity K_{t+1} of capital from the capital producers to be used in production in time $t+1$. After production in period $t+1$, the firm may sell capital back to the capital producer and/or refurbish depreciated capital. We assume that the cost of replacement is unity and that there are no adjustment costs. Thus, intermediate goods firms face a static problem, solving their profit maximization problem one period at a time rather than maximizing expected profit over the lifetime of the firm.

Goods producers finance physical capital by borrowing from financial intermediates[10]. Note that borrowers are not constrained by the quantity of claims Z_t they

[9]Following Gertler and Karadi (2011) we do not introduce price stickiness through intermediate goods producers, but rather do so by assuming that retailers are monopolistic.

[10]Private and public financial intermediaries are perfect substitutes in the eyes of the borrower.

want to purchase. However, as intermediate private banks are constrained by the quantity of funds they may obtain from households, there is an indirect effect of the interest rate $R_{k,t}$ on goods producer dynamics.

Each goods producer then purchases a quantity Z_t of capital claims, in which each claim equals one unit of capital $Z_t = K_{t+1}$ and that the price per unit capital is Q_t. It follows that $Q_t K_{t+1} = Q_t Z_t$.

Recall that goods producers are homogeneous and all behave in the same fashion. Then we can write the quantity of intermediate non-financial goods $Y_{m,t}$ produced by the representative physical goods producer at time t as a Cobb-Douglas production function involving capital and labor such that[11]:

$$Y_{m,t} := K_t^{\alpha} L_t^{1-\alpha} \tag{25}$$

where the subscript m differentiates intermediate goods ($Y_{m,t}$) from final goods (Y_t), and α is the elasticity of production with respect to capital. As we assume no stochastic shocks, we abstract here from quality capital shocks as in Merton (1973) and a total factor productivity shock as in classic DSGE models (Smets and Wouters (2007)).

Each goods producer chooses quantities of labor and capital in order to maximize its profit. The solution to this problem yields the following first order conditions:

$$W_t = (1-\alpha) P_{m,t} \frac{Y_{m,t}}{L_t} \tag{26}$$

$$R_{k,t} = \frac{\alpha P_{m,t} \frac{Y_{m,t}}{K_t} + (1-\delta) Q_t}{Q_{t-1}} \tag{27}$$

where δ is the capital depreciation rate. As we are in a perfect competitive frame-

[11] Since we assume that retailers are monopolistic, one unit of intermediate good $Y_{m,t}$ does not necessary equal one unit of final good Y_t. As shown in the Appendix, these quantities are related by the equation $Y_{m,t} = v_{p,t} Y_t$ at equilibrium, where $v_{p,t}$ is the price dispersion of the aggregated final good.

work, equations (26) and (27) establish that intermediate good producers choose the quantity of labor to equate real wages and the marginal product of labor, and quanty of capital such that the real price of capital equals the net return after depreciation.

3.6 Capital Producers

There exists a continuum of perfectly competitive, homogeneous capital production firms. At the end of each period t, capital producers may produce new capital by buying final goods from retailers $I_{n,t}$ (i.e. investing), purchase non-depreciated capital from intermediate good producers at price Q_t, repair depreciated capital at cost unity, and/or sell capital to intermediate goods producers at price Q_t. In doing so, total aggregate capital accumulates in the following fashion:

$$K_{t+1} := (1-\delta)\,K_t + I_{n,t} \tag{28}$$

where δ is the capital depreciation rate and $I_{n,t}$ is net/new capital investment.

Furthermore, we assume that there is no adjustment or investment cost associated with repairing capital. However, producing new capital does face an adjustment cost associated with changing the level of investment. Thus, capital producer profit can be written as[12]:

$$D_{k,t} = \left((Q_t - 1)I_{n,t} - f\left(\frac{I_{n,t}}{I_{n,t-1}}\right)I_{n,t}\right) \tag{29}$$

A representative capital producer chooses the quantity of net capital investment $I_{n,t}$ to maximize its discounted profits:

$$\mathbb{E}_t \sum_{i=0}^{\infty} \Lambda_{t,t+i}\left((Q_{t+i} - 1)I_{n,t+i} - f\left(\frac{I_{n,t+i}}{I_{n,t-1+i}}\right)I_{n,t+i}\right) \tag{30}$$

[12] See the Appendix for a detailed derivation.

where the adjustment cost function ($f(\cdot)$) depends on net capital investment at times t and $t-1$. Specifically, it is defined as:

$$f\left(\frac{I_{n,t}}{I_{n,t-1}}\right) = \frac{\kappa}{2}\left(\frac{I_{n,t}}{I_{n,t-1}} - 1\right)^2, \kappa > 0 \tag{31}$$

Remark that the adjustment cost is zero at the steady state, and that this cost is increasing with temporal changes in investment.

The first order condition for profit maximization yields:

$$Q_t = 1 + f\left(\frac{I_{n,t}}{I_{n,t-1}}\right) + f'\left(\frac{I_{n,t}}{I_{n,t-1}}\right)\frac{I_{n,t}}{I_{n,t-1}} - \mathbb{E}_t\Lambda_{t,t+1}f'\left(\frac{I_{n,t+1}}{I_{n,t}}\right)\left(\frac{I_{n,t+1}}{I_{n,t}}\right)^2 \tag{32}$$

This equation is the marginal Tobin's "Q" which, given asset prices, defines the optimal investment demand function. Remark that with no adjustment costs, $Q_t = 1$.

3.7 Retailers

Let there be a continuum of monopolistic *normal retailers* indexed by $h \in [0, 1]$, and a continuum of perfectly competitive *super retailers* that purchase and assemble final goods produced by *normal retailers* in order to produce an aggregate final good that will be sold at price P_t. We assume that *super retailers* are homogeneous and all behave in the same fashion (*normal retailers* are not treated as homogeneous).

The *super retailer* is characterized by the following CES production function:

$$Y_t := \left(\int_0^1 Y_t(h)^{\frac{\epsilon_p - 1}{\epsilon_p}} dh\right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \tag{33}$$

where $Y_t(h)$ is final good produced by *normal retailer* h, and ϵ_p is the elasticity of substitution of choosing between *normal retailer* goods.

Given the prices of normal retailer goods $P_t(h)_{h\in[0,1]}$ and the final aggregated good price P_t, the *super retailer* chooses the quantities of *normal retailers* goods

$(Y_t(h))_{h\in[0,1]}$ in order to maximize its profit. The solution yields the following demand function for good h:

$$Y_t(h) = \left(\frac{P_t(h)}{P_t}\right)^{-\epsilon_p} Y_t \quad \forall h \tag{34}$$

Notice that the production function of the *super retailer* includes constant returns to scale and that firms are perfectly competitive, meaning that firms experience zero profits at equilibrium. We therefore obtain the following equation for the price of the final aggregate good:

$$P_t = \left(\int_0^1 P_t(h)^{1-\epsilon_p} dh\right)^{\frac{1}{1-\epsilon_p}}. \tag{35}$$

Each *normal retailer* h uses intermediate goods, produced by the intermediate goods firms, to "pack" the intermediate goods and sell them to the *super retailers* at price $P_t(h)$. We assume that it takes one unit of intermediate good to produce one unit of normal final output. Thus, the marginal cost for each *normal retailer* is the intermediate price $P_{m,t}$, which is the same for all *normal retailers*.

We introduce nominal price rigidity as in Calvo (1983). In each period t, a fraction $(1-\theta_p)$ of *normal retailers* can re-optimize their nominal price $(P_t(h) = P_t^*(h))$, while the remaining fraction can only partially adjust their prices according to past inflation. If firm h cannot change its price for i periods, then its normalized price after i periods is:

$$\prod_{s=1}^{i} \Pi_{t+s-1}^{\chi} \frac{P_t(h)}{P_{t+i}} \tag{36}$$

where $\chi \in (0,1)$ reflects the price response to inflation and $\Pi_t := \frac{P_t}{P_{t-1}}$ represents the level of inflation from period $t-1$ to t.

Profits for *normal retailer* h at date t is then given by:

$$\left(\prod_{s=1}^{i} \Pi_{t+s-1}^{\chi} \frac{P_t(h)}{P_t} - P_{m,t}\right) Y_t(h) \tag{37}$$

Given the option, each *normal retailer* firm will choose to readjust its price. The choice of $P_t^*(h)$ does not depend on the specific household h because all firms that are able to choose their prices will do so in the same fashion. Furthermore, firms only consider future states in which re-optimization is not possible thus each firm h chooses $P_t(h)$ to maximize expected discounted profits:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta_p^i \Lambda_{t,t+i} \left(\prod_{s=1}^{i} \Pi_{t+s-1}^{\chi} \frac{P_t(h)}{P_{t+i}} - P_{m,t+i}\right) Y_{t+i}(h) \tag{38}$$

subject to equation (34).

The first order condition of this problem yields:

$$\mathbb{E}_t \sum_{i=0}^{+\infty} \theta_p^i \Lambda_{t,t+i} Y_{t+i}(h) \left(\frac{P_t^*}{P_{t+1}} \prod_{s=1}^{i} \Pi_{t+s-1}^{\chi} - \mathcal{M} P_{m,t+i}\right) = 0 \tag{39}$$

where $\mathcal{M} = \frac{\epsilon_p}{\epsilon_p - 1}$ is the desired price markup, absent from inflation. This equation gives the optimal price setting condition.

Finally, using the fact that a fraction $(1-\theta_p)$ of *normal retailers* can optimize prices while the rest index prices to past inflation, equation (35) can be written as:

$$P_t^{1-\epsilon} = \theta_p \left(\Pi_{t-1}^{\chi} P_{t-1}\right)^{1-\epsilon} + (1-\theta_p)\left(P_t^*\right)^{1-\epsilon} \tag{40}$$

3.8 Government, Monetary Policy and the Market Clearing Condition

The government distributes unemployment benefits b_t, issues public debt $B_{g,t}$ to households for which it pays a gross interest rate R_t, sells claims $Z_{g,t}$ to non-financial firms at price Q_t and gross interest rate of return of $R_{k,t}$, recovers/pays lump-sum taxes, and spends its own expenditures G_t.

As discussed previously, there is a portion of the population that is infected and is not part of the labor force. We assume that they receive at least partial unemployment benefits from the government. We define those benefits b_t as:

$$b_t = \zeta W_t, \quad \zeta \in [0,1) \tag{41}$$

where ζ is the rate of unemployment compensation and W_t real wages. Thus, unemployment benefits are proportional to wages earned from working.

As explained in Subsection 3.4, in each period, the government via the Central Bank, lends a fraction ψ_t of total credit to financial intermediates. However, government assets come with an inefficiency cost of $\tau \in [0,1]$ per claim. (Recall that private banks are more efficient in that they have better access to market information.) Then government expenditure on financial intermediates is given by $\tau\psi_t Q_t K_{t+1}$.

We assume as well that government consumption of final goods is always constant, $G_t := \omega_g Y_t$, where ω_g is the steady state share of GDP that the government uses for its own expenditures. Assuming that transfers automatically adjust at each date, the government faces the following budget constraint:

$$G_t + \tau\psi_t Q_t K_{t+1} + b_t(1 - L_t) + \psi_t Q_t Z_t = T_t + (R_{k,t} - R_t)\, B_{g,t} + B_{g,t+1} \tag{42}$$

Equation (42) equates all expenditures (final good consumption, expenditures to non-financial intermediaries, and unemployment benefits) to revenue (lump sum taxes, interest from debt).

Unconventional monetary policy ψ_t is set in the following manner:

$$\psi_t = \bar{\psi}_t + \omega \mathbb{E}_t \left[(logR_{k,t+1} - logR_{t+1}) - (logR_k - logR) \right] \tag{43}$$

where $\bar{\psi}_t$ is defined as our "*epi loans*", $\omega > 0$ is the Central Bank credit feedback parameter, and $logR_k - logR$ is the steady state risk-premium. The feedback parameter governs the intensity of the reaction of the Central Bank to changes in the spread relative to the steady state risk premium. When the risk-premium is larger than its steady state, the Central Bank expands its credit with the larger the ω, the greater the credit expansion. In our baseline simulations, we treat $\bar{\psi}_t$ as a constant equal to zero. We then relax this assumption, taking $\bar{\psi}_t$ as a deterministic, exogenous shock, to study the ability of our "*epi loans*" to alleviate the negative effects of the epidemic.

Suppose that the Central Bank also conducts conventional monetary policy by setting nominal interest rates, i_t, following a Taylor rule of the form:

$$1 + i_t = (1 + i_{t-1})^{\phi_i} \left(\frac{1}{\beta} \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_{ss}} \right)^{\phi_y} \right)^{1-\phi_i}, \tag{44}$$

where Π_t is the steady state of inflation and Y_{ss} is the steady state GDP in a scenario without disease. In this formulation the parameter ϕ_y measures the response of the Central Bank to the output gap, which contrary to other DSGE models, we define as the deviation of current GDP with respect to the steady state GDP without an epidemic[13].

Finally, we have the following Fisher relation that links nominal interest rates fixed by the Central Bank to the gross real interest rate fixed by the market:

$$1 + i_t = R_{t+1} \mathbb{E}_t \Pi_{t+1} \tag{45}$$

Market clearing conditions established that production is divided between consumption, net investment, government expenditures in goods, and government finan-

[13]Generally, in classic DSGE models, the output gap is defined as the deviation of current GDP with respect to its steady state. In our model, depending on the type of disease, it is possible to have different steady states values for Y. We believe that the real output gap should be measured as the deviation with respect to a fixed value of Y.

cial intervention.

$$Y_t = C_t + I_{n,t} + f\left(\frac{I_{n,t}}{I_{n,t-1}}\right) I_{n,t} + G + \tau \psi_t Q_t K_{t+1} \tag{46}$$

Equation (46) closes the model.

4 Parameter Calibration and Simulation Analysis

Details on model aggregation and calculation of the the steady state values are given in the Appendix. Each time period corresponds to a quarter. Baseline parameter values are summarized on Table 3. Calibration of our baseline parameters follows Smets and Wouters (2007) and Gertler and Karadi (2011) for the U.S. economy. Specifically, the discount factor β is set to ensure a 4% annual interest rate, with the elasticity of substitution among final goods taken to yield a steady-state price markup of 31%. The output of elasticity of capital α is calibrated assuming a "labor share" of approximately 2/3 and the bankers' survival rate is fixed at 0.975, which assumes that bankers remain bankers on average for 10 years. We fix the share of unemployment compensation ζ to 0.5. As in Gertler and Karadi (2011), the private banks' parameters λ and ϵ are fixed to meet the following targets: a risk-premium steady state of 100 basis points and a steady state leverage ratio of 4. Initial conditions and baseline epidemiological parameters were chosen to illustrate a full epidemic cycle, and are *not* meant to represent a specific disease.

Simulation of the model proceeds in two steps. First, we calculate the trajectories of the number of susceptible, infected, and recovered individuals given initial conditions and epidemic parameters. The dynamics of the epidemic were solved using a first-order Euler approximation for a time horizon of 150 periods, corresponding to the time scale of the economic model. We then used the trajectory of infected individuals as a deterministic, permanent shock to the real economy. In this way,

Table 1: State and control variables

Variable	Symbol	Type
Epidemic block		
Susceptible	S	State
Infected	$\tilde{I}$	State
Recovered	$\tilde{R}$	State
Households		
Labor	L	Control/State
Consumption	C	Control
Deposit = Government bonds	B	Control
Financial Intermediates		
Quantity of financial claims issued by private banks	Z_p	Control
Non-financial intermediates and capital producers		
Intermediate non-financial goods	Y_m	Control
Capital	K	Control/State
Labor	L	Control/State
Net capital investment	$I_{n,t}$	Control
Retailers and Capital Producers		
Normal retailed good price	$P(h)$	Control

Table 2: Model definitions and outcomes

Variable	Symbol
Households	
Total population	N
Real discount factor from date t to $t+1$	$\Lambda_{t,t+1}$
Good price = Aggregate retailer's price	P
Total real profits	D
Lump-sum taxes	T
Marginal lifetime discounted utility function	λ_c
Real wage	W
Financial Intermediates	
Total quantity of financial claims	Z
Bankers' net worth	Ω
Expected discounted terminal wealth	V
Leverage ratio of private banks	ϕ
Auxiliary variable	Γ
Risk-less gross real rate of return	R
Claims gross real rate of return = Capital rate of return	R_k
Financial claims price	Q
Total leverage ratio (public and private)	Φ
Marginal value of banker's gain w.r.t claim income	ν
Marginal value of banker's gain w.r.t wealth	η
Existing banker's net worth	Ω_e
New banker's net worth	Ω_n
Private deposits	B_p
Private bank profit	$D_{b,t}$
Non-financial intermediates and capital producers	
Intermediate non-financial good price	P_m
Intermediate non-financial profit	$D_{m,t}$
Capital producer profit	$D_{k,t}$
Adjustment cost function of investment	$f(\cdot)$
Retailers and Capital Producers	
Aggregate super retailed good	Y
Normal retailed good	$Y(h)$
Normal retailed good price	$P(h)$
Optimal normal retailed good price	P^*
Normal retailer profit	$D_{r,t}$
Price dispersion	$v_{p,t}$
Central Bank and Government	
Level of goods price inflation	Π
Fraction of total credits financed by the Central Bank	ψ
Quantity of financial claims issued by the Government	Z_g
Unemployment compensation	b
Government consumption	G
Nominal interest rate	i
GDP without disease	$\bar{Y}$
Inflation without disease	$\bar{\Pi}$
Government bonds	B_g
Exogenous fraction of publicly intermediate assets	$\bar{\psi}$

agents possess perfect foresight regarding the future states of the epidemic when computing their optimal solutions. We solve the economic block from a set of initial conditions to the steady-state of both economic and epidemic blocks[14].

In order to test the effectiveness of unconventional monetary policy to mitigate the epidemic crisis, we first establish a baseline model scenario with an epidemic and study the economic consequences of changes in the epidemic structure. We then implement unconventional monetary policy by testing the sensitivity of the model to the steady state leverage ratio for private banks, the intensity of the reaction of the Central Bank to changes in the spread, and our "*epi loans*" policy. All model simulations were conducted in Dynare 4.6.1. All source code and simulation data can be found on the Open Science Framework (osf.io/j7m65).

5 Results and Discussion

This section is divided in four parts. First, we present our baseline results of the model and the different pathways by which the epidemic affects the economy. Second, we describe the economic response to changes in epidemiological parameters (transmission and recovery rates). Third, we discuss the effects of unemployment compensation on the economy. Finally, we evaluate the potential of monetary policies to remedy the economic burden of the epidemic. For each of our results, we compare the trajectories of our economic variables to those in the absence of disease (or the "no-disease" case). When changing model parameters, we re-calculate the trajectories of the no-disease case to correspond to the new set of parameters.

[14]We solve the linearized version of the perfect foresight model with the Newton method, which uses sparse matrices to simultaneously solve all equations in every period.

Table 3: Parameter Calibration

Parameter	Symbol	Calibrated Value/Baseline
Epidemic block		
Initial condition of susceptible	S_0	0.9
Initial condition of infected	$\widetilde{I}_0$	0.1
Initial condition of recovered	$\widetilde{R}_0$	0
Transmission rate	α_v	0.4
Recovery rate	γ_v	0.1
Households		
Discount factor	β	0.99
Internal habit formation	h	0.71
Financial Intermediates		
Bankers' survival rate	θ	0.972
Fraction of claims income that can be diverted	λ	Function of risk premium at steady state, leverage ratio at steady state and θ
Proportional transfer to the new bankers	ϵ	Function of risk premium at steady state, leverage ratio at steady state, θ and $\bar{\psi}$
Risk premium at steady state	$R_k - R$	0.01/4
Leverage ratio at steady state	ϕ	4
Non-financial intermediates and capital producers		
Capital depreciation	δ	0.025
Price indexation to inflation	χ	0.24
Calvo price parameter	θ_p	0.66
Capital share	α	0.33
Retailers and Capital Producers		
Adjustment cost constant	κ	5.74
Elasticity of substitution between normal retailers	ϵ_p	4.167
Price markup	$\mathcal{M}$	Function of θ_p
Central Bank and Government		
Efficiency cost	τ	0.001
Unemployment rate compensation	ζ	0.5
Feedback parameter	ω	10
Taylor rule response to inflation	ϕ_π	2.04
Taylor rule response to output gap	ϕ_y	0.08
Taylor rule inertia	ϕ_i	0.81
Steady state share of GDP that Government expends	ω_g	0.18

5.1 Baseline Results

Our baseline results are summarized in Figures 3 and 4 . For brevity, we focus on a set of core variables of the model.

By assumption, the epidemic decreases the quantity of available labor (only healthy individuals are allowed to work), which at its maximum severity decreases the workforce by 45%. This effect on the labor market echoes throughout the economy, with declines in household consumption, non-financial intermediary capital, and capital producer investment following the trajectory of labor. The first is a consequence of lost wages and equality in the market clearing condition. The latter two follow declines in production due to a lower workforce.

Regarding financial intermediaries, the epidemic primarily affects their expected discounted terminal wealth (V). Both components of wealth - net worth (Ω) and claim selling (QZ) - are affected. This is because a decrease in capital translates to a decrease in claims demand ($K_{t+1} = Z_t$), which has a negative impact on claim prices (Q) compared to the no-disease case. We observe significant declines in GDP, reaching a maximum loss of 20% compared to the no-disease case.

What is particularly interesting is that as the crisis starts, the Central Bank increases its share of total credits that it finances (ψ) to compensate for losses in investment and production that follow declines in labor. This is because, while decreases in investment in capital and production of goods provoke decreases in interest rates (risk-less and capital rate of return), the observed spread in the interest rates is still higher than the steady-state.

Similarly, we observe an increase in inflation during the epidemic. In this model, the standard relationships between supply and demand and prices holds. If price increases (decreases), then the supply (demand) side dominates as the DSGE framework shifts back to equilibrium. In a perfectly competitive market, as overall production decreases with the epidemic, we would expect to see a larger than observed increase

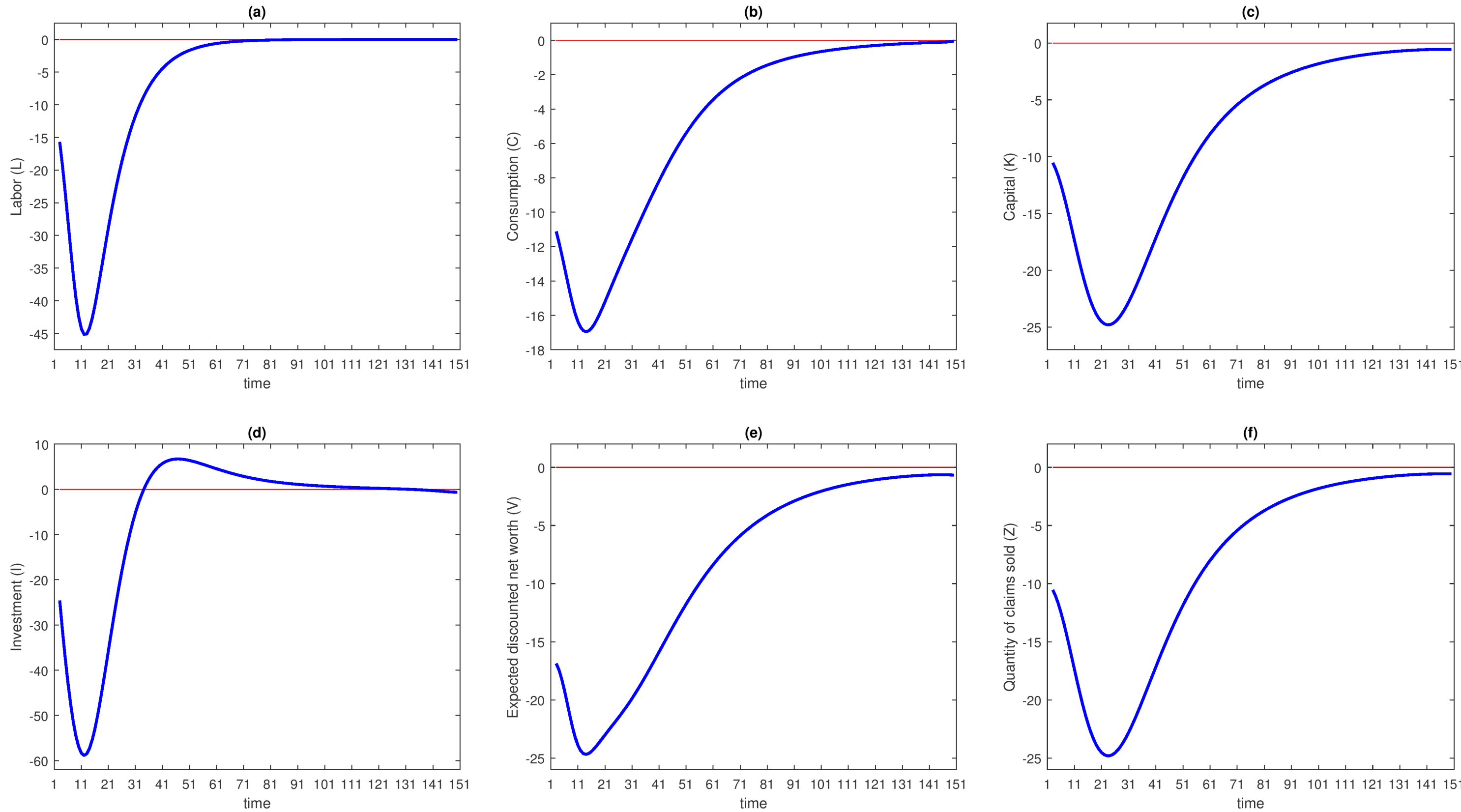

Figure 3: Baseline results for labor (a), consumption (b), capital (c), investment (d), expected discounted net worth (e), and the quantity of claims sold (f). Reported values are the percent deviation from the no-disease case. The red line corresponds to a zero percent change.

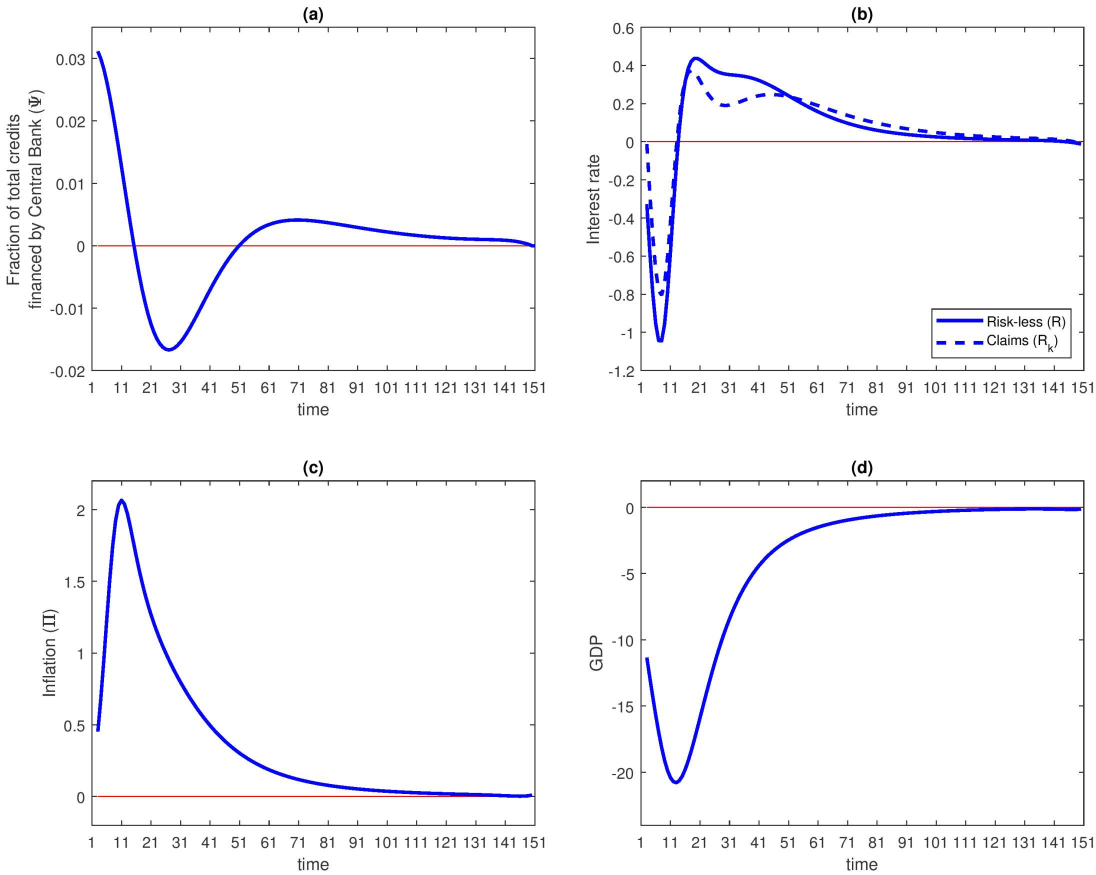

Figure 4: Baseline results for the fraction of total credits financed by the Central Bank (a), interest rates (b), inflation (c), and GDP (d). Reported values are the percent deviation from the no-disease case. For comparison, the red line corresponds to a zero percent change.

in prices (at least in the early stages of the epidemic). However, the increase in inflation is less than that of a perfectly competitive framework because of sticky prices.

5.2 Economic Response to Changes in Epidemic Structure

Holding all economic parameters constant, we vary the epidemiological parameters to understand how structural changes in the epidemic profile affect the economy. We find marked changes in cumulative GDP, with the recovery rate being the primary driver (Figure 5a). Indeed, at moderate to high recovery rates the model is relatively insensitive to the infection rate.

In our framework, the main burden of disease on the economy is in the labor supply: only healthy people are allowed to work. Therefore, an epidemic that persists for a long time in the population (low recovery rate) and, consequently, keeps people from working, will be the most costly. Even if we have a highly contagious epidemic (high infection rate), as long as it can pass through the population quickly (moderate or high recovery rate), then the overall burden in terms of GDP will be less.

This result has interesting implications for the relationship between disease's basic reproductive number (an epidemiological measure of the severity of a disease) and GDP (an economic measure of the well-being of an economy). The basic reproductive number (R_0) is defined as the average number of secondary infections that occur when a single individual is introduced into a population where everyone is susceptible (F.Brauer and Castillo-Chavez (2012), Hethcote (2000)). In general, if $R_0 > 1$ then the disease will spread through the population, and if $R_0 < 1$, then the disease will die out. The bigger the R_0, then the worse or more severe the disease. For a standard SIR model, it is defined as the ratio of the infection and recovery rates (α_v/γ_v) (Diekmann et al. (1990), Diekmann et al. (2010), Heffernan et al. (2005)).

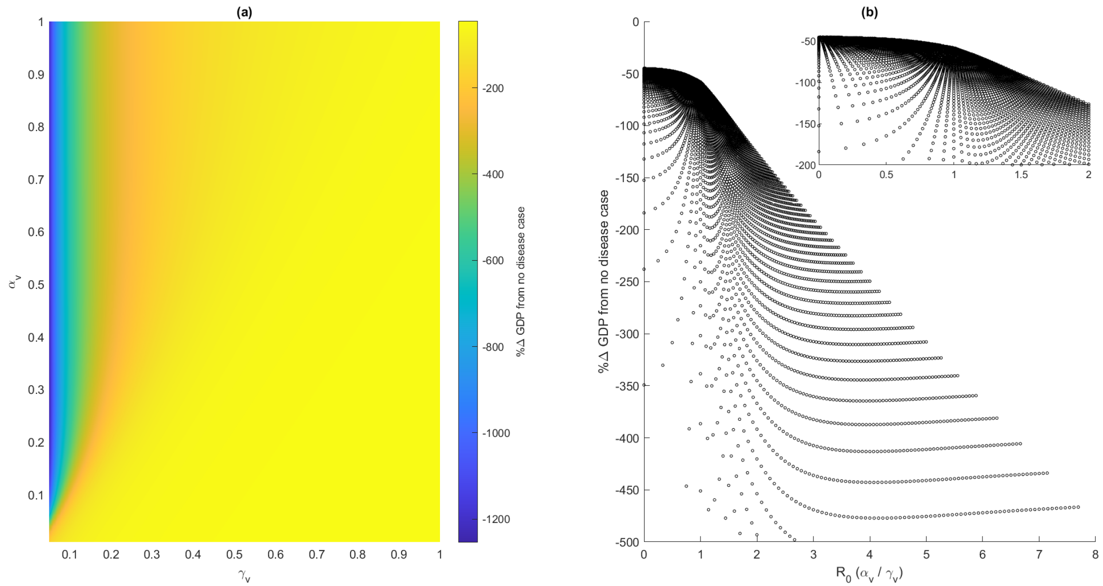

Figure 5: Sensitivity of GDP losses to epidemic parameters. Panel (a) presents the percent change in GDP for different combinations of disease transmission (α_v) and recovery (γ_v) rates. Color corresponds to the magnitude of GDP losses compared to the no-disease case. Dark blue (yellow) indicates greater (less) loss. Panel (b) relates the disease R_0 - generated from the epidemic parameters in panel (a) - to the percent change in GDP from the no-disease case. Though not tailored to a specific disease, for comparison the R_0 for COVID-19 is estimated to be between 1.4-6.5 (Cheng and Shan (2019)), 3.4 for H1N1 avian influenza (Chang et al. (2010)), 1.5-1.9 for Ebola (Khan et al. (2015)), and between 3.5-6 for smallpox (Hethcote (2000)).

Given the effects of the epidemiological parameters and GDP, a higher R_0 does not necessarily translate to greater GDP loss (Figure 5b). It is feasible to have a severe epidemic (in an epidemiological sense of the word) that does not result in a large economic loss, if the recovery rate is sufficiently high to allow workers to quickly return to the labor force. However, it is worth stressing that this result depends on a number of simplifying - albeit, we believe acceptable - assumptions. The model assumes a constant population size with homogeneous mixing, where the primary burden of disease is via the labor force. It does not account for deaths, vaccinations or treatments, nor quarantines or epidemic-related business closures. We leave further investigation to future work.

5.3 Unemployment Compensation

Next, we evaluate the quantity of unemployment benefits distributed to households who are unable to work due to infection. We find that, contrary to real-world expectations, distributing unemployment benefits generates no change in GDP compared to the baseline scenario. In a Keynesian framework, we would expect that compensating workers would help counterbalance the negative effects of the epidemic on GDP. The reason for this is that because households are Ricardian - a not unheard of phenomenon empirically (Evans and Hasan (1994)) - they are forward-looking and, in response to increases in government spending, choose to save today expecting to pay higher taxes later. This leads to no change in consumption. Ricardian consumer behavior is a common assumption in neoclassical models, which warrants future consideration when evaluating unemployment benefits as an economic policy.

5.4 Can monetary policy help fight the adverse effects of an epidemic?

In order to answer this research question, we individually vary a set of economic parameters, holding all the other parameters at their baseline values. We concentrate our analysis on financial parameters only, specifically focusing on three policy instruments. Remark that in this model, changing the economic parameters never provokes a change in labor. This is because we take labor as exogenously determined by the epidemic.

We start by first considering the steady-state leverage ratio for private banks (ϕ), defined as the total loans that a private bank can issue compared to its net worth (Figure 6). We find that the higher the leverage ratio, the higher the injection of funds from the Central Bank into the economy (ψ). This effect is observed because with a higher leverage ratio at the steady state, there is a greater probability of banks to sell claims. As this occurs, it causes the spread in the interest rates to increase, leading the Central Bank to further insert money into the economy. We also find a compositional shift in bankers' wealth, with income from selling claims (net worth) increasing (decreasing) with an increase in the steady-state leverage ratio. However, we do not observe a marked change in GDP compared to the baseline scenario.

Second, we test the sensitivity of Central Bank to a change in the spread via the feedback parameter ω (Figure 7). As the Central Bank responds more intensively to changes in the spread, it injects a higher quantity of funds into the economy during the beginning of the epidemic (when the difference in the spread is highest), and then drops off in the later stages. Volatility in the variation of the spread is greater with ω. This affects the quantity and composition of bankers' wealth, with higher wealth stemming from a smaller decrease in net worth. We find no effect on GDP losses. However, we observe that when the Central Bank reacts more intensively to changes in the spread, reductions in consumption are smaller than the baseline. This

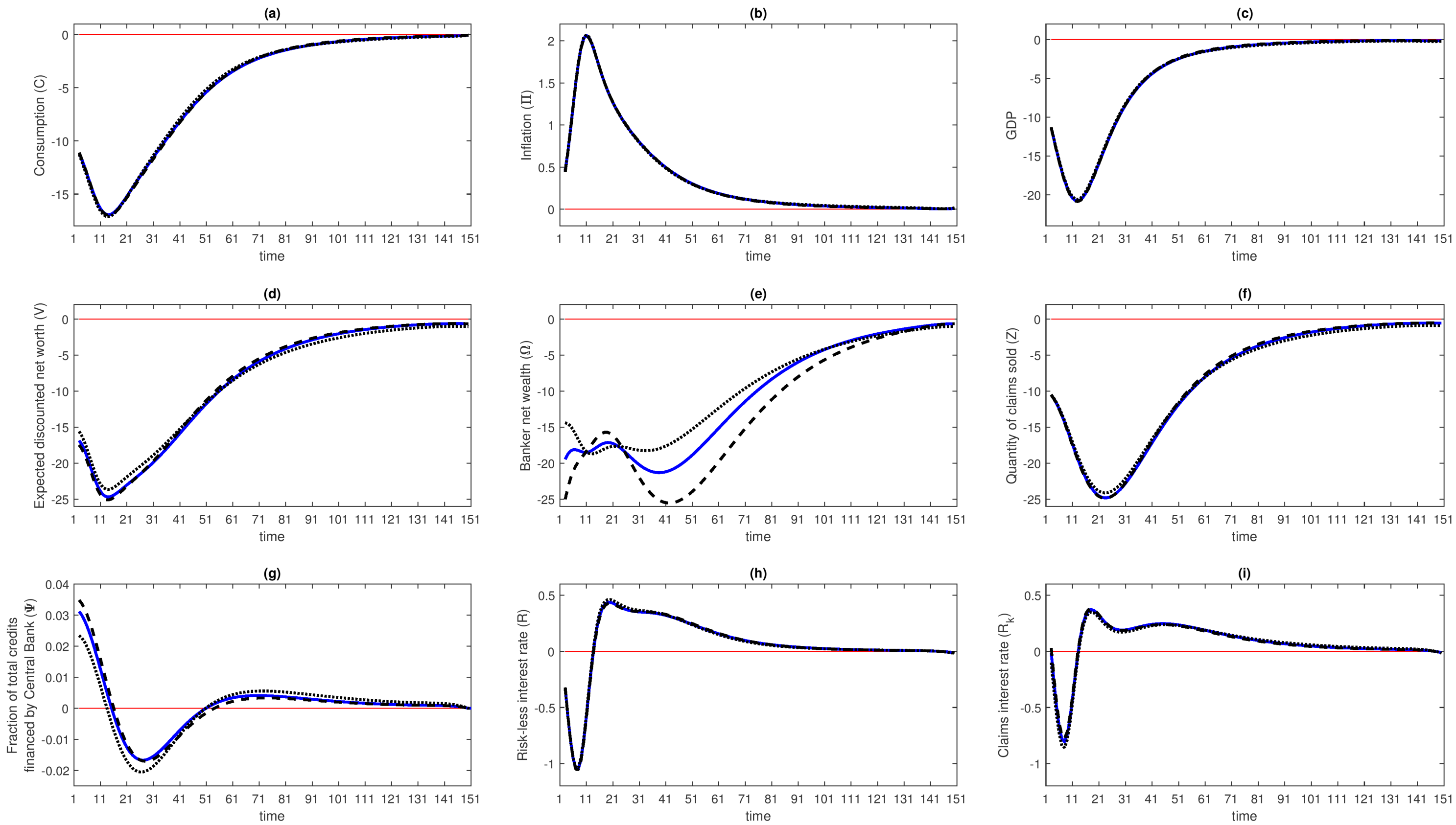

Figure 6: Model sensitivity to the steady state leverage ratio (ϕ). Recall that the results are reported as the percent change from the no-disease case. Line style and color indicates the value of the steady state leverage parameter: ϕ=2 (dotted, black), ϕ=4 (solid, blue; baseline), and ϕ=6 (dashed, black).

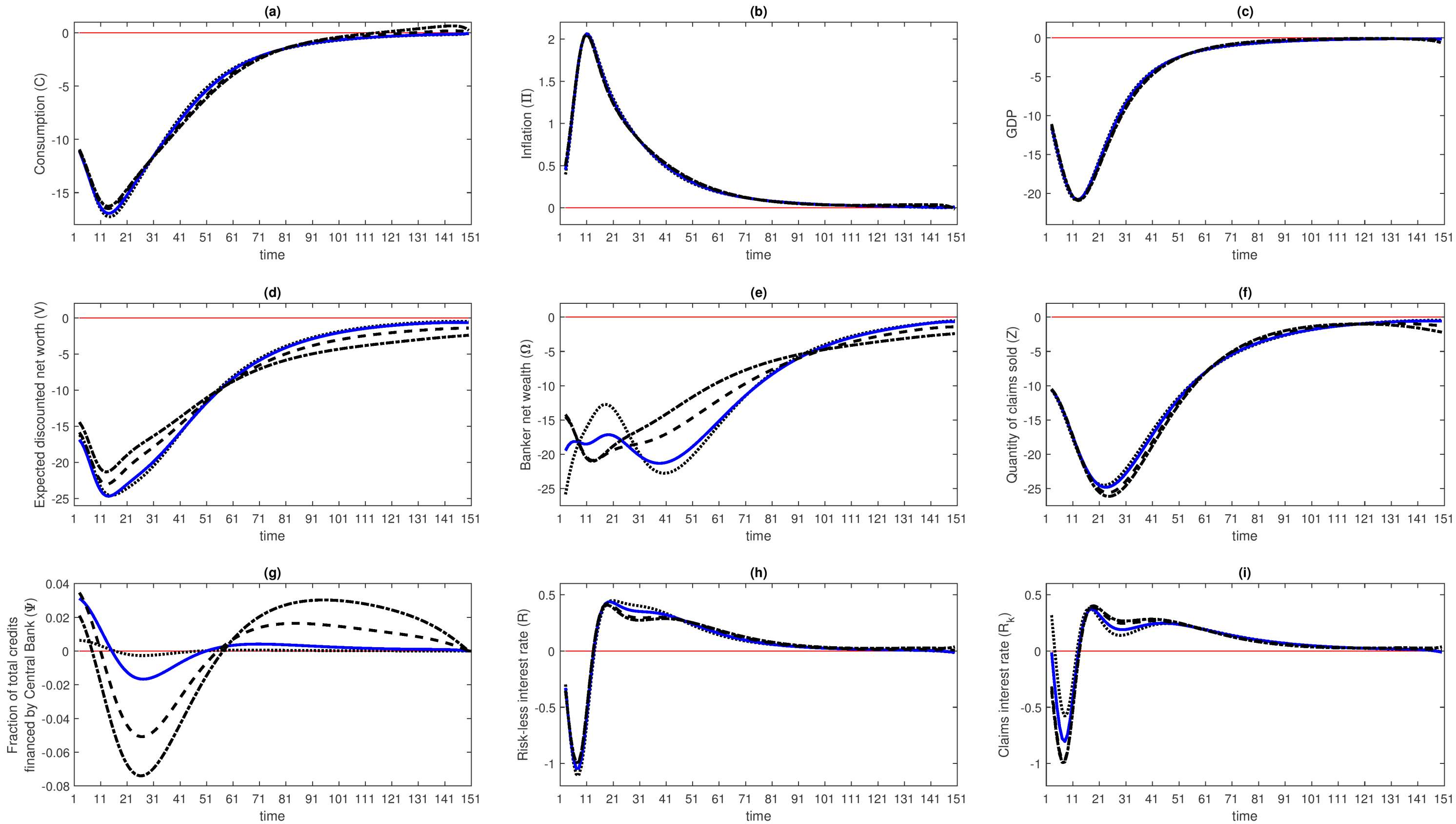

Figure 7: Model sensitivity to the feedback parameter (ω). Note that results are reported as the percent change from the no-disease case. Line style and color indicates the value of the feedback parameter: ω=1 (dotted, black), ω=10 (solid, blue; baseline), ω=100 (dashed, black), and ω=1000 (dot-dashed, black).

last result may suggest that, when talking about consumption, a stronger reaction to the spread is better for households.

Finally, we evaluate the use of "*epi loans*" to mitigate the effects of the epidemic (Figure 8). This takes the form of an exogenous shock on the steady state fraction of publicly intermediate assets $\bar{\psi}$, which affects the share of total claims the Central Bank finances (ψ). We assume that the Central Bank (with a cost) administers liquidity directly to the real economy in the form of claims that are transformed (one to one) into capital, and it does so from the beginning of the epidemic to its peak (in our case, this is about period 20).

Our definition of "*epi loans*" is an extreme form of a QE policy, but not exactly "helicopter money" as proposed by Friedman (1969). Instead of giving money directly to households with no expectation of being repaid, the Central Bank increases its share of total claims issued, and firms subsequently purchase capital without having to pass through private banks. Thus our "*epi loans*" directly affect demand by incentivizing investment, and should be thought of as expanding Central Bank intermediation rather than expanding the money supply.

With this policy we observe a smaller reduction in GDP compared to the baseline case. This should not come as a surprise given the fact that any increase in ψ will automatically increase GDP in the form of income obtained by the sale of claims. It is important to note, however, that although GDP loss is less than the baseline, the expected discounted terminal wealth of banks is reduced and the share of claims sold by private banks decreases. These are counterbalanced by an increase in the total quantity of claims sold such that the overall reduction of capital is smaller than the baseline. For households, this means that consumption is lower compared to the baseline case. An increase in claims reduces real rental interest rates and makes the acquisition of capital more attractive, incentivizing the investment in physical capital. As a side effect, we observe an expected increase in inflation. By reducing

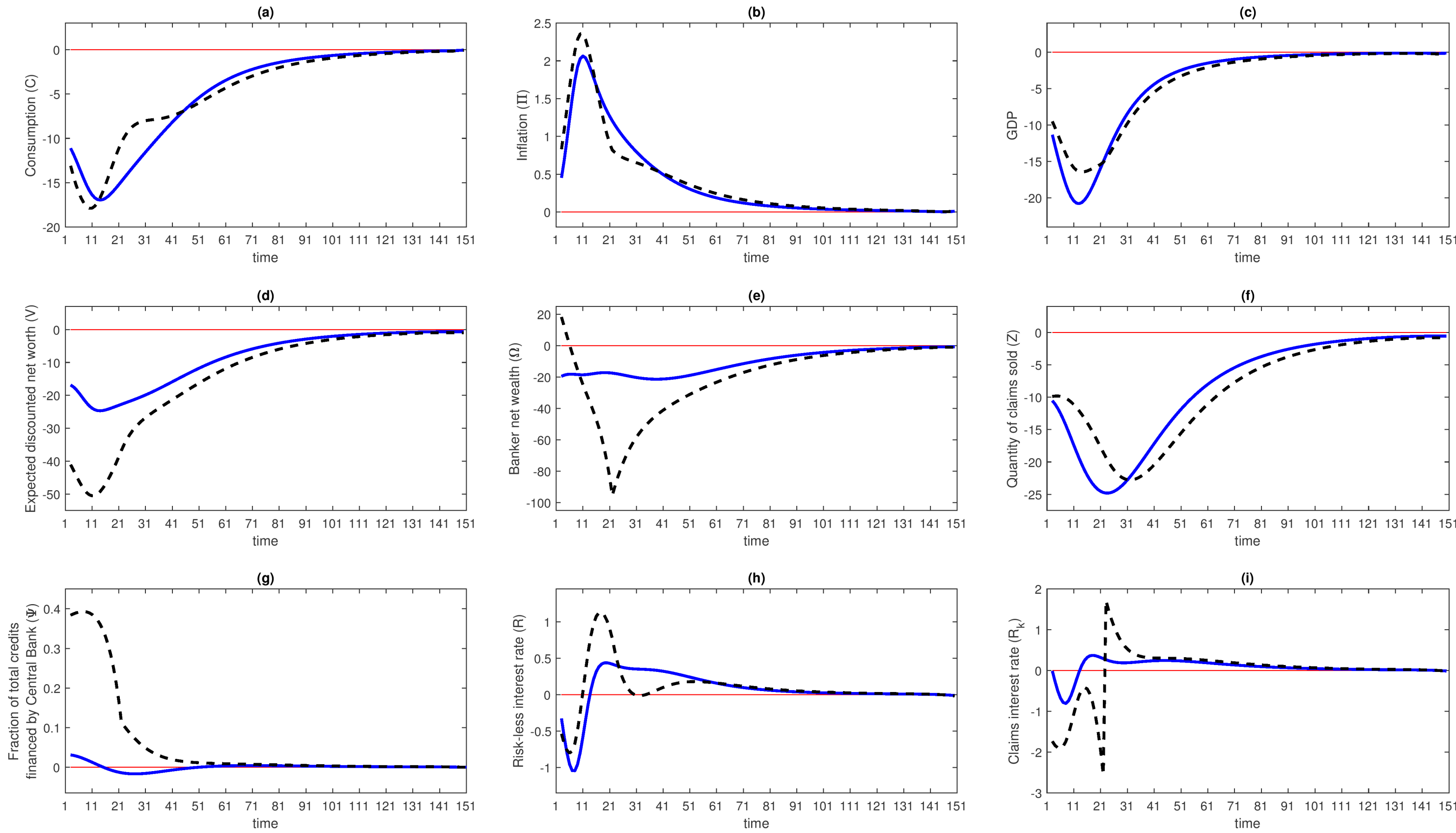

Figure 8: *Epi loans* ($\bar{\psi}$). Results are reported as the percent change from the no-disease case. The solid, blue line indicates the baseline model. The black, dotted line indicates a model implementing "*epi loans*" ($\bar{\psi}$=0.5). Note that the Central Bank administers "*epi loans*" from period 1 until peak of the epidemic (period 20).

demand, we drive up prices. However, it is important to remark that the increase in inflation, at its worst, is only 0.3% higher than that without an "*epi loans*" policy. Our results are in line with those proposed by Sharma et al. (2020), Céspedes et al. (2020), and Kiley (2020).

6 Conclusion

For the first time, we use a financial DSGE-SIR model to study the response of economy to an epidemic shock. We summarize our findings into three primary contributions.

First, due to the epidemic, the economy is likely to experience a deep recession. With our baseline calibration, we observe significant declines in GDP, reaching a maximum loss of 20% compared to the no-disease case. Although not directly comparable to other papers, for illustrative purposes Angelini et al. (2020), Chudik et al. (2020) and Bodenstein et al. (2020) found decreases in GDP post COVID-19 between 1.5% to 2.5%, 15%, and 20% to 30% respectively. However, our framework can be tailored to any combination of epidemiological models or economic parameters, making it possible to be calibrated to specific diseases and countries.[15]

Second, the profile of the epidemic has a significant effect on the shape of the recession. An epidemic that persists for a long time in the population (low recovery rate) and, consequently, keeps people from working, will be the most costly. Even if we have a highly contagious epidemic (high infection rate), as long as it can pass through the population quickly (moderate or high recovery rate), then the overall

[15] One could, for example, calibrate the epidemiological model to the COVID-19 epidemic. As COVID-19 is generally accepted to have an asymptomatic phase (Bi et al. (2020), He et al. (2020)), one would use a Susceptible-Asymptomatic-Infected-Recovered (SAIR) epidemiological model, which allows for asymptomatically-infectious individuals (F.Brauer and Castillo-Chavez (2012), Hethcote (2000)). Estimations of epidemiological model parameters have been conducted by Fanelli and Piazza (2020), Liangrong et al. (2020), Prem et al. (2020), and Yin et al. (2020), among others. However, it should be noted that there is uncertainty in estimations of these model parameters, as they will vary by country, the quality and timeframe of the data, the choice and timing of management strategies, accessibility to treatment and vaccines, as well as general assumptions inherent to disease models (such as homogeneous mixing or age structure).

recession will be less. This is because, in our model, as long as people are able to work, there should not be a reduction in production. We can infer that measures to decrease recovery time - such as treatments (which directly increases the recovery rate) and vaccination (which prevents individuals from getting sick) - could prove fruitful in minimizing economic losses of an epidemic. However, while straightforward to model in an epidemiological model (F.Brauer and Castillo-Chavez (2012), Hethcote (2000), Lenhart and Workman (2007)), these measures come with associated costs and the optimum usage is difficult to ascertain in a "macro-epidemic" framework (though see Lenhart and Workman (2007), Horan et al. (2010), and Toxvaerd and Rowthorn (2020) for examples in a microeconomics framework). We leave this for future work.

Finally, we found that, with the exception of increasing the share of claims from the Central Bank, our unconventional monetary policies cannot negate the negative economic effects of the crisis. However, as last resort lender, the Central Bank could use an unconventional monetary policy to exogenously increase its share of total claims issued ("*epi loans*"), which firms will then use to buy capital. This policy has the potential to lessen total losses in GDP, partially mitigating the economic recession, without being extremely inflationary, a side effect which has worried economists since the first use of unconventional monetary policies after the sub-prime crisis (e21 Staff (2010)). This is an encouraging thought as many industrialized countries have announced billions in stimulus to combat the COVID-19 crisis.

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On Monopolistic Competition and Involuntary Unemployment *

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Abstract

In a simple temporary general equilibrium model, it is shown that, if the number of firms is small, imperfect price competition in the markets for goods may be responsible for the existence of unemployment at any given positive wage. In our examples involving two firms facing their "true" demand curves, total monopolistic labor demand remains bounded as the wage rate goes to zero, and unemployment prevails for a sufficiently large inelastic labor supply. In the competitive case total labor demand would go to infinity and intersect labor supply at a positive wage.

1 Introduction

In a period and in a region where unemployment persists unwillingly at a very high rate, it might seem paradoxical that economists are still looking for an adequate definition of, and even for the theoretical possibility of, involuntary unemployment. Of course, such a possibility goes

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against the predictions of a perfect competition theory with complete markets in which agent is completely informed, takes as given all prices including wages, and knows that he will not be rationed. By contraposition, in an imperfect world with unemployment each of these conditions might be violated. No wonder that the theoretical explanations of involuntary unemployment are so many and incomplete, and that the resulting policy recommendations are so basically controversial.[1]

Classical explanations of unemployment are based upon various sources of downward wage rigidities – in particular, the market power of unions. In a well-known paper Hart [1982] argues again in favor of the idea that imperfect competition in the labor market is responsible for the existence of unemployment. The originality of his argument lies in his general equilibrium approach to imperfect competition in all markets. In his approach if the wage rate were to go to zero, the supply of goods would increase to infinity (as by assumption total revenue is always increasing in output), and so would the labor demand. Hence unemployment is due to the unions preventing the wage rate from falling.

More recent policy recommendations by Weitzman [1984, 1985] are based on a similar diagnosis. It is proposed to cure unemployment by adjusting the wage rate down to the positive level at which full employment is reached, and meanwhile by compensating the workers through some profit sharing. The approach is a general temporary equilibrium one, with monopolistic competition, using simple parameterized utility functions and a linear technology. The short-run equilibrium employment is shown to be a decreasing function of the wage rate, cutting the perfectly inelastic supply of labor at some positive wage.

Here we shall introduce a similar model, again taking prices as the strategic variables. The class of economies considered will appear to contain those analyzed by Weitzman [1985]. But, and this is our main point, it will also contain another set of economies for which Weitzman's policy recommendation does not fully work. No positive wage ensuring full employment at equilibrium will exist. In other words, only a zero wage could, possibly, clear the labor market. This

[1] For example, see the recent evaluation (with many references) by Lindbeck and Snower [1985].

we have called[2] a situation of "involuntary unemployment" in the spirit of Keynes, according to whom unemployment is involuntary when there is "*no* method available to labor as a whole" for attaining full employment "by making revised *money* bargains with the entrepreneurs" [Keynes, 1973, p. 13]. Moreover, such a situation is compatible with the existence of a Walrasian competitive equilibrium at which the labor market clears at a positive wage level and at a higher level of employment. As well commented on by Silvestre [1988, p. 1], involuntary unemployment in our sense corresponds to a severe exploitation of workers in the neoclassical sense[3] that the real wage is lower than the physical margin product of labor (at zero wage productive labor becomes a free good). The situation is well illustrated by Figure 1.

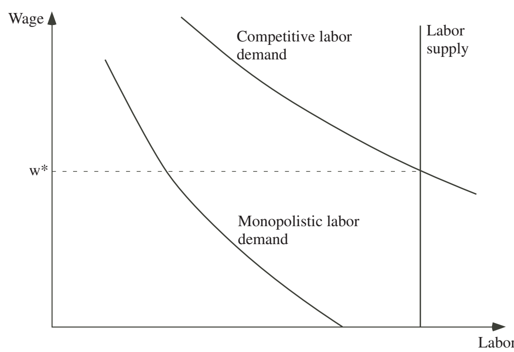

Figure 1:

We see that there are two "Labor Demand Curves" or, more precisely, two curves of equilibrium employment levels associated with all possible wages: the Competitive one, leading to a positive competitive equilibrium wage w^*; and the Monopolistic one, which does not intersect the Labor Supply at a positive wage. Our result will be to determine a set of economies in which such a figure is a true possibility. It will owe much to the fact that we abandon Hart's and Weitzman's assumption that the total revenue of each producer is an increasing function of output,

[2]See d'Aspremont, Dos Santos Ferreira, and Gérard-Varet [1984] and the subsequent development of a monopoly example by Dehez [1985]. Also see d'Aspremont, Dos Santos Ferreira, and Gérard-Varet [1989].

[3]See Pigou [1920], p. 51 or pp. 813–14. Robinson [1933] has well emphasized that such exploitation may occur, even with wage-taking behavior of the firm, because of monopolistic power in the output market [Ch. 25].

however large this output. Instead we shall assume, as in our previous work on a Cournot model (see also Dehez [1985] and Silvestre [1988]), that total revenue becomes decreasing in output as output becomes large. However, we shall keep the "objective" approach used by Hart and Weitzman to describe the demand for goods faced by the producers. As discussed in Nikaido's [1975] book and Hart's [1985] survey, this "objective" approach[4] (as opposed to the "subjective" approach of Negishi [1961]) supposes that the producers know the "true" demand curve they face. This implies that the indirect effects (or "feedback effects" in Hart's terminology) of the producers' decisions on their own demand, through aggregate wealth, be explicitly taken care of by some specific assumption. Several possibilities will emerge from our discussion.

The present paper is organized as follows. In Section 2 we describe the basic model and define equilibrium concepts. A class of examples is developed in Section 3. Then, in Section 4 the possibility of involuntary unemployment is demonstrated and contrasted with the conclusion of Weitzman [1985]. Different interpretations or extensions of the model are discussed in Section 5.

2 The model

We consider an economy with four goods: two produced goods, labor, and money[5] in a temporary equilibrium framework. There are two firms, each one specialized in producing, out of labor and with constant productivity, one of the two consumption goods. All prices are nonnegative monetary prices: we denote by w the wage rate, by p the price of one good (henceforth called the "Latin" good) and by π the price of the other (called the "Greek" good). Labor productivity in the Latin sector is denoted by $1/\ell$, $\ell > 0$, and, in the Greek sector, $1/\lambda$, $\lambda > 0$.

[4]Also see Gabszewicz and Vial [1972], Marschak and Selten [1972], Laffont and Laroque [1976], Roberts [1986, 1989], Jones and Manuelli [1987], and Benassy [1988].

[5]Although in the presentation of our results we use a temporary equilibrium framework (as described, for example, in Grandmont [1983]), we consider in Section 5 an alternative framework similar to the one in Hart [1982], where there is only one period (still divided into two stages) and where money is taken as a "nonproduced good".

There is a continuum of consumers with identical homothetic preferences. Labor has no disutility. Thus, every consumer θ in $[0,1]$ is described, for a given wealth $\omega_\theta(p,\pi,w) \geq 0$ varying with prices and the wage, by two multiplicatively separable continuous demand functions, respectively, denoted $h(p,\pi)\omega_\theta(p,\pi,w)$ for the Latin good and $\chi(p,\pi)\omega_\theta(p,\pi,w)$ for the Greek good. For a given distribution of consumers ν, aggregate wealth is given by the function $\Omega(p,\pi,w) = \int_0^1 \omega_\theta(p,\pi,w)\nu(d\theta)$, and the two aggregate demands are simply $h(p,\pi)\Omega(p,\pi,w)$ and $\chi(p,\pi)\Omega(p,\pi,w)$. In addition, every consumer θ is assumed to supply one unit of labor at every positive wage.[6] Hence the total supply of labor is $L = \int_0^1 \nu(d\theta) > 0$, whenever w is positive.

From a strategic viewpoint, the (Latin and Greek) producers have to base their decisions on ex ante conjectures about the consumers' demand for their product. We want that these conjectures be, in some sense, objectively founded. In our homothetic preference case, we assume that the functions h and χ are correctly perceived. As to the aggregate wealth function Ω, several specifications are possible, however, and have been used in the literature. They differ according to the degree in which producers are assumed to take into account the impact of their own decisions upon the value of Ω. To cover these different cases, let $r(p,\pi,w)$ and $\rho(p,\pi,w)$ denote the wealth functions conjectured, respectively, by the Latin and Greek producers.

Moreover, let us divide r and ρ into an autonomous wealth part, say $A > 0$, taken as given by the producers, and an induced part, depending upon the producers' decisions. The decomposition into these two parts relies on alternative behavioral assumptions. Here are three examples.

1. A first (extreme) specification is to take the whole wealth as autonomous, i.e., for all $p,\pi,w > 0$,

$$r(p,\pi,w) = \rho(p,\pi,w) = A,$$

[6] At a zero wage we can consider different types of behavior leading to different interpretations. In the examples introduced below we assume that every individual is indifferent between working or not at a zero wage. In this limit case unemployment may still arise in the sense that only part of the labor force is employed – identical individuals being unequally treated.

and to adjust A parametrically so that conjectures are fulfilled in equilibrium. This is one of the solutions proposed by Marschak and Selten [1972] and used, for instance, in Hart [1982] and Silvestre [1988]. For each producer, it amounts to neglecting all the effects of his decisions on aggregate wealth.

2. Another specification (also proposed by Marschak and Selten [1972] and used by d'Aspremont, Dos Santos Ferreira, and Gérard-Varet [1984]) is to suppose that the producers take into account the effects of their decisions on the total wage bill, but not on the distributed profits, which are included in the autonomous part.[7] Accordingly, the conjectured wealth function is the sum of the total wage bill and the autonomous wealth; i.e.,

$$r(p,\pi,w) = w[\ell h(p,\pi) + \lambda\chi(p,\pi)]r(p,\pi,w) + A$$

and

$$\rho(p,\pi,w) = w[\ell h(p,\pi) + \lambda\chi(p,\pi)]\rho(p,\pi,w) + A$$

or, with $w[\ell h(p,\pi) + \lambda\chi(p,\pi)] < 1$,

$$r(p,\pi,w) = \rho(p,\pi,w) = A/[1 - w(\ell h(p,\pi) + \lambda\chi(p,\pi))].$$

Again A can be adjusted parametrically to get fulfillment of conjectures in equilibrium.

3. A third specification (at another extreme) is to consider that the producers take into account the effects of their decisions both on the total wage bill and on the distributed profits (assuming that all profits are distributed). We thus get

$$\begin{aligned} r(p,\pi,w) &= w[\ell h(p,\pi) + \lambda\chi(p,\pi)]r(p,\pi,w) \\ &\quad +(p - w\ell)h(p,\pi)r(p,\pi,w) \\ &\quad +(\pi - w\lambda)\chi(p,\pi)r(p,\pi,w) + A \\ &= [ph(p,\pi) + \pi\chi(p,\pi)]r(p,\pi,w) + A \end{aligned}$$

and, similarly,

$$\rho(p,\pi,w) = [ph(p,\pi) + \pi\chi(p,\pi)]\rho(p,\pi,w) + A,$$

[7] This amounts to considering a conjectured demand including what we have called "Ford effects", following the idea developed by Henry Ford [1922].

leading to

$$r(p,\pi,w) = \rho(p,\pi,w) = A/[1 - ph(p,\pi) - \pi\chi(p,\pi)]. \tag{1}$$

To write this last expression, we need to assume that the marginal propensity to consume in the present period $[ph(p,\pi) + \pi\chi(p,\pi)]$ is less than one.

Also, given our temporary equilibrium framework, equation (1) will apply in equilibrium with $A = M + \hat{I}$, where M is the supply of money and $\hat{I}$ stands for the total expected future income. We thus have A positive and adjusted to fulfill conjectures in equilibrium. Notice that there is in this framework a "money market" which, by Walras' law, always clears when the goods markets clear.

Whatever the adopted specification of the conjectured wealth functions r and ρ, we see that they can always be written as a product,

$$r(p,\pi,w) = \rho(p,\pi,w) = AK(p,\pi,w),$$

where $K(p,\pi,w)$ is a multiplier, the definition of which varies accordingly. For any positive wage, we can construct a game between the two producers with prices as strategies ($p \geq w\ell$ and $\pi \geq w\lambda$, respectively), and payoff functions given by

$$\begin{aligned} F(p,\pi,w) &= (p - w\ell)h(p,\pi)K(p,\pi,w)A \\ \Phi(p,\pi,w) &= (\pi - w\lambda)\chi(p,\pi)K(p,\pi,w)A. \end{aligned}$$

In the following, we shall concentrate mainly on the game obtained from the third specification of the wealth functions. Also, we shall put aside the way in which the wage is determined. Actually, there are several ways to close the model. One is to consider the labor market as competitive with the producers as wage-takers and to adjust the wage parametrically for equilibrium.[8] Another way is to introduce two stages. In a first stage the labor market is organized, and the wage w is determined.[9] Then, at the second stage the goods prices are selected by the producers, taking the wage as given.

[8] Then, of course, the behavior of the consumers at a zero wage must be determined, since the wage may have to be adjusted at zero.

[9] This includes the way proposed by Hart [1982], in which the workers fix the nominal wage through their union.

Anyway, we can define a pair of nonnegative prices (p^*, π^*) as an *equilibrium in the goods markets at a given wage* w if

$$\text{(i)} \qquad p^* \in \arg\sup_{p \geq w\ell} F(p, \pi^*, w) \text{ and } \pi^* \in \arg\sup_{\pi \geq w\lambda} \Phi(p^*, \pi, w);$$

$$\text{(ii)} \qquad AK(p^*, \pi^*, w) = \Omega(p^*, \pi^*, w).$$

The first property is a Nash-equilibrium property for the game, and the second property imposes that conjectures be fulfilled. This definition, however, is incomplete, since it neglects a feasibility constraint. Indeed by letting the total employment required by the producers be

$$Z(p, \pi, w) = [\ell h(p, \pi) + \lambda\chi(p, \pi)]AK(p, \pi, w),$$

we need to impose that $Z(p, \pi, w) \leq L$, for $w \geq 0$. In fact, in the objective approach we have adopted, it is more reasonable to impose this feasibility constraint on the producers' strategy spaces. Consider the following correspondences:

$$P(\pi, w) = \{p \geq w\ell : Z(p, \pi, w) \leq L\}$$

$$\Pi(p, w) = \{\pi \geq w\lambda : Z(p, \pi, w) \leq L\}.$$

We call a pair of nonnegative prices (p^*, π^*) a *multisectoral equilibrium at given wage* $w \geq 0$ if

$$\text{(i)} \qquad p^* \in \arg\sup_{p \in P(\pi^*, w)} F(p, \pi^*, w) \text{ and } \pi^* \in \arg\sup_{\pi \in \Pi(p^*, w)} \Phi(p^*, \pi, w);$$

$$\text{(ii)} \qquad AK(p^*, \pi^*, w) = \Omega(p^*, \pi^*, w).$$

Notice that an equilibrium in the goods markets (p^*, π^*) at a given w is a multisectoral equilibrium given the same wage whenever

$$Z(p^*, \pi^*, w) \leq L.$$

In the next section we shall study these equilibrium concepts in a set of examples. For that purpose it will be useful to simplify notation. Aggregate objective demand conjectures will be written as multiplicative forms, namely, $H(p, \pi, w)A$ and $\chi(p, \pi, w)A$, where

$$H(p, \pi, w) = h(p, \pi)K(p, \pi, w)$$

$$X(p, \pi, w) = \chi(p, \pi)K(p, \pi, w).$$

Moreover, since we shall be essentially restricted to the third specification, as given by (1), the w argument will be omitted in H and X.

3 A class of examples

Let us illustrate the model by a class of economies arising in a two-period world. Suppose that each consumer θ has a C^2 strongly quasi-concave utility function that is homogeneous and intertemporally separable. For consumer θ, let $(c_{\theta t}, \gamma_{\theta t})$ be his consumptions of the Latin good and the Greek good in period $t (t = 1, 2)$, m_θ and $\tilde{m}_\theta$ be his initial money balance and his savings, i_θ and $\hat{i}_\theta$ be his current income and his expected future income, respectively. Also let $(\hat{p}, \hat{\pi})$ be the expected future prices. The consumer's program can now be written as

$$\max_{c_\theta, \gamma_\theta, \tilde{m}_\theta)} U(u_1(c_{\theta 1}, \gamma_{\theta 1}), u_2(c_{\theta 2}, \gamma_{\theta 2}))$$

subject to

$$pc_{\theta 1} + \pi \gamma_{\theta 1} + \tilde{m}_\theta \leq m_\theta + i_\theta$$

with

$$c_{\theta 1}, \gamma_{\theta 1} \geq 0$$

and

$$\hat{p} c_{\theta 2} + \hat{\pi} \gamma_{\theta 2} \leq \tilde{m}_\theta + \hat{i}_\theta$$

with

$$c_{\theta 2}, \gamma_{\theta 2} \geq 0.$$

We shall assume in the following that the function u_t is C.E.S. and constant over time:

$$u(c_{\theta t}, \gamma_{\theta t}) = (c_{\theta t}^{(s-1)/s} + \gamma_{\theta t}^{(s-1)/s})^{s/(s-1)},$$

with an intersectoral elasticity of substitution $s > 0$ $(s \neq 1)$.

Maximizing in two stages, first the two arguments of U, conditional on $\tilde{m}_\theta$, then[10] over $\tilde{m}_\theta$ and defining the marginal propensity to consume in period 1:

$$a = (pc_{\theta 1} \pi \gamma_{\gamma 1})/(m_\theta + i_\theta + \hat{\imath}) = (m_\theta + i_\theta - \tilde{m}_\theta)/(m_\theta + i_\theta + \hat{\imath}_\theta),$$

[10]Here we neglect the nonnegativity constraint on $\tilde{m}_\theta$. An individual's savings might be negative (he is then borrowing at zero interest rate).In any case the total savings $\tilde{M}$ are equal to the total money supply $M > 0$.

one easily gets the following solution for present consumption:

$$\begin{aligned} c^*_{\theta 1} &= \frac{1}{2(p/P)^s}\frac{a}{P}(m_\theta + i_\theta + \hat{\imath}_\theta) \\ \gamma^*_{\theta 1} &= \frac{1}{2(\pi/p)^s}\frac{A}{P}(m_\theta + i_\theta + \hat{\imath}_\theta), \end{aligned} \tag{2}$$

where a becomes simply a function of $P/\hat{P}$, with P and $\hat{P}$ denoting the price averages:

$$P = \left(\frac{p^{1-s} + \pi^{1-s}}{2}\right)^{1/(1-s)} \quad \text{and} \quad \hat{P} = \left(\frac{\hat{p}^{1-s} + \hat{\pi}^{1-s}}{2}\right)^{1/(1-s)}.$$

To be more specific, we may take U to be a C.E.S. function with elasticity of intertemporal substitution $\sigma > 0$ and relative weight of current utility $\delta \in]0, 1[$, getting

$$a = [1 + (\delta/(1-\delta))^{-\sigma}(P/\hat{P})^{\sigma-1}]^{-1}.$$

Considering the solution values $c^*_{\theta 1}$ and $\gamma^*_{\theta 1}$ for $m_\theta + i_\theta + \hat{\imath}_\theta = 1$, we obtain the functions (identical for each θ):

$$\begin{aligned} h(p,\pi) &= \frac{1}{2p(p/P)^{s-1}[1 + (\delta/(1-\delta))^{-\sigma}(P/\hat{P})^{\sigma-1}]} \\ \chi(p,\pi) &= \frac{1}{2\pi(\pi/P)^{s-1}[1 + (\delta/(1-\delta))^{-\sigma}(P/\hat{P})^{\sigma-1}]}. \end{aligned}$$

Taking the third specification of producers' conjectures as given by (1), we finally get, for $\alpha = \frac{1}{2}(\delta(1-\delta))^\sigma$,

$$\begin{aligned} H(p,\pi) &= \alpha p^{-s} P^{s-\sigma} \hat{P}^{\sigma-1} \\ X(p,\pi) &= \alpha \pi^{-s} P^{s-\sigma} \hat{P}^{\sigma-1}. \end{aligned} \tag{3}$$

We need also to choose some class of price expectations. For simplicity, we assume the price expected in one sector to depend only on the current price in that sector. Let these price expectation functions $\hat{p}(p)$ and $\hat{\pi}(\pi)$ be twice continuously differentiable, strictly increasing, and such that $\lim_{p\to 0}\hat{p}(p) = \lim_{\pi\to 0}\hat{\pi}(\pi) = 0$ and $\lim_{p\to\infty}\hat{p}(p) = \lim_{\pi\to\infty}\hat{\pi}(\pi) = \infty$. Denote, respectively, by $b(p)$ and $\beta(\pi)$ the elasticities of $\hat{p}(p)$ and $\hat{\pi}(\pi)$. We shall actually restrict our attention to two cases:

(a) Pure Substitutability. $1 < \sigma \leq s$, with $b(p)$ and $\beta(\pi)$ nonincreasing, upper-bounded by $\sigma/(\sigma-1)$, and taking values below $1 + (s-1)/(\sigma-1)$.

(b) Pure Complementarity. $s \leq \sigma < 1$, with $b(p)$ and $\beta(\pi)$ nondecreasing and taking values above $1 + ((1-s)/(1-\sigma))$.

These restrictions are put on the parameters of our example in order to get some general properties of the functions H and X that will be used to prove the existence of an equilibrium in the goods markets. Further restrictions on the parameters will be added to demonstrate the possibility of involuntary unemployment in Section 4.

We now describe these general properties.

Property A. (1) The function H (respectively, X) is positive, decreasing, and twice continuously differentiable in p (respectively, in π). It is continuous in both variables. (2) Both functions H and X are asymptotically finite for any sequence of prices bounded away from the axes.

In the example, positivity, continuity, and twice-differentiability of H are obvious (see (2) and (3). Also, in the pure complementarity case (b), where $(s-\sigma) \leq 0$ and $(\sigma - 1) < 0$, we see that H and X are decreasing in both prices so that Property A is verified. To treat the pure substitutability case (a), let us compute the price elasticity of H:

$$e(p,\pi) = -\frac{H'_p(p,\pi)}{H(p,\pi)}p = s - \frac{s-\sigma}{1+(p/\pi)^{s-1}} - \frac{(\sigma-1)b(p)}{1+(\hat{p}/\hat{\pi})^{s-1}}. \tag{4}$$

(The price elasticity of X is computed similarly and denoted $\eta(p,\pi)$.) Using (4), we have

$$H'_p(p,\pi) = (H(p,\pi)/p)(-e(p,\pi)) < 0,$$

since $-e(p,\pi) < -s + (s-\sigma) + (sigma-1)b(p) \leq 0$, according to the upper bound imposed on b. Hence (A1) holds in case (a) As for (A2), notice that

$$\begin{aligned} \sup_{\pi>0} H(p,\pi) &= \sup_{\pi>0} \alpha p^{-s} \left[\frac{2}{p^{1-s}+\pi^{1-s}}\right]^{(s-\sigma)/(s-1)} \left[\frac{2}{\hat{p}^{1-s}+\hat{p}^{1-s}}\right]^{(\sigma-1)/(s-1)} \\ &= 2\alpha p^{-\sigma}\hat{p}^{\sigma-1} \quad (\text{setting } \pi = \infty). \end{aligned}$$

Clearly, this last expression is finite for every $p > 0$. The same is true when p tends to infinity if

$$\lim_{p\to\infty} \ln(p^{-\sigma}\hat{p}^{\sigma-1}) = \lim_{p\to\infty} -\sigma \ln p \left[1 - \frac{(\sigma-1)\ln\hat{p}}{}\sigma \ln p\right] \leq 0 < \infty.$$

But this is implied by

$$\lim_{p\to\infty} \frac{(\sigma-1)\ln \hat{p}(p)}{\sigma \ln p} = \frac{\sigma-1}{\sigma} \lim_{p\to\infty} b(p) \leq 1.$$

A similar argument applies to X.

PROPERTY B. The profit function $F(p,\pi,w) = (p - w\ell)H(p,\pi)A$ (respectively, $\Phi(p,\pi,w) = (\pi - w\lambda)\, X(p,\pi)A$) is strictly quasi-concave in p (respectively, in π).

In the example, we have for $p > w\ell$:

$$F'_p(p,\pi,w) = AH(p,\pi)[(p-w\ell)/p](p/(p-w\ell)) - e(p,\pi)], \tag{5}$$

where $e(p,\pi)$ is the price elasticity given by (4).

Since $b'(p) \leq 0$ in case (a) and $b'(p) \geq 0$ in case (b), $[(p/(p-w\ell)) - e(p,\pi)]$ is strictly decreasing in p, and F'_p remains negative once it becomes so. Property B is thus verified.

PROPERTY C. For any sequence of positive price vectors such that some price tends to infinity, the corresponding total revenue function becomes decreasing in this price for at least one producer.

First, let $(p^\tau, \pi^\tau)_{\tau\geq 1}$ be a sequence such that $\lim_{\tau\to\infty} \pi^\tau < \lim_{\tau\to\infty} p^\tau = \infty$. We have to check that $e(p^\tau,\pi^\tau)$ is larger than one infinitely often (or, equivalently, that $p^\tau H(p^\tau,\pi^\tau)$ is decreasing in p). Clearly, by (4) and the upper-boundedness of b, $\lim_{\tau\to\infty} e(p^\tau,\pi^\tau) = s > 1$, in case (a). Using the restriction $\lim_{p\to\infty} b(p) > 1 + ((1-s)/(1-\sigma))$, we get, in case (b),

$$\lim_{\tau\to\infty} e(p^\tau,\pi^\pi) = \sigma - (\sigma-1) \lim_{\tau\to\infty} b(p^\tau) > 1 + 1 - s > 1.$$

Second, supposing that both prices tend to infinity, it is enough to check that $e(p^\tau,\pi^\tau) + \eta(p^\tau,\pi^\tau) > 2$ infinitely often. Now,

$$\begin{aligned} e(p,\pi) + \eta(p,\pi) &= s + \sigma - (\sigma-1)[(\hat{p}^{1-s}/(\hat{p}^{1-s} + \hat{\pi}^{1-s}))b(p) \\ &+ (\hat{\pi}^{1-s}/(\hat{p}^{1-s} + \hat{\pi}^{1-s}))\beta(\pi)]. \end{aligned} \tag{6}$$

The conclusion is obtained under the restrictions,

$$\lim_{p\to\infty} b(p) < 1 + \frac{s-1}{\sigma-1} \quad \text{and} \quad \lim_{\pi\to\infty} \beta(\pi) < 1 + \frac{s-1}{\sigma-1},$$

in case (a), and the reverse restrictions in case (b).

These first three properties, satisfied by our class of examples, are enough to derive a preliminary result, namely, the existence, at any positive wage, of two continuous "reaction curves" that intersect at some pair of prices, thus establishing existence of an equilibrium in the goods markets.

Proposition 1 *Under Properties A1, B, and C, there exists an equilibrium in the goods markets at any positive wage.*

Proof: First, using standard arguments, it is possible to show that the solution to $\max_{p \geq w\ell} F(p, \pi, w)$ is a well-defined continuous function $\pi(p, w)$ (for a detailed proof, see CORE DP 8635). We can define in a symmetric way the function $\tilde{\pi}(p, w)$. Second, suppose for some positive w that no $(p, \pi) \geq (w\ell, w\lambda)$ satisfies $p\tilde{p}(\pi, w)$ and $\pi = \tilde{p}(p, w)$; i.e., there is no equilibrium in the goods markets. Then it is possible to find a sequence of prices $(p^\tau, \pi^\tau)_{\tau \geq 1}$ such that, for any $\tau, w\ell \leq p^\tau < \tilde{p}(\pi^\tau, w)$ and $w\lambda \leq \pi^\tau \leq \tilde{\pi}(p^\tau, w)$ and such that at least one of the two prices, say p^τ, tends to infinity. By Property C, the corresponding total revenue function becomes decreasing for at least one producer, say the Latin. Hence the profit function,

$$F(p^\tau, \pi^\tau, w) = (1 - (w/ell/p^\tau))p^\tau H(p^\tau, \pi^\tau)A,$$

which tends to the total revenue function as p^τ tends to infinity, becomes decreasing, a contradiction to $p^\tau \leq \tilde{p}(\pi^\tau, w)$, by Property B. ■

Notice that the proposition does not ensure the existence of a multisectoral equilibrium at a given positive wage, since the equilibrium in the goods markets may violate the labor market constraint. However, it is possible to demonstrate the existence of a multisectoral equilibrium at any positive wage by requiring in addition that the set of admissible prices,

$$\tilde{Z}(w) = \{(p, \pi) : p \geq w\ell, \pi \geq w\lambda, [\ell H(p, \pi) + \lambda X(p, \pi)]A \leq L\},$$

be nonempty and strictly biconvex (or, $\ell H + \lambda X$ strictly quasi-convex in each price separately).[11]

[11] In the pure complementarity case this is always true because $\ell H + \lambda X$ is decreasing in both arguments

In the case where for some positive wage the two reaction curves intersect outside the admissible region, we find full employment multisectoral equilibria on the boundary of $\tilde{Z}(w)$, the labor constraint being binding for at least one producer. One possible case is given by point E in Figure 2, where $\tilde{Z}(w)$ is the hatched region, and $\tilde{p}(\pi, w)$ and $\tilde{\pi}(p, w)$ are the reaction curves of the two producers. Arrows at point E indicate the directions of increasing profits. It is interesting to observe that this Figure 2, with the type of equilibria it illustrates, is similar to the one (Figure 10) presented by Cournot [1838] in Chapter VIII analyzing price competition between two producers of complementary inputs.

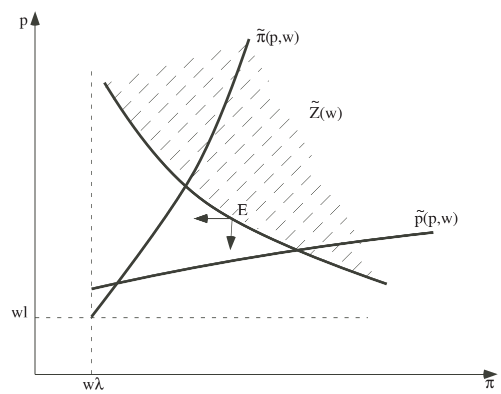

Figure 2:

We now turn to the main objective of this paper, and that is the possibility of involuntary unemployment.

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separately and goes to zero as at least one of the two prices goes to infinity. In the pure substitutability case nonemptiness of $\tilde{Z}(w)$ is entailed by the restrictions, $\lim_{p\to\infty} \hat{p}(p)/p < \infty$ and $\lim_{z\to\infty} \hat{\pi}(\pi)/\pi < \infty$, as again $\ell H + \lambda X$ goes to zero when both prices go to infinity. For strict quasi convexity of $\ell H + \lambda X$ in each price separately, a sufficient condition, for the simple case where $s = \sigma$, is that

$$-\frac{b'(p)p}{b(p)} < (\sigma - 1)\left[\frac{\sigma}{\sigma - 1} - b(p)\right] \text{ for any } p$$

and correspondingly for the greek sector, as shown in CORE DP 8635.

4 The possibility of involuntary unemployment

The class of examples that we have introduced is related to the class used by Weitzman [1985] in his analysis of the impact of profit sharing on unemployment. Weitzman considers, as we do, an economy where the goods markets are imperfectly competitive. Then he shows that, for such economies, it is always possible to find a positive wage at which full employment obtains at equilibrium. We challenge this conclusion by exhibiting economies where, because of the oligopolistic features of the goods markets, there is unemployment at any positive wage.

Let us first take the case of a given wage w and consider Weitzman's model. It starts with a composite utility function which is simply the particular specification of the above taking $\sigma = 1$, namely, an intertemporal Cobb-Douglas function of parameter $\delta \in]0, 1[$ but still an intersectoral CES-function. Restricting his model to the case $n = 2$ and using our notation, we get

$$h(p, \pi) = \frac{1}{2}\delta p^{-s} P^{s-1} \quad \text{and} \quad \chi(p, \pi) = \frac{1}{2}\delta \pi^{-s} P^{s-1}.$$

It is readily checked that the marginal propensity to consume $a = [ph(p, \pi) + \pi\chi(p, \pi)]$ is constant and equal to δ Hence, in this case, the two extreme specifications – completely autonomous versus completely endogenous wealth – are formally equivalent. Indeed if, for the sake of our comparison, we equalize to zero all the government and overhead-labor[12] variables introduced by Weitzman as well as our variable $\hat{I} = \int_0^1 \hat{\imath}_\theta \nu(d\theta)$, we find for both specifications that

$$AK(p, \pi) = M/(1 - \delta),$$

so that we simply take $A = M$ and

$$H(p, \pi) = \frac{1}{2}\frac{\delta}{1 - \delta} p^{-s} P^{s-1} \quad \text{and} \quad X(p, \pi) = \frac{1}{2}\frac{\delta}{1 - \delta}\pi^{-s} P^{s-1}.$$

Computing the equilibrium in the goods markets given w, (p^*, π^*), we get (taking $\ell = \lambda$ as in Weitzman):

$$p^* = \pi^* = ((s + 1)/(s - 1))w\ell.$$

[12]The overhead labor variables here do not play the essential role they played in Weitzman [1982].

Correspondingly, total labor demanded at equilibrium as a function of w is

$$Z^*(w) = (s-1)\delta M/(s+1)(1-\delta)w.$$

It is strictly decreasing and unbounded.[13] Now this fact is justly seen by Weitzman as a way of reducing unemployment as much as wanted. Weitzman suggests doing it by introducing a pay function whereby, in addition to the base wage rate w, the workers should receive a share of the profit. More precisely, the payoff functions of the two producers become, for some profit-sharing parameter $\tau \in]0,1[$:

$$\begin{aligned} F_\tau(p,\pi,w) &= (1-\tau)(p-w\ell)H(p,\pi)M, \qquad \text{for } p \geq w\ell, \\ \Phi_\tau(p,\pi,w) &= (1-\tau)(\pi-w\lambda)X(p,\pi)M, \qquad \text{for } \pi \geq w\lambda. \end{aligned}$$

Since these are positive linear transformations of the previous payoff functions, we get the same equilibrium prices as before and, hence, the same labor demand as a function of w. Therefore, a possible policy to reach full employment, without affecting the workers' income (in a world without uncertainty), is to decrease the base wage w enough, and increase the profit-sharing parameter accordingly.

We want to challenge this conclusion. It is crucially based on the fact that total labor demanded at equilibrium is a function of w and that this function goes to infinity when w vanishes. This is a particular case. In general, Z^* is a correspondence, and it may be bounded, or it may contain bounded selections. This leads to our formal definition of involuntary unemployment in a strong or in a weak sense.

For a given economy, we say that there is *weak involuntary unemployment* whenever there exists $u_0 \in]0,1[$ such that for each positive wage w there is a multisectoral equilibrium (p^*,π^*),

[13] Weitzman [1985] speaks of a "symmetric Nash equilibrium in prices", and since he supposes a large number of firms, he computes it by assuming "that each firm i is justified in regarding its demand ... as a true function of only its own price p_i, with aggregate variables P ... parametrically fixed beyond its control". Then the equilibrium prices are $p^* = \pi^* = (s/(s-1))w\ell$ and $Z^*(w) = (s-1)\delta M/s(1-\delta)w$ is simply a linear transformation of the above expression. This means that Weitzman's argument still holds when using the regular Nash equilibrium concept.

depending upon w, verifying

$$L - Z(p^*, \pi^*, w) \geq u_0 L. \tag{7}$$

Alternatively, we say that there is strong *involuntary unemployment* whenever the same conclusion holds for *every* multisectoral equilibrium at any positive wage.

To get weak involuntary unemployment, we need to further restrict our class of examples. It must verify:

Property D. For any sequence of positive price vectors such that some price tends to zero, the corresponding total revenue function becomes increasing in this price for at least one producer.

To ensure this property, we simply introduce, by symmetry with C, the following restrictions on the parameters of the example:

$$\lim_{p\to 0} b(p) > 1 + \frac{s-1}{\sigma - 1} \text{ and } \lim_{\pi \to 0} \beta(\pi) > 1 + \frac{s-1}{\sigma-1}, \text{ in case(a)},$$

and the reverse inequalities in case (b).

We may now prove

Proposition 2 *Under Properties A to D there exists $\varepsilon > 0$ such that, for every economy with mean autonomous wealth A/L less than ε, there is weak involuntary unemployment.*

Proof: By contradiction, assume that, however small the mean autonomous wealth A/L may be, there exists an economy which does not display weak involuntary unemployment. Then there exists a sequence $(u_0^\tau, L^\tau, A^\tau, w^\tau, p^\tau, \pi^\tau)_{\tau\geq 1}$ such that u_0^τ and A^τ/L^τ both tend to zero, and, for all τ, (p^τ, π^τ) is an equilibrium in the goods markets at w^τ, violating inequality (7) or, equivalently, verifying the inequality:

$$1 - [\ell H(p^\tau, \pi^\tau) + \lambda X(p^\tau, \pi^\tau)](A^\tau/L^\tau) < u_0^\tau.$$

This implies that, at least for one good, demand is unbounded along that sequence. Hence by Property A2 some price must vanish. By Property D the corresponding demand elasticity becomes less than one for at least one sector, leading to an increasing profit function (see (5) and thus contradicting the fact that (p^τ, π^τ) is always an equilibrium in the goods markets. ■

In fact, the argument we have used to prove Proposition 2 entails a stronger result. It proves that, for a mean autonomous wealth small enough and any positive wage, *all* equilibria in the goods markets satisfy inequality (7). So, in order to get strong involuntary unemployment, it suffices to exclude multisectoral equilibria such that the feasibility constraint is binding for some producer (making an increasing profit function compatible with equilibrium). An example of such an equilibrium is given by the point E in Figure 3, where we have represented the limiting case of a zero wage.[14] The set of admissible prices $\tilde{Z}(0)$ is the hatched region, and $\tilde{p}(\pi, 0)$ and $\tilde{\pi}(p, 0)$ are the two producers' reaction curves (which intersect in the interior of $\tilde{Z}(0)$ at an unemployment multisectoral equilibrium point). Arrows at point E indicate the directions of increasing profits.

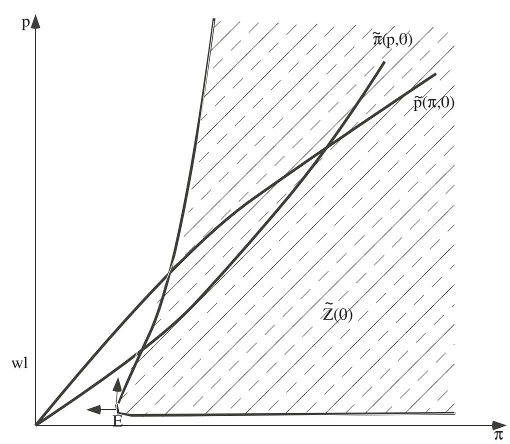

Figure 3:

In order to rule out multisectoral equilibria such as point E in Figure 3, still an additional property is needed.

Property E. For any sequence of positive price vectors such that some price tends to zero, the function $\ell H + \lambda X$ becomes decreasing in the price of a sector where the total revenue function

[14]The curves in Figure 3 have been calculated for the following values of the parameters: $\ell = 1, \lambda = 3$, and $s = \sigma = 3/2$. The price expectation functions were assumed to be identical and given by $\hat{p}(p) = (p^{-1} + p^{-5/2})^{-1}$.

becomes increasing in this price.

This property leads immediately to our main result.

Proposition 3 *Under Properties A to E there exists $\varepsilon > 0$ such that, for every economy with mean autonomous wealth A/L less than ε, there is strong involuntary unemployment.*

Proof: By contradiction, again take a sequence $(u_0^\tau, L^\tau, A^\tau, w^\tau, p^\tau, \pi^\tau)_{\tau \geq 1}$, as defined above, but where (p^τ, π^τ) is now a multisectoral equilibrium such that the labor constraint is binding for at least one producer. Since some demand is unbounded along that sequence, some price must tend to zero, and the profit function must become increasing in the corresponding sector, as seen in the proof of Proposition 2. By Property E the function $\ell H + \lambda X$ becomes decreasing in the price of such a sector. Thus, the producer will want to increase his price and will not be constrained by the labor supply, so that, for τ large enough (p^τ, π^τ) cannot be a multisectoral equilibrium. ■

It remains to verify that Property E is satisfied for some values of the parameters of our class of examples.

In case (b) this is simply because H and X are decreasing in both prices. Property E holds immediately. In case (a) we assume that the price expectations satisfy the following additional restrictions: $b(0) = \beta(0) = B$ and for $0 < c < \infty$, $0 < \gamma < \infty$, $\lim_{p \to \infty} [\hat{p}(p)/p^B] = c$ and $\lim_{\pi \to 0} [\hat{p}(\pi)/\pi^B] = \gamma$.

Consider the elasticity of $\ell H + \lambda X$ in p:

$$\frac{\ell H_p' + \lambda X_p'}{\ell H + \lambda X} p = \frac{(s - \sigma)\pi^{s-1}}{p^{s-1} + \pi^{s-1}} + \frac{(\sigma - 1) b(p) \hat{\pi}^{s-1}}{\hat{p}^{s-1} + \hat{\pi}^{s-1}} - \frac{s \ell \pi^s}{\lambda p^s + \ell \pi^s}. \tag{8}$$

The elasticity of $\ell H + \lambda X$ in π is defined analogously.

If only one price tends to zero, then the corresponding elasticity tends to $[-\sigma + (\sigma - 1)B]$ which has been assumed to be nonpositive. In fact, the case where it is zero can be neglected since

$$\lim_{p \to 0} (\sup_{\pi > 0} H(p, \pi)) = \lim_{p \to 0} (2\alpha(\hat{p}^{\sigma-1}/p^\sigma)) = 2\alpha c^{\sigma-1} < \infty,$$

and similarly,

$$\lim_{\pi \to 0} (\sup_{p>0} X(p, \pi)) < \infty,$$

so that we have demand saturation at a nil price, leading trivially to strong involuntary unemployment. The case where it is negative gives what we need by Property D.

Therefore, to verify Property E, the difficult case is when we have a sequence $(p^\tau, \pi^\tau)_{\tau \geq 1}$ of prices converging both to zero with, say, $\lim_{\tau\to\infty}(p^\tau/\pi^\tau) = q$. Since, by Property D, at least one of the total revenue functions becomes increasing, (i.e., e or η becomes less than one), it is enough to show that $\eta(p^\tau, \pi^\tau) \geq 1$ implies $\ell H'_p + \lambda X'_p < 0$, for infinitely many τ (and the symmetric implication). By (4) and (8) this amounts to showing, assuming $\sigma = s$ for simplicity, that

$$\frac{\beta(\pi^\tau)[p^\tau(\pi^\tau)/\hat{\pi}(\pi^\tau)]^{s-1}}{1 + [\hat{p}(p^\tau)/\hat{\pi}(\pi^\tau)]^{s-1}} \leq 1$$

implies

$$\frac{(s-1)b(p^\tau)}{1 + [\hat{p}(p^\tau)/\hat{\pi}(\pi^\tau)]^{s-1}} < \frac{s}{1 + (\lambda/\ell)(p^\tau/\pi^\tau)^s}$$

for infinitely many τ. Since $\lim_{\tau\to\infty}[\hat{p}(p^\tau)/\hat{\pi}(\pi^\tau)] = (c/\gamma)q^B$, the above implication leads, at the limit, to

$$q \leq \left(\frac{\gamma}{c}\right)^{1/B} [B-1]^{-1/B(s-1)}$$

implies

$$\frac{1 + (\gamma/\ell)q^s}{1 + (c/\gamma)^{s-1} q^{B(s-1)}} < \frac{s}{B(s-1)}.$$

The last inequality is satisfied for $q = 0$ (if not, demand is saturated at a nil price and strong involuntary unemployment is a trivial result). As its left-hand side is a quasi-convex function of q, it suffices to impose that it also be satisfied when q is equal to its maximum admissible value. By symmetry this finally leads to the required condition,

$$\left[1 + \frac{s}{s-1}) - B\right]^{-1} [B-1]^{1-(s/B(s-1))} < \frac{\lambda}{\ell}\left(\frac{\gamma}{c}\right)^{s/B} < \left[1 + \frac{s}{s-1} - B\right] [B-1]^{(s/B(s-1))-1}. \quad (9)$$

The admissible range for $(\lambda/\ell)(\gamma/c)^{s/B}$ is decreasing in s. As s increases from 1 to 2, more and more symmetry is imposed upon unit labor costs and price expectations at zero. Without this symmetry one cannot exclude the persistence of equilibria in which the feasibility constraint is binding for at least one producer (such as the one represented by point E in Figure 3) as A/L tends to zero.

5 Possible extensions

In order to give an idea of the robustness of the preceding results about involuntary unemployment, let us consider some natural extensions (or modifications) of our model.

1. First, we may increase the number of sectors, without invalidating Propositions 1 to 3. But, in order that Properties C and D hold, we need the elasticities of price expectation to take values below and above $1 + (n-1)((s-1)/(\sigma-1))$. This implies that the elasticity of price expectation takes, for some range of prices, higher and higher values as n increases. In the complementarity case Property C (and hence existence) becomes more and more difficult to obtain. In the substitutability case Property D becomes more restrictive and, combined with Property A, leads to

$$1 + (n-1)\frac{s-1}{\sigma-1} < \frac{\sigma}{\sigma-1} \text{ implying that } 1 < s < \frac{n}{n-1}.$$

Clearly, the admissible interval decreases as n increases. More competition makes the occurrence of involuntary unemployment more unlikely.

2. In the class of examples we have discussed, we have maintained a constant elasticity of intertemporal substitution and imposed strong restrictions on price expectations. These restrictions could be weakened by allowing a variable elasticity of intertemporal substitution. For instance, take the case of rigid price expectations, i.e., $\hat{p}$ and $\hat{\pi}$ exogenously given. By (1) and (2) we may write

$$H(p,\pi) = \frac{1}{2(p/P)^s P} \cdot \frac{a}{1-a} = \frac{1}{2(p/P)^s \hat{P}} \cdot \frac{a/P}{(1-a)/\hat{P}},$$

where a is a function of $P/\hat{P}$.

Since the term

$$\frac{a/P}{(1-a)/\hat{P}}$$

is the ratio of present real consumption to future real consumption, its elasticity with respect to $\hat{P}/P$ is nothing else than the elasticity of intertemporal substitution σ, which is now a function of $P/\hat{P}$. From this (or using equation (4) with $b(p) \equiv 0$) it is easy to check that

$$e(p,\pi) = s - s\frac{\partial P}{\partial p}\frac{p}{P} + \sigma\frac{\partial P}{\partial p}\frac{p}{P} = \frac{\pi^{1-s}}{p^{1-s}+\pi^{1-s}}s + \frac{p^{1-s}}{p^{1-s}+\pi^{1-s}}\sigma;$$

i.e., the price elasticity of H is a convex combination of the (constant) elasticity of intersectoral substitution s and the elasticity of intertemporal substitution σ. Therefore, imposing that σ be a differentiable nondecreasing function of $P/\hat{P}$, taking values below and above $2 - s$ (in the general case: $n - (n-1)s$), and upperbounded by s if $s > 1$, we can verify Properties A, B, C, and D. As for Property E, this is still the case for $s < 1$. If $s > 1$, a longer calculation along the lines of the preceding section shows that, denoting by σ_0 the limit of σ when $P/\hat{P}$ tends to zero, the condition for Property E to hold is,

$$\left[1+\frac{\sigma_0}{s-1}\right]^{-1}\left[\frac{1-\sigma_0}{s-1}\right]^{-1/(s-1)} < \frac{\lambda}{\ell} < \left[1+\frac{\sigma_0}{s-1}\right]\left[\frac{1-\sigma_0}{s-1}\right]^{1/(s-1)}.$$

3. A further extension of our results is to apply them to a purely atemporal version of our model, thereby avoiding the use of any kind of arbitrary price expectations. This consists in taking, as in Hart [1982], a utility function depending for every *theta* on the consumption of the produced goods $(c_\theta, \gamma_\theta)$ and of a nonproduced good k_θ. If this function is of the form $U(u(c_\theta,\gamma_\theta),k_\theta)$, with U and u having the same properties as before, the preceding derivations can be reinterpreted straightforwardly: let $\hat{P}$ be the price of the nonproduced good normalized to one and, for consumer θ, m_θ be the endowment of consumer θ in the nonproduced good, $\tilde{m}_\theta$ be replaced by k_θ (the consumption of the nonproduced good), and of course, $\hat{\imath}\theta$ be zero. Reinterpreting σ as the elasticity of substitution between the produced goods and the nonproduced one, we are immediately led to Properties A-E by the same restrictions. An example[15] of such a utility function is given by Silvestre [1988], where U is a C.E.S. function in u and k_θ modified by a linear term in k_θ.

[15] Another example, not derived from a C.E.S. function, is given in d'Aspremont, Dos Santos Ferreira, and Gérard-Varet [1989].

4. Finally, one can use a modified version of this model to derive results analogous to those in Weitzman [1982]. The modification consists of suppressing the nonproduced good, of adding fixed costs for each producer in the form of a given quantity of overhead labor needed to produce a positive amount, and introducing a zero-profit condition. One way to treat this modified model is to use the second specification of the feedback effects, namely those limited to the wage income (as discussed in Section 2 above), with A equal to the sum of fixed labor costs.

In the absence of a nonproduced good, labor can be taken as the numeraire and w put equal to one. Also the propensity to consume the produced goods is now one. Hence we get

$$h(p, \pi) = \frac{1}{2p(p/P)^{s-1}}.$$

Using a similar expression for χ, we may compute the multiplier to be

$$\frac{1}{1-(\ell h + \lambda \chi)} = \frac{2P^{1-s}}{(p-\ell)p^{-s} + (\pi - \lambda)\pi^{-s}},$$

leading to

$$H(p, \pi) = \frac{\pi^s}{(p-\ell)\pi^s + (\pi - \lambda)p^s}$$

and

$$e(p, \pi) = \frac{(p-\ell)\pi^s}{(p-\ell)\pi^s + (\pi - \lambda)^s} \frac{p}{p-\ell} + \frac{(\pi - \lambda)\pi^s}{(p-\ell)\pi^s + (\pi - \lambda)p^s} s.$$

Properties A1, B, and C are readily verified if $s > 1$, so that existence of an equilibrium in the goods markets is ensured. Taking A/L low enough, we get unemployment.

6 Conclusion

This work has presented a simple general equilibrium model of imperfect competition in prices. The purpose was to find a class of examples in which involuntary unemployment occurs and can be unambiguously attributed to oligopolistic competition in the goods markets. The exercise has been made difficult in several respects. First, trivial cases, due to bounded productive capacities or due to saturated demand, have been excluded. Second, the different producers have been assumed to conjecture objective demand curves and, unlike previous work of this kind,[16] to allow both for cross-sectoral price effects and for all kinds of income feedback effects. Third, we have stuck to utility functions with a constant elasticity of intersectoral substitutions. When the elasticity of intertemporal substitution is also constant, sensitivity is obtained by varying the elasticity of expectation of future prices. Alternatively, allowing for a variable elasticity of intertemporal substitution, while keeping the elasticity of expectation null, permits the same results to be reached, which therefore apply even to an atemporal economy (where future consumption is replaced by a nonproduced good). A clear distinction has been made between the complementarity case and the substitutability case. It is in this latter case, where the cross-sectoral price effect is positive, that the difficulty is greatest.

Among the properties that carry the results through, the main one is Property D. It ensures that the total revenue of some producer becomes increasing in price (or decreasing in quantity) when this or both prices go to zero.[17] Hence, as the wage vanishes, the equilibrium prices (in the goods markets) will not go to zero, thus excluding a complete Pigou effect and the achievement of full employment. This is enough to get weak involuntary unemployment, i.e. unemployment at some multisectoral equilibrium given any positive wage; and even, in the case of complementary goods, strong involuntary unemployment, i.e., unemployment at all multisectoral equilibria given any positive wage. In contrast, under perfect competition, where prices equal marginal costs, the Walrasian equilibrium would realize full employment at some positive equilibrium wage. To obtain strong involuntary unemployment in the substitutability case, one needs the additional Property E, implying some kind of symmetry (and the more so, the greater is the substitutability) in order to exclude multisectoral equilibria where some producer is off his reaction curve because the labor supply constraint is binding.

It is worth mentioning that these results are still meaningful when the total labor supply is not perfectly inelastic and is not positive (or undetermined) at zero wage. Because of imperfect

[16] We are thinking of Hart [1982]; d'Aspremont, Dos Santos Ferreira, and Gérard-Varet [1984, 1989]; Dehez [1985]; and Silvestre [1988].

[17] As well stressed by Silvestre [1988], Hart [1982] makes the opposite assumption that total revenue should always be increasing in quantity. The same property is implied by a Cobb-Douglas intertemporal utility function, combined with a constant elasticity of intersectoral substitution larger than one, as in Weitzman [1985].

competition, the real wage, adjusted to clear the labor market, would still be inferior to labor marginal productivity, and to the Walrasian equilibrium wage, and any multisectoral equilibrium employment would be less than the Walrasian employment level. Moreover, as we have seen when discussing our weak involuntary unemployment concept, we could get at the same adjusted positive wage different multisectoral equilibria, some implying full employment but (and this is the point) others not. The market failure would then result from the multiplicity of equilibria (as in Heller [1986] or Roberts [1989]) leading, by lack of coordination, to persistent underemployment.

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Biodiversity, infectious diseases and the dilution effect*

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Abstract

Biologists point out that biodiversity loss contributes to promote the transmission of diseases. In epidemiology, this phenomenon is known as dilution effect. Our paper aims to introduce this effect in an economic model where the spread of an infectious disease is considered. More precisely, we embed a SIS model into a Ramsey model (1928) where a pollution externality coming from production affects the evolution of biodiversity. Biodiversity is assimilated to a renewable resource and affects the infectivity of the disease (dilution effect). A green tax is levied on production at the firm level to finance depollution according to a balanced budget rule. In the long run, a disease-free and an endemic regime are possible. We focus only on the second case and we find that the magnitude of the dilution effect determines the number of steady states. When the dilution effect remains low, there are two cases depending on the environmental impact of production: (1) a low impact implies two steady states with high and low biodiversity respectively; (2) a large impact rules out any steady state. Conversely, when the dilution effect becomes high, a (unique) steady state always exists**:** a strong dilution effect works as a buffer and prevents the human pressure from being lethal for biodiversity in the long run. Moreover, under a low dilution effect, a higher green-tax rate always impairs biodiversity at the low steady state, while this green paradox is over under a high dilution effect. In the short run, we show that a limit cycle can arise around the high biodiversity steady state when the dilution effect is low. Surprisingly, the limit cycle is preserved under a high dilution effect. In other words, even if a strong dilution effect preserves the biodiversity in the long run and prevents the economy from the green paradox, it does not shelter the economy from the occurrence of biodiversity fluctuations.

Keywords: dilution effect, pollution, SIS model, Ramsey model, local bifurcations of codimension one and two.

JEL Classification: C61, E32, O44.

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1 Introduction

During the past decades, the number of infectious diseases outbreaks has increased worldwide as well as the number of emerging infectious diseases (Smith et al., 2014). Diseases are human-specific when contagions are only between humans and they are zoonotic when contagions are from animals to humans. Following Lloyd-Smith et al. (2009), zoonoses represent 60% to 76% of emerging diseases and, in this respect, they constitute a major concern for public health. Moreover, since 1970, the World Wide Fund for Nature (WWF, 2014) reports that the size of population of vertebrate species has been divided by two. A negative relation seems to link the emergence of infectious diseases (in particular zoonoses) and biodiversity. In the literature, this negative link is known as dilution effect.[1]

The dilution effect can be simply explained. According to Civitello et al. (2015), a high level of biodiversity inhibits the proliferation of parasites and prevents the spread of infectious diseases. An interesting example of dilution effect is reported by Keesing et al. (2010) about the Lyme disease transmission in the northeastern USA. Humans contract the Lyme disease by the bite of an infected blacklegged tick (*Ixodes scapularis*). The blacklegged ticks become infected when they feed on infected hosts which are primarily the white-footed mouse (*Peromyscus leucopus*) and the Virginia opossum (*Didelphis virginiana*). Interestingly, blacklegged ticks feeding on the Virginia opossum are much more likely to be uninfected than the ones feeding on the white-footed mouse. This is due to the fact that the Virginia opossum is a poor host for the Lyme disease while the white-footed mouse is a much better host. As reported in Keesing et al. (2010), the Virginia opossum acts as a buffer for the Lyme disease and deflects away the disease from the white-footed mouse. That is, the reported reduction in the population of Virginia opossums promotes the Lyme diseases. In this example, a lower biodiversity level promotes the Lyme disease: a dilution effect occurs.

According to Keesing et al. (2010), the dilution effect was observed for various infectious diseases such as hantavirus disease, malaria, schistosomiasis or West Nile fever. However, as reported by Civitello et al. (2015), the generality of the dilution effect remains controversial in the literature. On the one hand, as seen above, biodiversity loss increases the pathogen's concentration in the remaining species which can increase the disease transmission (dilution effect), especially when the remaining species are good hosts. However, on the other hand, as reported by Salkeld et al. (2013), ecosystems with high biodiversity can be richer in parasite diversity which can promote disease transmission. Keesing et al. (2006) show that, if the net overall effect of biodiversity on disease risk is negative, a dilution effect occurs while, if the net overall effect is positive, an amplification effect takes place. Therefore, the debate is about the sign of the overall net effect (positive or negative) and the generality of the dilution effect.

In their study, Keesing et al. (2010) advocate for the generality of the dilu-

[1]See Keesing et al. (2010) among others.

tion effect. This conclusion is also supported by recent empirical evidences by Civitello et al. (2015). They show broad evidences for the dilution effect and conclude that anthropogenic declines in biodiversity increase human and wildlife diseases. The existence and the generality of the dilution effect is of great importance because it means that environmental and biodiversity preservation allows to preserve human health. This complementarity reveals also interdependences between biodiversity and the whole economy. Indeed, illness is recognized as an important source of both productivity loss (Mitchell and Bates, 2011; or Fouad et al., 2017) and workers absenteeism (Akazawa et al., 2003; and Fouad et al., 2017). That is, economic activities pollute and imply both climate change and biodiversity loss which promotes in turn infectious diseases (dilution effect) and impairs economic activities (productivity loss, workers absenteeism). In this paper, we aim to develop a theoretical framework to take into account this complementarity between biodiversity and economic activities induced by the dilution effect. To this purpose, an interdisciplinary approach is needed.

In epidemiology, there is a panoply of theoretical models to study the spread of an infectious disease through a population (Hethcote, 2009). To keep things as simple as possible, we have decided to consider the simplest model to represent the change in the share of healthy people over time: the Susceptible-Infective-Susceptible (SIS) model. In this stylized epidemiological framework, two parameters capture the dynamics: (1) the probability of a susceptible individual to become ill after a contact with an infected individual and (2) the recovery rate driving the lapse of time the infected individual spends to recover from the disease. Dynamics are straightforward. A disease-free steady state coexists with an endemic one. When (1) exceeds (2), the endemic steady state is stable while the other, unstable.

To bridge the gap between economy, ecology and epidemiology, we propose to embed the SIS model into a Ramsey growth model where the production activities pollute and impair the biodiversity. The Ramsey framework is better than an overlapping generations (OLG) model to represent short-run epidemiological dynamics. Indeed, in a two-period OLG model, a period covers the half-life of an individual, says 35 years according to the average expectancy of life across the world. Such a long period is inappropriate to reproduce the short-run cycles of most common diseases.

To simplify the architecture, we assimilate biodiversity to a renewable resource affecting the household's immune system. We introduce a two-sided dilution effect assuming the probability to become ill as a decreasing function of biodiversity and the recovery rate as an increasing function. As in Goenka et al. (2014), we consider a labor force constituted only of healthy people. We introduce a government who levies a simple Pigouvian tax on production at the firm level in order to finance depollution according to a balanced budget rule. Such a unified framework gives us the opportunity to observe the effects of environmental policies on economic variables, the biodiversity and the pollution level.

The integration of the SIS model into a Ramsey model is not new and dates back to Goenka and Liu (2012). They have considered a discrete-time model in which healthy people tune their labor supply through a consumption-leisure arbitrage. In a continuous version of the model where labor supply is exclusively driven by the number of healthy people, Goenka et al. (2014) address the issue of optimal health expenditures. More recently, Bosi and Desmarchelier (2018b) have reconsidered the continuous-time version developed by Goenka et al. (2014) to take in account the interplay between a flow of pollution and infectious diseases. Bosi and Desmarchelier (2018b) have pointed out that, when pollution becomes excessive, two limit cycles can appear (stable and unstable) near the endemic steady state through a Hopf bifurcation.

We study the equilibrium either in the long or the short run. In the long run, as in the standard SIS model, a disease-free regime coexists with the endemic one. The disease-free regime is characterized by the existence of two steady states: the one experiences a low biodiversity level; the other, a high level. Interestingly, when the environmental impact of production becomes excessive, the two steady states collide and disappear. This case captures the possibility of a mass extinction under human pressure on the environment. In the endemic regime, under a low dilution effect, we recover the key features of the disease-free regime. Conversely, under a strong dilution effect, there is always a unique steady state whatever the environmental impact of production. Therefore, very importantly, the complementarity between biodiversity and production activities induced by a large dilution effect seems to prevent the occurrence of a mass extinction. Moreover, a paradox emerges under a moderate dilution effect at the steady state with low biodiversity: a higher green-tax rate lowers the biodiversity. This counter-intuitive effect is similar to the static green paradox pointed out by Bosi and Desmarchelier (2017).[2] Conversely, a green-tax rate always increases the biodiversity level at the steady state with higher biodiversity. Interestingly, the static green paradox is ruled out by a strong dilution effect. In the short run, we show that both the dilution effects (low and high) are compatible with the existence of a limit cycle (arising through a Hopf bifurcation) around the high-biodiversity steady state when preferences exhibit a complementarity between biodiversity and consumption. To sum up, a high dilution effect seems to have a double benefit: (1) it preserves biodiversity in the long run and (2) it prevents the economy from the green paradox. Nevertheless, it is not able to avoid the fluctuations of biodiversity (limit cycle) around the steady state.

The rest of the paper is organized as follows. Section 2 introduces the model. Sections 3 derives the equilibrium system. Sections 4 and 5 focus on the long and short-run dynamics. A numerical illustration with isoelastic fundamentals is provided in section 6. Section 7 concludes.

[2] In Bosi and Desmarchelier (2017), a static green paradox is a positive relation between the green-tax rate and the pollution level at the steady state while, in the seminal contribution by Sinn (2008), this paradox is a positive relation along the transition path.

2 Model

In this paper, we are interested in the relationship between biodiversity loss and transmission of infectious diseases in an economic context. Our mathematical approach, despite some technical aspects, leads to a deep understanding of economic and ecological feedbacks, and unveils their practical consequences. In the following section, we introduce the general framework to develop in the rest of the paper.

2.1 Disease

Epidemiologists use the SIS model to study the spread of endemic diseases. Population (N_t) is divided in two classes: susceptible (S_t) and infective (I_t) with $S_t + I_t = N_t$. We consider a wide range of infectious diseases, not a specific one. So the infective class covers many different illnesses. The proportion of susceptible and infective are given by $s_t = S_t/N_t$ and $i_t = I_t/N_t$. $\beta > 0$ denotes the average number of adequate contacts (sufficient to transmit the disease) of an infective per unit of time, and S_t/N_t the probability to face a susceptible during a contact. β increases in the transmissibility due to the virulence and pathogenicity of microbes, which increases in turn in the loss of biodiversity. Thus, $\beta S_t/N_t$ is the average number of adequate contacts with susceptibles of one infective per unit of time, while the number of new infectives per unit of time is given by $\beta I_t S_t/N_t$. An infective is seek during a period of time after which he recovers and becomes a new susceptible ($\gamma = -\dot{I}_t/I_t$ is the recovery rate in absence of new contamination, a sort of exponential decay rate from infection). The recovery rate decreases with the virulence and, so, with the loss of biodiversity. Notice that the SIS model postulates that the infection does not confer immunity.

In an oversimplified world with no births, no deaths, no migrations, the population remains constant over time ($\dot{N}_t = 0$). The evolution of S_t and I_t over time is simply given by:

$$\dot{S}_t = -\beta \frac{I_t}{N_t} S_t + \gamma I_t, \tag{1}$$

$$\dot{I}_t = \beta \frac{I_t}{N_t} S_t - \gamma I_t. \tag{2}$$

Since $\dot{N}_t = 0$, it follows that $\dot{S}_t + \dot{I}_t = 0$ and equation (1) becomes:

$$\dot{s}_t = (1 - s_t)(\gamma - \beta s_t). \tag{3}$$

As in Goenka et al. (2014), we assume that the labor force (L_t) consists only of healthy people: $L_t = S_t$. Since $l_t = L_t/N_t \leq 1$, l inherits the dynamics of s:

$$\dot{l}_t = (1 - l_t)(\gamma - \beta l_t). \tag{4}$$

We can see that (4) exhibits two steady states: $l = 1$ and $l = l^* = \gamma/\beta$. The first one is called disease-free because the disease disappears while the other is called endemic because the disease persists. Clearly, the endemic steady state becomes meaningless when $\gamma > \beta$ because in this case $l^* > 1$. In other words, the endemic steady state is admissible from an economic point of view if and only if $\gamma < \beta$.

As empirically observed by Keesing et al. (2010) or Johnson and Thieltges (2010) among others, a biodiversity loss promotes infectious diseases. This is the dilution effect. The goal of our model is to understand its macroeconomic consequences. Since the dilution effect captures a negative correlation between the disease prevalence and the biodiversity level, we assume that a higher biodiversity level (B_t) reduces the disease transmissibility (β) and increases the recovery rate (γ). The following hypotheses sum up this idea.

Assumption 1 *$\beta'(B_t) < 0$ and $\gamma'(B_t) > 0$ with* $\lim_{B_t \to 0} \beta(B_t) = \infty$, $\lim_{B_t \to \infty} \beta(B_t) = 0$, $\lim_{B_t \to 0} \gamma(B_t) = 0$ and $\lim_{B_t \to \infty} \gamma(B_t) = \infty$.

We introduce a new formal definition of dilution effect, encompassing both the impacts of biodiversity on the disease transmissibility (β) and on the recovery rate (γ).

Definition 1 (dilution effect) *The dilution effect is given by*

$$d(B_t) \equiv \varepsilon_\gamma(B_t) - \varepsilon_\beta(B_t) > 0, \tag{5}$$

where

$$\varepsilon_\beta(B_t) \equiv \frac{B_t \beta'(B_t)}{\beta(B_t)} < 0 \text{ and } \varepsilon_\gamma(B_t) \equiv \frac{B_t \gamma'(B_t)}{\gamma(B_t)} > 0, \tag{6}$$

are the first-order elasticities.

We observe that, at the endemic steady state, the dilution effect captures also the sensitivity (elasticity) of labor supply with respect to biodiversity, that is, $d(B_t) = B_t l'(B_t) / l(B_t) > 0$. As we will see later, the dilution effect has serious consequences on epidemiological and economic dynamics. To simplify the presentation, we normalize the population size to the unity: $N_t = 1$.

2.2 Preferences

The household earns a capital income $r_t h_t$ and a labor income $\bar{\omega}_t$, where r_t and h_t denote respectively the real interest rate and the individual wealth at time t. Income is consumed (c_t), saved and invested according to the budget constraint:

$$\dot{h}_t \leq (r_t - \delta) h_t + \bar{\omega}_t - c_t. \tag{7}$$

In this model, healthy people work while sick people don't. However, for simplicity, we assume a perfect social security, that is a full unemployment insurance in the case of illness. Healthy and sick agents earn the same labor income $\bar{\omega}_t$. L_t healthy people supply one unit of labor at a wage w_t. Under a balanced-budget rule for social security, we obtain $\bar{\omega}_t N_t = w_t L_t$. Therefore, $\bar{\omega}_t = w_t l_t$.

Gross investments include the capital depreciation at the rate δ. Since $N_t = 1$, we obtain $L_t = N_t l_t = l_t$, $K_t = N_t h_t = h_t$ and $h_t = K_t/N_t = k_t l_t$.

Let $u(c_t, B_t)$ be the utility function of the representative household. We assume that biodiversity affects marginal utility of consumption ($u_{cB} \neq 0$) which means that it affects the consumption behavior. Indeed, biodiversity has not only a positive impact on physical health through the dilution effect, it has also a positive influence on mental health. For instance, as reported by Dean et al. (2011), biodiversity in cities has some psychosocial benefits: recovery from stress, self-regulation of emotions, restoration of attention fatigue and enhanced sense of community. In their study, Dean et al. (2011) point out that these psychosocial benefits preserve mental health and prevent a depressive behavior. Even if, to the best of our knowledge, there is no empirical evidence about the effect of biodiversity on consumption demand, its benefits on mental health suggest also a role in consumption. Indeed, less biodiversity makes agents more depressive and can reduce their consumption demand ($u_{cB} > 0$). Conversely, they can compensate the loss of pleasure by increasing their consumption demand ($u_{cB} < 0$). The ambiguous effects of biodiversity on this demand remind us those of pollution on consumption highlighted by Michel and Rotillon (1995). The following assumption sums up the properties of the utility function.

Assumption 2 *Preferences are rationalized by a non-separable utility function $u(c_t, B_t)$. First and second-order restrictions hold on the sign of derivatives: $u_c > 0$, $u_B > 0$ and $u_{cc} < 0$, and the limit conditions:*

$$\lim_{c_t \to 0^+} u_c(c_t, B_t) = \infty \text{ and } \lim_{c_t \to +\infty} u_c(c_t, B_t) = 0.$$

We introduce the second-order elasticities:

$$\begin{bmatrix} \varepsilon_{cc} & \varepsilon_{cB} \\ \varepsilon_{Bc} & \varepsilon_{BB} \end{bmatrix} \equiv \begin{bmatrix} \frac{c_t u_{cc}}{u_c} & \frac{B_t u_{cB}}{u_c} \\ \frac{c_t u_{Bc}}{u_B} & \frac{B_t u_{BB}}{u_B} \end{bmatrix}, \tag{8}$$

$-1/\varepsilon_{cc}$ represents the intertemporal elasticity of substitution in consumption while ε_{cB} captures the effect of biodiversity on the marginal utility of consumption. Typically, if $\varepsilon_{cB} > 0$ (< 0), biodiversity and consumption are complements (substitutes) for households.

The illness lowers labor supply and the individual income in turn. The agent maximizes the intertemporal utility function

$$\int_0^\infty e^{-\theta t} u(c_t, B_t)\, dt, \tag{9}$$

under the budget constraint (7), where $\theta > 0$ is the rate of time preference.

Proposition 2 *The first-order conditions of the consumer's program are given by a static relation*

$$\mu_t = u_c(c_t, B_t), \tag{10}$$

a dynamic Euler equation and the budget constraint (1), now binding:

$$\dot{\mu}_t = \mu_t \left(\theta + \delta - r_t\right), \tag{11}$$

$$\dot{h}_t = \left(r_t - \delta\right) h_t + w_t l_t - c_t, \tag{12}$$

jointly with the transversality condition $\lim_{t\to\infty} e^{-\theta t}\mu_t h_t = 0$. μ_t *denotes the multiplier associated to the budget constraint.*

Proof. See the Appendix. ■

Applying the Implicit Function Theorem to the static relation $\mu_t = u_c\left(c_t, B_t\right)$, we obtain the consumption function $c_t \equiv c\left(\mu_t, B_t\right)$ with elasticities

$$\frac{\mu_t}{c_t}\frac{\partial c}{\partial \mu_t} = \frac{1}{\varepsilon_{cc}} < 0 \text{ and } \frac{B_t}{c_t}\frac{\partial c}{\partial B_t} = -\frac{\varepsilon_{cB}}{\varepsilon_{cc}}, \tag{13}$$

where ε_{cB} captures the effect of biodiversity on consumption demand. More precisely, when the household's preferences display complementarity (substitutability) between biodiversity and consumption, a higher biodiversity level entails a higher (a lower) consumption demand.

To sum up, biodiversity affects the households in two respects: (1) through the labor income (a lower biodiversity implies more persistent infectious diseases (dilution effect) lowering the labor supply and the labor income in turn); (2) directly through the utility function ($\varepsilon_{cB} \lesseqgtr 0$).

2.3 Production

A single firm[3] chooses the amount of capital (K_t) and labor (L_t) to maximize the profit taking as given the real interest rate r_t as well as the wage rate w_t. In addition, the government levies a proportional tax $\tau \in (0,1)$ on polluting production $F\left(K_t, L_t\right)$ to finance depollution expenditures. The following assumption sums up the properties of the production function.

Assumption 3 *The production function* $F : \mathbb{R}_+^2 \to \mathbb{R}_+$ *is* C^2*, homogeneous of degree one, strictly increasing and concave. Inada conditions hold.*

The profit maximization $\max_{K_t, L_t}\left[F\left(K_t, L_t\right) - r_t K_t - w_t L_t - \tau F\left(K_t, L_t\right)\right]$ entails the following first-order conditions:

$$r_t = (1-\tau)\,\rho\left(k_t\right) \text{ and } w_t = (1-\tau)\,\omega\left(k_t\right), \tag{14}$$

where $k_t \equiv K_t/L_t$ is the capital intensity and $f\left(k_t\right) \equiv F\left(k_t, 1\right)$ the average productivity, $\rho\left(k_t\right) \equiv f'\left(k_t\right)$ and $\omega\left(k_t\right) \equiv f\left(k_t\right) - k_t f'\left(k_t\right)$.

We introduce the capital share in total disposable income $\alpha\left(k_t\right) \equiv k_t f'\left(k_t\right)/f\left(k_t\right)$ and the elasticity of capital-labor substitution $\sigma\left(k_t\right) \equiv \alpha\left(k_t\right)\omega\left(k_t\right)/\left[k_t\omega'\left(k_t\right)\right]$. We obtain the elasticities of factor prices: $k_t\rho'\left(k_t\right)/\rho\left(k_t\right) = \left[\alpha\left(k_t\right) - 1\right]/\sigma\left(k_t\right)$ and $k_t\omega'\left(k_t\right)/\omega\left(k_t\right) = \alpha\left(k_t\right)/\sigma\left(k_t\right)$.

[3] Price-taker producers share the same technology. Because of the constant returns to scale, their individual profit maximization is equivalent to our aggregate profit maximization.

2.4 Government

The government uses all the tax revenues to finance depollution expenditures (G_t) according to a balanced budget rule:

$$G_t = \tau F(K_t, L_t). \tag{15}$$

Despite the simplicity of this tax scheme, we will see that tax rate τ has important environmental effects in the long run.

2.5 Biodiversity

To keep things as simple as possible, we assimilate the biodiversity to a renewable natural resource. Following Ayong Le Kama (2001) and Wirl (2004) the dynamics of natural resource is given by

$$\dot{B}_t = g(B_t) - P_t, \tag{16}$$

where $g(B_t)$ and P_t represent the reproduction function and the pollution level respectively. In the following, we will refer to (16) as reproduction function in a broad sense. In the spirit of Wirl (2004) and Bella (2010), we specify $g(B_t)$ as a Pearl-Verhulst logistic function: $g(B_t) \equiv B_t(1 - B_t)$. Clearly, if $B_t < 1$, then $g(B_t) > 0$ and $\dot{B}_t \lesseqgtr 0$ depending on the pollution level (P_t) while $B_t > 1$ implies $g(B_t) < 0$ and then $\dot{B}_t < 0$.

Interestingly, since $g'(B_t) = 1 - 2B_t$, the maximal sustainable yield occurs at $B_t = 1/2$. In other words, for every $B_t > 1/2$, g decreases, which means that B_t is bounded and can not tend to infinity. Interestingly, Wirl (2004) has pointed out that limit cycles can occur if and only if $B_t < 1/2$ (the maximal sustainable yield) at the steady state.

To simplify the presentation, we assume as in Itaya (2008) or in Fernandez et al. (2012) that pollution is a flow coming from production activity:

$$P_t = aY_t - bG_t, \tag{17}$$

where a and b capture respectively the environmental impact of production and the depollution efficacity.

Considering (15), (16) and (17), we find the natural resource accumulation law.

$$\dot{B}_t = B_t(1 - B_t) + (b\tau - a) F(K_t, L_t). \tag{18}$$

A non-negative net pollution requires an additional assumption.

Assumption 4 *$a > b\tau$.*

3 Equilibrium

Since $N_t = 1$, the natural resource accumulation law becomes $\dot{B}_t = B_t(1 - B_t) + (b\tau - a) l_t f(k_t)$. At the equilibrium, all the markets clear. This leads to the following proposition.

Proposition 3 *Dynamics are driven by a four-dimensional dynamic system:*

$$\dot{\mu}_t = \mu_t \left[\theta + \delta - (1-\tau)\,\rho(k_t)\right], \tag{19}$$

$$\dot{k}_t = \left[(1-\tau)\,\rho(k_t) - \delta\right] k_t + (1-\tau)\,\omega(k_t) - \frac{c(\mu_t, B_t)}{l_t} - k_t z(l_t, B_t), \tag{20}$$

$$\dot{l}_t = l_t z(l_t, B_t), \tag{21}$$

$$\dot{B}_t = B_t(1-B_t) + (b\tau - a)\, l_t f(k_t), \tag{22}$$

with $z(l_t, B_t) \equiv [\gamma(B_t) - \beta(B_t)\, l_t](1-l_t)/l_t$, *jointly with the transversality condition.*

Proof. See the Appendix. ■

(19)-(22) is a bioeconomic system. More precisely, equations (19) and (20) capture the economic part (the Ramsey model) while equations (21) and (22) represent respectively the epidemiological and the ecological part of the model. Figure 1 summarizes all relations between firms, households, biodiversity and the government.

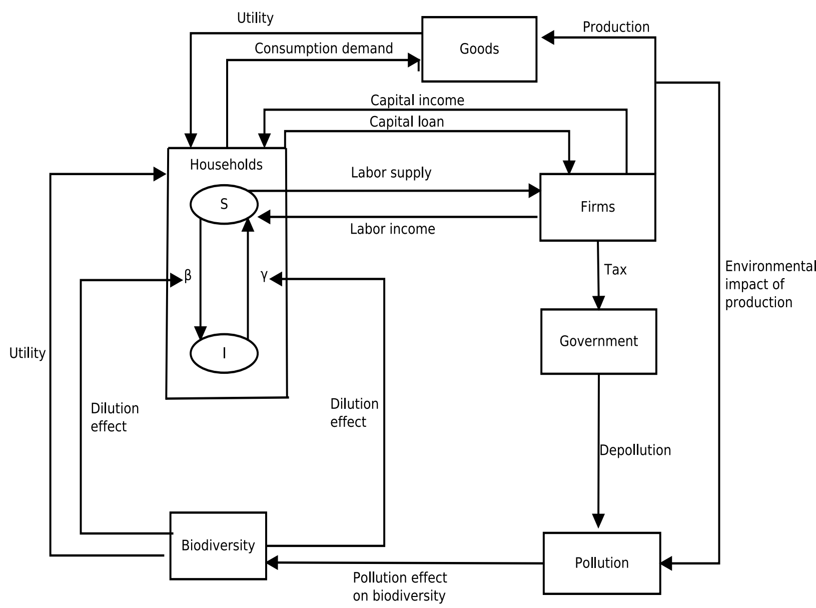

Fig.1: Model structure.

In the next sections, we will analyze the behavior of the dynamical system (19)-(22) to observe how the dilution effect affects the economy in the long-run (steady state) and in the short-run (local dynamics).

4 Steady state

At the steady state, all the variables remain constant: $\dot{\mu}_t = \dot{k}_t = \dot{l}_t = \dot{B}_t = 0$. From equation (19), we obtain the Modified Golden Rule (MGR):

$$\rho\left(k^*\right) = \frac{\theta + \delta}{1 - \tau}. \tag{23}$$

As usual, the MGR gives the capital intensity k^* at the steady state. Indeed, Assumption 3 ensures the invertibility of ρ. Thus, the capital intensity at the steady state is given by $k^* = \rho^{-1}\left(\left(\theta + \delta\right) / \left(1 - \tau\right)\right) > 0$. It is interesting to remark that both biodiversity and infectious disease have no effects on the capital intensity in the long run. Focus now on equation (21). At the steady state, $z\left(l, B\right) = 0$, that is, $\left(1 - l\right)\left[\gamma\left(B\right) - \beta\left(B\right) l\right] = 0$ with solutions, $l = 1$ or $l \equiv l^* = \gamma\left(B\right) / \beta\left(B\right)$. Hence, we recover one of the main feature of the SIS model; indeed two steady states coexist: $l = 1$ is the disease-free steady state, while $l = l^*$ is the steady state with an endemic disease.

Since $z\left(l, B\right) = 0$ at the steady state, equation (20) gives simply the consumption level:

$$c^* = l\left[\left(1 - \tau\right) f\left(k^*\right) - \delta k^*\right], \tag{24}$$

with $l = 1$ or $l = l^*$.

According to (24), we see that biodiversity and the infectious disease affect the consumption level in the long run when $l = l^*$, even if (23) shows that the capital intensity of steady state is not affected: because of the dilution effect, a decrease in biodiversity implies a reduction in labor supply entailing in turn a drop in consumption at the steady state.

We know that k^* is unique and positive at the steady state. Thus, given l, according to equation (24), there is a unique and positive value c^* of c at the steady state. Moreover, (10) implies that, given c^* and B, there is a unique and positive shadow price μ^*.

Finally, at the steady state, the natural resource accumulation law (22) becomes

$$B^2 - B + \left(a - b\tau\right) l f\left(k^*\right) = 0, \tag{25}$$

with $l = 1$ or $l = l^*$.

The existence and the uniqueness/multiplicity of the endemic steady state l^* (as well as of the disease-free steady state) depend upon the number of B satisfying equation (25). In the following, we compare the disease-free and the endemic regime.

4.1 Disease-free steady state

At the disease-free steady state, the disease no longer exists and all the labor force is employed, that is $l = 1$. In this case, equation (25) becomes:

$$g\left(B\right) \equiv B\left(1 - B\right) = \left(a - b\tau\right) f\left(k^*\right). \tag{26}$$

At the disease-free steady state, B satisfies equation (26). Under Assumption 4, $B \in (0,1)$ at the steady state. Moreover, given k^*, the environmental impact of production (a) drives the number of steady state. Indeed consider (26), when $a \to +\infty$, the RHS never crosses the LHS while, when $a \to 0^+$, the RHS crosses the LHS for two distinct values of B. Of course, multiple steady states arise because of the bell-shaped reproduction function $g(B)$.

Let us set a^* the threshold value of a:

$$a^* \equiv b\tau + \frac{1}{4f(k^*)},$$

The roots of equation (26) are simply given by:

$$B_1 \equiv \frac{1}{2}\left(1 - \sqrt{\Delta}\right), \tag{27}$$

$$B_2 \equiv \frac{1}{2}\left(1 + \sqrt{\Delta}\right), \tag{28}$$

where $\Delta \equiv 1 - 4(a - b\tau) f(k^*)$. Distinct biodiversity levels B_1 and B_2 exist if and only if $a < a^*$ (or, equivalently, $\Delta > 0$). In this case, $0 < B_1 < 1/2 < B_2 < 1$. The following proposition sums up these results.

Proposition 4 *Let $l = 1$.*
(1) If $a < a^*$, there are two steady states with $0 < B_1 < 1/2 < B_2 < 1$.
(2) If $a > a^*$, there are no steady states.
And, if $a = a^*$, $B_1 = B_2 = 1/2$.

a represents the environmental impact of production and reflects the human pressure on Nature. When the pressure exceeds the threshold ($a > a^*$), biodiversity fails to exist in the long run (there are no steady states). The dynamic properties of this case, where the disease does not persist in the long run ($l = 1$), has been studied by Bosi and Desmarchelier (2018c).

4.2 Endemic steady state

Focus on the endemic steady state[4] (that is, $l = l^* \equiv \gamma(B)/\beta(B)$). From (25), the stationary biodiversity level satisfies

$$\varphi(B) \equiv B(1-B)\frac{\beta(B)}{\gamma(B)} = (a - b\tau) f(k^*) > 0, \tag{29}$$

which implies $B \in (0,1)$.

For simplicity, in order to obtain a dilution effect independent of the endogenous biodiversity level B, focus on the following isoelastic functions:

$$\beta(B_t) \equiv A_\beta B_t^{\varepsilon_\beta} \text{ and } \gamma(B_t) \equiv A_\gamma B_t^{\varepsilon_\gamma}, \tag{30}$$

[4]Since γ and β are endogenously determined by the biodiversity level, nothing guarantees that $\gamma(B) < \beta(B)$. The following section studies the case where this inequality holds. In section 6, we provide a numerical example to show that this case is relevant.

with $A_\beta, A_\gamma > 0$, $\varepsilon_\beta < 0$ and $\varepsilon_\gamma > 0$ (see (6)). According to (5) and (29), we get $d = \varepsilon_\gamma - \varepsilon_\beta$ and

$$\varphi(B) = (1 - B) B^{1-d} \frac{A_\beta}{A_\gamma}. \tag{31}$$

The first-order elasticity of φ is given by

$$\frac{B\varphi'(B)}{\varphi(B)} = \frac{1 - 2B}{1 - B} - d. \tag{32}$$

Let

$$\bar{B} \equiv \frac{1 - d}{2 - d} \text{ and } \bar{a} \equiv b\tau + \frac{\varphi(\bar{B})}{f(k^*)}, \tag{33}$$

where k^* is the capital of Modified Golden Rule. When the dilution effect is low ($d < 1$), $\varphi'(B) > 0$ if and only if $B < \bar{B}$. Thus, when the dilution effect is low, φ is bell-shaped with $\varphi(0) = \varphi(1) = 0$ and a maximum at $B = \bar{B} \in (0, 1)$. According to (29), two steady states, say B_3 and B_4 with $B_3 < B_4$, exist if and only if

$$(a - b\tau) f(k^*) < \varphi(\bar{B}), \tag{34}$$

or, equivalently, if and only if $a < \bar{a}$. Clearly, $0 < B_3 < \bar{B} < B_4 < 1$. In other words, when the dilution effect is low, two steady state exist, if and only if the environmental impact of production is not too high ($a < \bar{a}$). Conversely, the steady state fails to exist when the environmental impact of production is excessive ($a > \bar{a}$). Of course, the steady state becomes unique ($B_3 = B_4 = \bar{B}$) when $a = \bar{a}$. Let us provide a formal statement.

Proposition 5 (small dilution effect) *Focus on the isoelastic case (30) with* $0 < d < 1$ *and the endemic steady state* $l = l^*$.

(1) If $a < \bar{a}$*, there are two steady states* B_3 *and* B_4 *such that* $0 < B_3 < \bar{B} < B_4 < 1$.

(2) If $a = \bar{a}$*, there is a unique steady state* $\bar{B}$.

(3) If $a > \bar{a}$*, there are no steady states.*

Thus, under a low dilution effect ($d < 1$), we recover in the endemic regime ($l = l^*$) the properties of the disease-free regime ($l = 1$) (Proposition 4): when human pressure on Nature becomes excessive ($a > a^*$), the steady state vanishes and the biodiversity disappears in the long run.

The parallel between these two regimes existing under a low dilution no longer holds when the dilution effect becomes high ($d > 1$). Indeed, according to (32) and (33), we find $\varphi'(B) < 0$ for any $B \in (0, 1)$ with $\lim_{B \to 0} \varphi(B) = +\infty$ and $\lim_{B \to 1} \varphi(B) = 0$. Hence, there is a unique solution of equation (29), say B_5.

Proposition 6 (large dilution effect) *Consider the endemic steady state* $l = l^*$ *in the isoelastic case (30) with* $d > 1$*. Then, there exists a unique steady state* $B_5 \in (0, 1)$.

How could we explain the different consequences of low and high dilution effects? As seen in Proposition (5), the steady state fails to exist under a low dilution effect ($d < 1$) if the environmental impact of production becomes excessive ($a > \bar{a}$). Production activities are so bad for environment that biodiversity no longer survives in the long run. Conversely, under a strong dilution effect ($d > 1$), a slight decrease of biodiversity has a large negative effect on labor supply and production in turn. Thus, a strong dilution effect works as a buffer and prevents the human pressure from being lethal for biodiversity in the long run. The dilution effect introduces a kind of complementarity between the level of economic activity and Nature: when $d > 1$, the complementarity becomes sufficiently powerful to preserve the biodiversity from a mass extinction. Since production affects biodiversity through a flow of pollution, we can reasonably expect that the effectiveness of a green tax rate (τ) largely depends on the magnitude of dilution.

4.3 Comparative statics

In this section we are interested in long-run impacts of the green-tax rate on both economic and biological variables at the endemic steady state ($l = l^*$). The next proposition sums up the results under low ($d < 1$) and high ($d > 1$) dilution effect.

Proposition 7 *Let Assumption 4 hold and $B \in (0, 1)$. Focus on the qualitative impact of τ on the endemic steady state ($l = \gamma/\beta$). We have*

$$\frac{\tau}{k^*}\frac{dk^*}{d\tau} < 0. \tag{35}$$

Moreover,
(1) if $0 < d < 1$,

$$\frac{\tau}{B_3}\frac{dB_3}{d\tau} < 0 \text{ and } \frac{\tau}{B_4}\frac{dB_4}{d\tau} > 0,$$
$$\frac{\tau}{l^*}\frac{dl^*}{d\tau} < 0 \text{ if } B = B_3, \text{ and } \frac{\tau}{l^*}\frac{dl^*}{d\tau} > 0 \text{ if } B = B_4,$$
$$\frac{\tau}{\mu^*}\frac{d\mu^*}{d\tau} > 0 \text{ if } (B = B_3 \text{ and } d > -\frac{\varepsilon_{cB}}{\varepsilon_{cc}}) \text{ or } (B = B_4 \text{ and } d < -\frac{\varepsilon_{cB}}{\varepsilon_{cc}}),$$

(2) if $d > 1$,

$$\frac{\tau}{B_5}\frac{dB_5}{d\tau} > 0,$$
$$\frac{\tau}{l^*}\frac{dl^*}{d\tau} > 0,$$
$$\frac{\tau}{\mu^*}\frac{d\mu^*}{d\tau} > 0 \text{ if } d < -\frac{\varepsilon_{cB}}{\varepsilon_{cc}}.$$

Proof. See the Appendix. ■

Interestingly, the critical value of the dilution effect remains 1 as in Propositions 5 and 6. Proposition 7 deserves some economic interpretations.

The negative impact of τ on k^* is far from surprising. Indeed, since τ is levied on the production level, a higher green-tax rate reduces production and income, entailing a lower capital level at the end. Conversely, the effect of τ on biodiversity is ambiguous and depends on two key elements: the shape of the reproduction function and the magnitude of the dilution effect. First, let us consider the case of a low dilution effect ($0 < d < 1$) and remark that, since a higher green tax always reduces the capital level, it also lowers the right-hand side of (29). That is, if at the steady state, the economy is located along the upward-sloping branch of φ (that is $B = B_3$), a higher green tax always lowers the biodiversity level. Conversely, if the economy is located on the downward-sloping branch of φ (that is $B = B_4$), a higher tax level lowers the right-hand side of (29) and hence, increases the biodiversity level at the steady state. The ambiguous effect of the green tax on the biodiversity reveals that the shape of the reproduction function matters when the dilution effect is low ($0 < d < 1$). Furthermore, the fact that a higher green-tax rate impairs the biodiversity level (when $B < \bar{B}$) is counter-intuitive and refers to the static green paradox introduced in Bosi and Desmarchelier (2017 and 2018c).

The impact of the green tax on the labor supply mimics that on the biodiversity level because of Assumption 1. Indeed, because of the dilution effect, a lower biodiversity level implies a more infective disease (a higher β jointly with a lower γ) which reduces the labor supply ($l = \gamma/\beta$).

Now, consider the case of a higher dilution effect ($d > 1$). In this case, φ is always a decreasing function of B (see the proof of Proposition 6). Hence, a higher tax rate lowers the right-hand side of (29) which always leads to a higher biodiversity level.

Summing up, we observe that the green paradox (that is the negative impact of the green tax on the biodiversity level) only occurs for low levels of biodiversity ($B < \bar{B}$) and dilution effect ($0 < d < 1$).

It is worthy to notice that when the biodiversity becomes a prominent determinant of human health because of a sufficiently large dilution effect ($d > 1$), the unpleasant (static) green paradox is ruled out. Moreover, in the previous section, we have observed that the complementarity between economic activities (production) and Nature entails a biodiversity preservation in the long run through a large dilution effect. It follows that a large dilution effect ($d > 1$) has a double benefit: (1) it preserves the biodiversity in the long run and (2) it rules out the green paradox.

μ is a shadow price (marginal utility of consumption: $\mu = u_c(c, B)$). From an economic point of view, it is more interesting to characterize the impact of the green tax on consumption than on this unobservable variable.

We observe that

$$c^*(\tau) = \frac{\theta + [1 - \alpha(k^*(\tau))]\,\delta}{\alpha(k^*(\tau))} k^*(\tau)\, l^*(\tau)\,. \tag{36}$$

In order to provide a clear-cut comparative statics, we focus on the Cobb-Douglas case. In this case, the capital share $\alpha(k^*)$ becomes a constant and (36) entails

$$\frac{\tau}{c^*}\frac{\partial c^*}{\partial \tau} = \frac{\tau}{k^*}\frac{\partial k^*}{\partial \tau} + \frac{\tau}{l^*}\frac{\partial l^*}{\partial \tau}. \tag{37}$$

Therefore, the impact of the green tax on consumption can be disentangled in its effects on the production factors.

Proposition 8 *Consider the endemic steady state* $l = \gamma/\beta$ *with* $a = b$ *and* $\sigma = 1$ *(Cobb-Douglas technology).*

(1) If $0 < d < 1$ *(low dilution effect),*

$$\frac{\partial c^*}{\partial \tau} > 0 \textit{ iff } B_4 < \frac{1}{2}.$$

(2) If $d > 1$ *(high dilution effect),*

$$\frac{\partial c^*}{\partial \tau} > 0 \textit{ iff } B_5 < 1/2.$$

Proof. See the Appendix. ■

The last proposition shows that the effect of τ on the consumption demand is ambiguous in the long run. This deserves some economic interpretations.

Consider the case of a low dilution effect ($0 < d < 1$) and suppose, for simplicity, no capital depreciation. As seen before, a higher green-tax rate always lowers the capital level (see (35)). In addition, since the economy is located along the increasing branch of the reproduction function ($B = B_3$), this implies a drop in the biodiversity level, rendering the disease more infective, which lowers the labor supply. Focus on expression (24): $c^* = l^*(1-\tau) f(k^*)$. Since τ increases and k^* and l^* decrease, c^* decreases.

Now, assume that the economy is located along the decreasing branch of the reproduction function ($B = B_4$). In this case, a higher green-tax rate implies more biodiversity making the disease less infective (dilution effect) and raising the labor supply. In contrast, as above, the capital intensity lowers. Then, l^* increases, while $(1-\tau) f(k^*)$ decreases. The total impact of τ on $c^* = l^*(1-\tau) f(k^*)$ is ambiguous. In order to know whether the increase in the labor supply dominates the decrease of $(1-\tau) f(k^*)$, we have to focus on the elasticity (32) of the reproduction function φ. $B = B_4$ jointly with $0 < d < 1$ implies $\varphi'(B) < 0$. The slope of φ becomes flatter when $B_4 < 1/2$ and steeper when $B_4 > 1/2$. In other words, an increase in the green-tax rate has a larger effect on biodiversity and labor supply when $B_4 < 1/2$. Therefore, since $B_4 > \bar{B}$, if $B_4 < 1/2$, the increase in labor supply l^* dominates the drop of $(1-\tau) f(k^*)$ which implies a higher consumption level in the long run. Conversely, if $B_4 > 1/2$, the increase in labor income is dominated and consumption lowers in the long run.

Similar interpretations hold in the case of a strong dilution effect ($d > 1$).

5 Local dynamics around the endemic steady state

As seen above, the disease-free regime is the same as the one considered by Bosi and Desmarchelier (2018c). Then, the novelty is the study of the endemic regime since Bosi and Desmarchelier (2018c) do not consider the spread of an infectious disease. In their paper, the authors point out that a limit cycle (Hopf bifurcation) can occur around the steady state with the highest biodiversity level (denoted by B_2 in the current paper) since biodiversity affects the marginal utility of consumption. As seen above, a kind of relationship seems to exist between the disease-free and the endemic regimes when the dilution effect is low ($d < 1$). More precisely, in these two cases, under a low environmental impact of production, two steady states exist (Propositions 4 and 5): the one with a low biodiversity level (B_1 and B_3 respectively for the disease-free regime and for the endemic one) and the other with a high biodiversity level (B_2 and B_4 respectively for the disease-free regime and for the endemic one). Moreover, when the environmental impact of production becomes excessive, we have observed that the two steady states collide and disappear (B_1 and B_2 for the disease-free regime and B_3 and B_4 for the endemic regime). Because of this relationship, we expect to recover around B_4 the limit cycle surrounding B_2 pointed out by Bosi and Desmarchelier (2018c). Nevertheless, we have observed earlier that the situation becomes very different under a large dilution effect ($d > 1$). Indeed, the complementarity between production activities and biodiversity induced by a strong dilution effect ensures that there exist always a unique positive steady state (Proposition 6): biodiversity is always preserved in the long run.

Therefore, two questions arise: (1) is the limit cycle surrounding B_2 (see Bosi and Desmarchelier, 2018c) preserved around B_4?; (2) is the complementarity between production activities and biodiversity induced by a large dilution effect able to prevent the existence of a limit cycle around B_5? The current section will focus on these two questions.

To study the equilibrium transition, we linearize the dynamic system (19)-(22) around the endemic steady state $l = l^*$. Noticing that $\omega(k)/[k\rho(k)] = [1-\alpha(k)]/\alpha(k)$, we get the Jacobian matrix:

$$J = \begin{bmatrix} 0 & n\frac{\mu^*}{k^*} & 0 & 0 \\ -\frac{q}{\varepsilon_{cc}}\frac{k^*}{\mu^*} & \theta & (m+q)\frac{k^*}{l^*} & \frac{k^*}{B}\left(q\frac{\varepsilon_{cB}}{\varepsilon_{cc}} - md\right) \\ 0 & 0 & -m & \frac{l}{B}md \\ 0 & \alpha\frac{B}{k^*}(B-1) & \frac{B}{l^*}(B-1) & 1-2B \end{bmatrix},$$

where $q \equiv [\theta + (1-\alpha)\delta]/\alpha = c^*/(k^* l^*)$, $m \equiv \beta(1-l^*)$ and $n \equiv (\theta+\delta)(1-\alpha)/\sigma$. To study the local dynamics of this four-dimensional system, we apply the methodology developed by Bosi and Desmarchelier (2019) and based on the sums of principal minors of the Jacobian. The characteristic polynomial is given by $P(\lambda) \equiv \lambda^4 - T\lambda^3 + S_2\lambda^2 - S_3\lambda + D$ where

$$S_1 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = T, \quad (38)$$
$$S_2 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4, \quad (39)$$
$$S_3 = \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4 + \lambda_1\lambda_2\lambda_3, \quad (40)$$
$$S_4 = \lambda_1\lambda_2\lambda_3\lambda_4 = D, \quad (41)$$

T and D denote the trace and the determinant of J while S_2 and S_3 represent the sum of principal minors of order two and three.

In our model,

$$\begin{aligned} T &= \theta - m - 2B + 1, \\ S_2 &= [2B - 1 + (1 - B)\, d]\, m + (1 - 2B)\,\theta - (1 - B)\, dm\alpha + q\alpha\frac{\varepsilon_{cB}}{\varepsilon_{cc}}(1 - B) - m\theta + \frac{nq}{\varepsilon_{cc}}, \\ S_3 &= \frac{1}{\varepsilon_{cc}}\left[(1 - 2B - m)\, nq + (B - 1)\, mq\alpha\varepsilon_{cB}\right] + (B - 1)\, dmq\alpha + [2B - 1 + (1 - B)\, d]\, m\theta, \\ D &= \frac{mnq}{\varepsilon_{cc}}[2B - 1 + (1 - B)\, d]\,. \end{aligned} \quad (42)$$

According to Bosi and Desmarchelier (2018c), a Hopf bifurcation can arise around the higher disease-free steady state B_2 because $\varepsilon_{cB} \neq 0$. To characterize the occurrence of limit cycles in our model, we introduce, for simplicity, a isoelastic utility function:

$$u(c_t, B_t) = \frac{\left(c_t B_t^{\eta}\right)^{1-\varepsilon}}{1 - \varepsilon}. \quad (43)$$

The functional form (43) is widely used in economics.[5] $1/\varepsilon > 0$ is the intertemporal elasticity of substitution of the composite good $c_t B_t^{\eta}$ while $\eta > 0$ represents the weight of biodiversity in the household's utility. Interestingly, in this case, the elasticities of preferences (8) are invariant to the steady state:

$$\begin{bmatrix} \varepsilon_{cc} & \varepsilon_{cB} \\ \varepsilon_{Bc} & \varepsilon_{BB} \end{bmatrix} = \begin{bmatrix} -\varepsilon & \eta(1 - \varepsilon) \\ 1 - \varepsilon & \eta(1 - \varepsilon) - 1 \end{bmatrix}.$$

We observe that $\varepsilon_{cB} > 0$ (< 0) if and only if $\varepsilon < 1$ (> 1)**.** Let $\varepsilon_1 \equiv \varepsilon_{cc} = -\varepsilon < 0$, $\varepsilon_2 \equiv \varepsilon_{cB} = \eta(1 - \varepsilon)$ and

$$\varepsilon_H \equiv \varepsilon_1 \frac{z_3 - T\frac{Z + \sqrt{Z^2 - 4mD(m+T)}}{2(m+T)}}{m\alpha q(1 - B)}, \quad (44)$$

with

$$\begin{aligned} z_3 &\equiv qm\alpha d(B - 1) + (T - \theta)\, nq/\varepsilon_1 + \theta D\varepsilon_1/(nq)\,, \\ Z &\equiv m\left[m\alpha d(B - 1) + nq/\varepsilon_1 + D\varepsilon_1/(nq) + (T - \theta)\,\theta\right] + z_3. \end{aligned}$$

The following proposition characterizes the occurrence of a Hopf bifurcation under a low $(d < 1)$ and large $(d > 1)$ dilution effect.

[5]See d'Autume et al. (2010) or Bosi and Desmarchelier (2018c) among others.

Proposition 9 *Let $B < (1+\theta)/2$. If*
(1) $0 < d < 1$ (low dilution effect) and $B = B_4$ (large biodiversity) or
(2) $d > 1$ (large dilution effect),
then a limit cycle occur near the endemic steady state if and only if $\varepsilon_2 = \varepsilon_H$.

Proof. See the Appendix. ■

Proposition 9 shows the possibility of limit cycle around the endemic steady state with the higher biodiversity (B_4) in the spirit of Bosi and Desmarchelier (2018c) where the limit cycle surrounds the (disease-free) steady state with the higher biodiversity (B_2). The only difference between B_2 and B_4 is the existence of a (low) dilution effect. Qualitatively, nonlinear dynamics under a low dilution effect ($d < 1$) look like those with no dilution effect. However, now, the limit cycle is preserved also when the dilution effect becomes large ($d > 1$). Even if, as seen above, a sufficiently large dilution effect entails the preservation of biodiversity in the long run, it does not shelter the bioeconomic system from the existence of (limit) cycles.

To understand the preservation limit cycles under a large dilution effect ($d > 1$), we have to consider the sign of ε_H. Since $\varepsilon_{cB} = \varepsilon_H$ at the Hopf bifurcation point, we assume a positive cross elasticity $\varepsilon_{cB} > 0$ (complementarity between consumption and biodiversity) to avoid the unpleasant and meaningless case $\varepsilon_H < 0$. In the next section, to convince the reader, we will provide a numerical simulation with a limit cycle around B_4 with $\varepsilon_H > 0$ (that is a large elasticity of intertemporal substitution: $1/\varepsilon > 1$).

Now, let the economy be at the steady state today and assume an exogenous rise in the pollution level. (16) implies a drop in biodiversity with two consequences: (1) a lower labor supply ($l_t = \gamma(B_t)/\beta(B_t)$ decreases: dilution effect) and (2) a lower consumption demand due to complementarity ($\mu_t = u_c(c_t, B_t)$ decreases). The Euler equation (intertemporal consumption smoothing) implies $\dot{\mu}_t/\mu_t = \theta + \delta - (1-\tau)\rho(k_t) < 0$. Thus, $\rho(k_t)$ increases from the Modified Golden Rule $(\theta+\delta)/(1-\tau)$ today to the new transition value tomorrow and, since ρ is a decreasing function, the capital intensity k_t lowers and the average productivity $f(k_t)$ as well. Pollution is given by $P_t = (a - b\tau) l_t f(k_t)$. Hence, under Assumption 4, the drops in labor supply l_t and in productivity $f(k_t)$ entail a lower pollution level and hence a higher biodiversity level. Thus, a higher pollution today entails a weaker pollution tomorrow giving rise to an endogenous fluctuation. According to this interpretation, the dilution effect (a biodiversity loss implies less labor supply) amplifies the drop of production induced by the drop of consumption (because of the complementarity: $\varepsilon_{cB} < 0$) at the origin of the endogenous fluctuation. In other terms, a larger dilution effect magnifies the cycle instead of preventing it.

The existence of a limit cycle makes sense from a biological point of view. The biodiversity level fluctuates over time: a period with a larger biodiversity can be followed by another age with lower biodiversity and so on. This is reminiscent of the five recurrent mass extinctions experienced by planet Earth in the past 540 million years (Barnosky et al., 2011).

6 Simulations

The analytical study of local dynamics has revealed the existence of limit cycles under either a low or a high dilution effect. Now, let us convince the reader through a numerical illustration. This simulation will convince her not only about the occurrence of cycles but also about the existence of more sophisticated dynamics (of codimension two) such as the Bogdanov-Takens bifurcation generated by the system (19)-(22).

Since the Bogdanov-Takens bifurcation arises only under a low dilution effect, for brevity's sake, we will focus only on the case of a low dilution effect. More precisely, we will consider numerically the dynamics around the steady state with a higher biodiversity (μ^*, k^*, l^*, B_4).

We reconsider the isoelastic utility (43) and the isoelastic functions (30). For simplicity, we use also a Cobb-Douglas production function: $f(k_t) = Ak_t^\alpha$. According to the MGR (23), we find the stationary capital level:

$$k^* = \left[\frac{\alpha A(1-\tau)}{\theta+\delta}\right]^{\frac{1}{1-\alpha}}.$$

Replacing k^* in (29), we obtain the biodiversity level as solution of

$$(1-B)B^{1-d} = A(a-b\tau)\frac{A_\gamma}{A_\beta}k^{*\alpha}. \tag{45}$$

According to Proposition 5, equation (45) possesses two solutions if and only if

$$a < \bar{a} = b\tau + \frac{A_\beta}{A_\gamma}\frac{(1-\bar{B})\bar{B}^{1-d}}{Ak^{*\alpha}},$$

where $d \equiv \varepsilon_\gamma - \varepsilon_\beta > 0$.

Consider now the calibration in Table 1.

α, δ and θ are set at their usual quarterly values[6] while b and τ[7] are fixed as in Bosi and Desmarchelier (2018a). According to Table 1, we observe that $\varepsilon_H > 0$ since $\varepsilon < 1$.[8] That is, we have fixed ε to ensure that $\varepsilon < 1$. In addition, as explaining before, we focus on the low dilution effect case ($d < 1$). By fixing $\varepsilon_\gamma = -\varepsilon_\beta = 0.25$ we ensure that $d \equiv \varepsilon_\gamma - \varepsilon_\beta = 1/2 < 1$. Finally, since there are no critical values concerning the scaling parameters A, A_β and A_γ for the existence of the Hopf bifurcation (see Proposition 9), we choose to fix each of them equal to the unity.

[6] See for instance Nourry et al. (2013).

[7] In this economy, τ captures the public air protection expenditures. Indeed $G/Y = \tau Y/Y = \tau = 0.2\%$. According to the OECD Environmental Performance reviews for France (2016) (p. 149), the public air protection expenditures amount to less than 5 billions of euros (2013 prices), which represents less than 0.25% of France GDP. Then, our calibration for τ is in accordance with data.

[8] In economic terms, $\varepsilon < 1$ means that consumption and biodiversity are complements in the household's preferences. As explained in the previous section, such a complementarity can generate a limit cycle through a Hopf bifurcation, that is fluctuations of biodiversity.

parameter	**symbol**	**value**
total factor productivity	A	1
capital share in total income	α	0.33
capital depreciation rate	δ	0.025
rate of time preference	θ	0.01
inverse of elast. of intertemp. subst.	ε	0.5
abatement efficiency	b	0.0015
green-tax rate	τ	0.002
disease transmission rate without dilution effect	A_γ	1
disease recovery rate without dilution effect	A_β	1
disease transmission rate sensitivity w.r.t. biodiversity	ε_γ	0.25
disease recovery rate sensitivity w.r.t. biodiversity	ε_β	-0.25

Table 1: Parameter values.

The calibration provided in Table 1 yields $\bar{a} = 0.1276$. We fix $a = 0.127 < \bar{a}$ and we solve (45) for B. As expected (see Proposition 5), we obtain two roots: $B_3 = 0.2968$ and $B_4 = 0.3713$ (see Proposition 5).We observe that the necessary (but not sufficient) condition for the occurrence of a Hopf bifurcation in Proposition 9 is satisfied: $\bar{B} = 0.333 < B_4 = 0.3713 < (1+\theta)/2 = 0.505$.

Focusing on $B_4 = 0.3713$, we compute η such that $\varepsilon_2 = \eta(1-\varepsilon) = \varepsilon_H$. We obtain $\eta_H = 0.27569 > 0$. Therefore, under this calibration (Table 1), when $a = 0.127$, the system undergoes a Hopf bifurcation at η_H and experiences a limit cycle near $B_4 = 0.3713$.

Summing up, under the calibration of Table 1, we get a Hopf bifurcation at $a = 0.127$ and a saddle-node bifurcation at $a = 0.1276$ near the higher endemic steady state ($B = B_4$ with $l = \gamma/\beta$ and $0 < d < 1$).

After having seen how to calibrate the model to find a Hopf bifurcation, we deepen our approach considering an equilibrium continuation.[9] We aim to plot the Hopf bifurcation curve and the saddle-node bifurcation curve in the (a, η)-plane and to show the occurrence of the Bogdanov-Takens bifurcation when these bifurcation curves meet each others. We will refer to Figure 2 where LP, H, BT and GH stand for Limit Point (elementary saddle-node), Hopf, Bogdanov-Takens and Generalized Hopf. These points are computed and represented by MATCONT when the corresponding bifurcations occur near the steady state.

To perform the equilibrium continuation using MATCONT, we consider first the bifurcation of codimension one (Hopf and the saddle-node). We fix $\eta = 0.27569$ as above and we set an arbitrary value for a at which no bifurcation occurs near the endemic steady state: $a = 0.1268 \equiv a_0$. In this case, according to Table 1, the endemic steady state becomes

$$(\mu^*, k^*, l^*, B_4) = (0.7354, 28.385671, 0.61423845, 0.37728887).$$

[9]To this purpose, we use the MATCONT package for MATLAB.

Let MATCONT raise a from $a_0 \equiv 0.1268$ to $\bar{a} = 0.1276$ keeping $\eta = 0.27569$ as constant. In Figure 2, we are moving to the right along the horizontal line HLP. MATCONT detects a Hopf bifurcation (H) at $a = a_H = 0.127$ and a saddle-node bifurcation (LP) at $a = \bar{a}$.

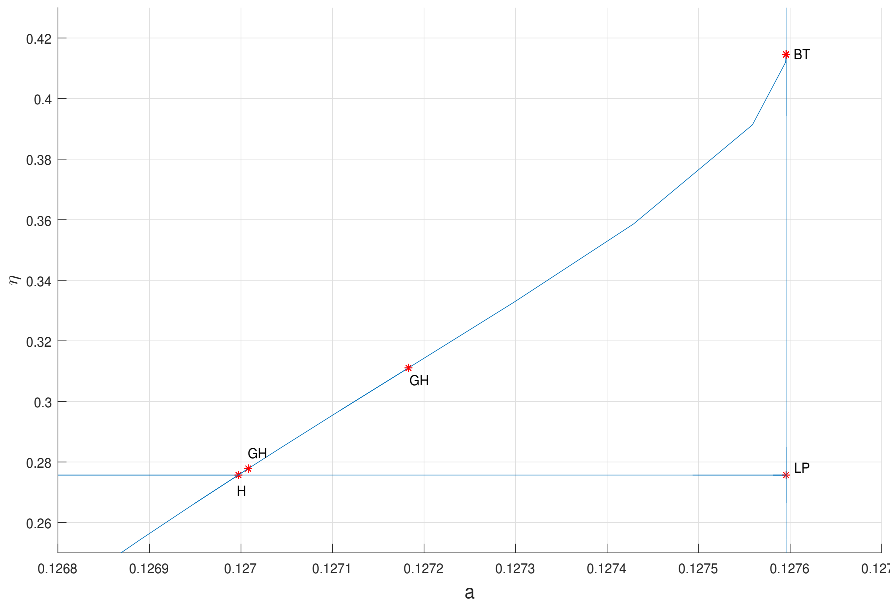

Fig 2: The equilibrium continuation.

Now, let us move from H to BT along the locus of all the Hopf bifurcations: for any a, there is a Hopf critical value

$$\eta_H(a) \equiv -\frac{\varepsilon}{1-\varepsilon} \frac{z_3(a) - T(a)\frac{Z(a)+\sqrt{Z(a)^2-4m(a)D(a)[m(a)+T(a)]}}{2[m(a)+T(a)]}}{m(a)\alpha q[1-B(a)]}.$$

The Hopf-bifurcation curve $\{(a, \eta_H(a))\}$ is precisely represented in Figure 2 by the curve HBT.

For any η, the elementary saddle-node bifurcation value for a is $\bar{a} = 0.1276$ (the line $LPBT$ is vertical because $\bar{a}$ does not depend on η). In particular, the Limit Point corresponding to $\eta = 0.275\,69$ is $LP = (0.1276, 0.275\,69)$. The vertical line $LPBT$ represents a third equilibrium continuation, the set of all the pairs $(a, \eta) = (\bar{a}, \eta)$ for which a saddle-node bifurcation occurs.

To sum up, increasing a from $a_0 = 0.1268$ to $\bar{a} = 0.1276$, we obtain all the Hopf bifurcations along the curve $HBT \equiv \{(a, \eta_H(a))\}_{a \in [a_0, \bar{a}]}$ going from H to BT. Moreover, in the range $[a_0, \bar{a}) \ni a$, we find two distinct steady states. When a attains the maximal value $\bar{a}$ these two steady states coalesce and the

Hopf bifurcation point $(a, \eta_H(a))$ reaches the ending point BT along the curve HBT while the economy experiences a Bogdanov-Takens bifurcation. Indeed, a Bogdanov-Takens bifurcation generically arises when a Hopf bifurcation curve crosses a locus of saddle-node bifurcations.

Along the locus of Hopf bifurcations, two Generalized Hopf (Bautin) bifurcations also appear. A Hopf bifurcation can be subcritical or supercritical, leading respectively to an unstable or stable limit cycle. The Generalized Hopf bifurcation point implies a change in the stability of the limit cycle arising near the steady state, that is, the bifurcation from subcritical becomes supercritical or viceversa. If the first Lyapunov coefficient (l_1) is negative (positive), the bifurcation is said to be supercritical (subcritical), leading to a stable (unstable) limit cycle near the steady state. At the Generalized Hopf bifurcation point, l_1 vanishes.

Let us explain the relation between a double-Hopf and a generalized Hopf bifurcation.[10] At the double-Hopf bifurcation, two limit cycles emerge simultaneously. The interaction between these two limit cycles can produce a wide range of dynamics depending upon higher-order terms of the Taylor series, such as a torus or local chaos. Concerning the Jacobian matrix, a double pair of purely imaginary eigenvalues appears at the double-Hopf bifurcation. In the case of a generalized Hopf bifurcation, the arising limit cycle is unique, as for a standard Hopf bifurcation: the Jacobian possesses a single pair of purely imaginary eigenvalues. The distinction between a standard or a generalized Hopf bifurcation rests on the value of the first-order Lyapunov coefficient and, thus, on the higher-order terms of the Taylor representation of the dynamical system. Indeed, at the generalized Hopf bifurcation, the first Lyapunov coefficient is equal to zero which means a change of stability for the limit cycle.

At the Hopf bifurcation point (H), the steady state is given by:

$$(\mu^*, k^*, l^*, B_4) = (0.736723, 28.38567, 0.609338, 0.371292),$$

with eigenvalues:

$$\lambda_1 = -0.34187,\ \lambda_2 = 0.108813 \text{ and } \lambda_3 = 0.0539646i = -\lambda_4.$$

In order to visualize the limit cycle arising at the Hopf bifurcation point, we project the four-dimensional dynamics on a three-dimensional space. Since the shadow price μ is not a directly observable variable, we prefer to represent the trajectory in the (k_t, l_t, B_t)-space.

The corresponding first Lyapunov coefficient is given by $l_1 = 6.182045 * 10^{-5} > 0$. Its positivity means that the Hopf bifurcation is subcritical, that is

[10] The reader is referred to pages 307 and 349 in Kuznetsov (1998) for the generalized Hopf bifurcation and the double-Hopf bifurcation respectively.

the limit cycle arising near the steady state is unstable (Figure 3).

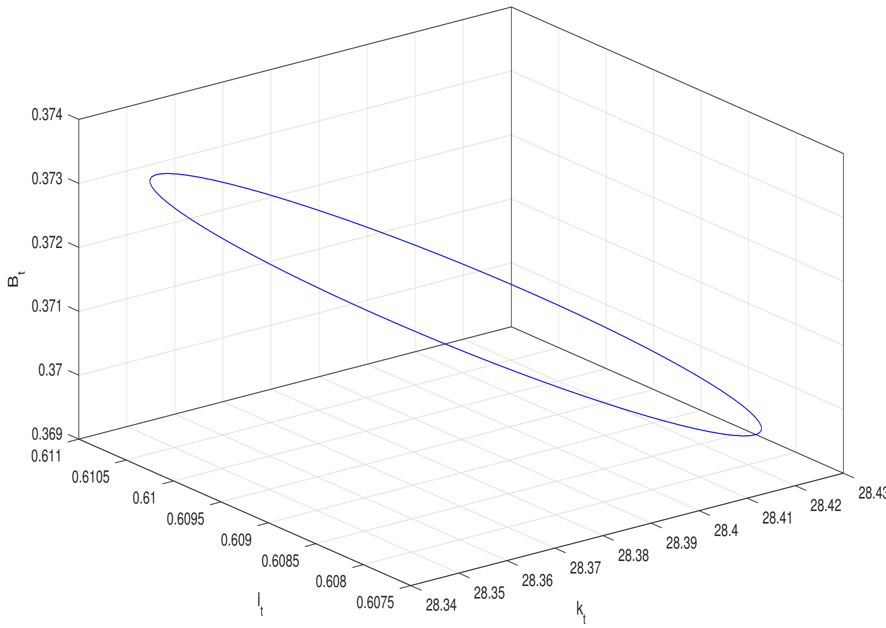

Fig. 3: The unstable limit cycle.

At the saddle-node bifurcation (LP), the steady state becomes:

$$(\mu^*, k^*, l^*, B_4) = (0.745\,69, 28.38567, 0.577347, 0.333333)\,,$$

with eigenvalues:

$$\lambda_1 = -0.401245,\ \lambda_2 = 0,\ \lambda_3 = 0.0204607 \text{ and } \lambda_4 = 0.16789.$$

At the Bogdanov-Takens bifurcation (BT), when $a = \bar{a} = 0.1276$ jointly with $\eta = \eta_{BT} = 0.414576$, the steady state becomes:

$$(\mu^*, k^*, l^*, B_4) = (0.690915, 28.385671, 0.577350, 0.333333)\,,$$

with eigenvalues:

$$\lambda_1 = -0.399\,61,\ \lambda_2 = \lambda_3 = 0 \text{ and } \lambda_4 = 0.186\,71.$$

The Bogdanov-Takens bifurcation occurs when conditions for the elementary saddle-node bifurcation and for the Hopf bifurcation meet each other.

As in Kuznetsov et al. (2014), at the Bogdanov-Takens point, the orbit describes a parasitic loop near the saddle-point (Figure 4). The parasitic loop typically arises when the limit cycle and the saddle-point coalesce.

As above, to represent the trajectory, we project the four-dimensional dynamics on the three-dimensional (μ_t, l_t, B_t)-space, where the parasitic loop appears (Figure 4).

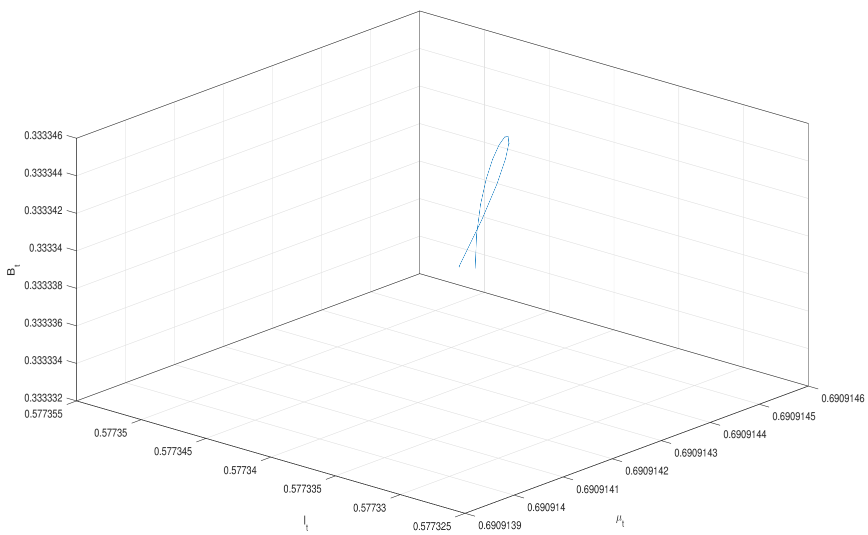

Fig. 4: The parasitic loop.

At the Generalized-Hopf bifurcations (GH), we obtain:

parameters	steady state	eigenvalues	l_2
$a = 0.12700794$ $\eta = 0.2778149$	$\mu^* = 0.73602393$ $k^* = 28.38567$ $l^* = 0.60905296$ $B_4 = 0.37094551$	$\lambda_1 = -0.342374$ $\lambda_2 = 0.109537$ $\lambda_3 = -0.0535261i$ $\lambda_4 = 0.0535261i$	$2.61837 * 10^{-3}$
$a = 0.12718298$ $\eta = 0.31106998$	$\mu^* = 0.7251378$ $k^* = 28.38567$ $l^* = 0.60395327$ $B_4 = 0.36475955$	$\lambda_1 = -0.351468$ $\lambda_2 = 0.122331$ $\lambda_3 = -0.0460886i$ $\lambda_4 = 0.0460886i$	$1.744669 * 10^{-2}$

A Generalized-Hopf bifurcation implies a change in the stability of the limit cycle arising through the Hopf bifurcation. Typically, such a bifurcation occurs when the first Lyapunov coefficient vanishes. This phenomenon can not be detected through a simple analysis of the eigenvalues. According to Kuznetsov (1998), a GH bifurcation is non-degenerated bifurcation if the second-order Lyapunov coefficient is different from zero ($l_2 \neq 0$). It is the case under our calibration for both the GH bifurcations.

7 Conclusion

We have provided a unified framework at the crossroad of economics, ecology and epidemiology, and studied how the negative relation between biodiversity and disease transmission (the so-called dilution effect) affects the economy in the long and the short run. More precisely, we have embedded a SIS model into a Ramsey model where a pollution externality coming from production impairs a biodiversity measure. For the sake of simplicity, we have assimilated biodiversity to a renewable resource and introduced a two-sided dilution effect assuming that both the probability to become ill and the recovery rate from the infectious disease depend on the biodiversity level. To complete the model, we have considered a proportional tax levied on production at the firm level to finance depollution.

In long run, we have recovered a standard feature of the SIS model: a disease-free regime coexists with an endemic one. In the endemic case, the number of steady states depends on both the magnitude of the dilution effect and on the environmental impact of production. More precisely, under a low dilution effect, two cases are possible: (1) two steady states coexist if the environmental impact of production is low (the one with low biodiversity and the other with high biodiversity); (2) there is no steady state if the environmental impact of production is large enough. In case (2), biodiversity is not preserved in the long run because of the excessive human pressure on Nature. When the dilution effect is high, the bioeconomic system works very differently: there is always a unique steady state whatever the environmental impact of production. In this case, the complementarity between economy and biodiversity induced by a strong dilution effect prevents human pressure from becoming lethal for Nature. In addition, we have highlighted a kind of green paradox in the endemic regime: under a low dilution effect, a higher green-tax rate always impairs biodiversity at the low steady state. This counter-intuitive result is comparable to the static green paradox considered in Bosi and Desmarchelier (2017). Conversely, the green paradox is over under a large dilution effect.

In the short run, limit cycles can arise under both the low and the high dilution effect through a Hopf bifurcation near the steady state. This happens in the endemic case when preferences exhibit a complementarity between biodiversity and consumption.

To sum up, a large dilution effect seems to imply a double benefit: (1) it preserves biodiversity in the long run and (2) it prevents the economy from the green paradox. However, a large dilution effect can not shelter the economy from unpleasant biodiversity fluctuations shaped as a limit cycle around the steady state when the household's preferences exhibit complementarity between biodiversity and consumption.

8 Appendix

Proof of Proposition 2

The agent maximizes the intertemporal utility function (9) under the budget constraint (7). Setting the Hamiltonian $H_t = e^{-\theta t} u(c_t, B_t) + \lambda_t [(r_t - \delta) h_t + \bar{\omega}_t - c_t]$, deriving the first-order conditions $\partial H_t / \partial c_t = 0$, $\partial H_t / \partial h_t = -\dot{\lambda}_t$ and $\partial H_t / \partial \mu_t = \dot{h}_t$, and defining $\mu_t \equiv \lambda_t e^{\theta t}$, we get (10), (11) and (12). ■

Proof of Proposition 3

Consider (4), (14), (18) and Proposition 2. ■

Proof of Proposition 7

We differentiate system (19)-(22) to find

$$\begin{bmatrix} \frac{\tau}{\mu^*}\frac{d\mu^*}{d\tau} \\ \frac{\tau}{k^*}\frac{dk^*}{d\tau} \\ \frac{\tau}{l^*}\frac{dl^*}{d\tau} \\ \frac{\tau}{B}\frac{dB}{d\tau} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1-\alpha}{\sigma} & 0 & 0 \\ -\frac{\phi}{\varepsilon_{cc}} & \theta & \phi + \beta(1-l) & \phi\frac{\varepsilon_{cB}}{\varepsilon_{cc}} - \gamma d \frac{1-l}{l} \\ 0 & 0 & 1 & -d \\ 0 & \alpha & 1 & -\frac{1-2B}{1-B} \end{bmatrix}^{-1} \begin{bmatrix} -\frac{\tau}{1-\tau} \\ \frac{\theta+\delta}{\alpha}\frac{\tau}{1-\tau} \\ 0 \\ \frac{b\tau}{a-b\tau} \end{bmatrix},$$

where

$$\phi \equiv \frac{\theta + (1-\alpha)\delta}{\alpha} = \frac{c^*}{k^* l^*},$$

that is

$$\begin{bmatrix} \frac{\tau}{\mu^*}\frac{d\mu^*}{d\tau} \\ \frac{\tau}{k^*}\frac{dk^*}{d\tau} \\ \frac{\tau}{l^*}\frac{dl^*}{d\tau} \\ \frac{\tau}{B}\frac{dB}{d\tau} \end{bmatrix} = \begin{bmatrix} \varepsilon_{cc}\left[\left(\frac{b\tau}{a-b\tau} + \frac{\alpha\sigma}{1-\alpha}\frac{\tau}{1-\tau}\right)\frac{1-B}{\bar{B}-B}\frac{d+\frac{\varepsilon_{cB}}{\varepsilon_{cc}}}{d-2} - \left[1 + \frac{\alpha}{1-\alpha}\frac{\sigma\theta+(1-\alpha)\delta}{\theta+(1-\alpha)\delta}\right]\frac{\tau}{1-\tau}\right] \\ -\frac{\sigma}{1-\alpha}\frac{\tau}{1-\tau} \\ \left(\frac{\alpha\sigma}{1-\alpha}\frac{\tau}{1-\tau} + \frac{b\tau}{a-b\tau}\right)\frac{1-B}{\bar{B}-B}\frac{d}{d-2} \\ \left(\frac{\alpha\sigma}{1-\alpha}\frac{\tau}{1-\tau} + \frac{b\tau}{a-b\tau}\right)\frac{1-B}{\bar{B}-B}\frac{1}{d-2} \end{bmatrix}. \tag{46}$$

Under Assumption 4 and $B \in (0,1)$, we obtain easily Proposition **7**. ■

Proof of Proposition 8

In the Cobb-Douglas case, $\sigma = 1$ and, according to expressions (46), (37) yields

$$\frac{\tau}{c^*}\frac{\partial c^*}{\partial \tau} = -\frac{1}{1-\alpha}\frac{\tau}{1-\tau} + \left(\frac{\alpha}{1-\alpha}\frac{\tau}{1-\tau} + \frac{b\tau}{a-b\tau}\right)\frac{1-B}{\bar{B}-B}\frac{d}{d-2}.$$

In the case $a = b$, we obtain

$$\frac{\tau}{c^*}\frac{\partial c^*}{\partial \tau} = \frac{1}{1-\alpha}\frac{\tau}{1-\tau}\left(\frac{1-B}{\bar{B}-B}\frac{d}{d-2} - 1\right) = \frac{1}{1-\alpha}\frac{\tau}{1-\tau}\frac{1-2B}{(d-2)\left(\bar{B}-B\right)}.$$

Proposition 8 immediately follows. ■

Proof of Proposition 9

According to Corollary 15 of Bosi and Desmarchelier (2019), a Hopf bifurcation arises iff $S_2 = S_3/T + DT/S_3$ and T and S_3 have the same sign.

Let us rewrite S_2 and S_3 as follows:

$$S_2 = \frac{Z}{m} - \frac{T}{m}\frac{S_3}{T}, \tag{47}$$

$$S_3 = z_3 - m\alpha q(1-B)\frac{\varepsilon_2}{\varepsilon_1}. \tag{48}$$

Replacing (47), equation

$$S_2 = \frac{S_3}{T} + D\frac{T}{S_3},$$

becomes

$$\frac{S_3}{T} = \frac{Z \pm \sqrt{Z^2 - 4mD\left(m+T\right)}}{2\left(m+T\right)}. \tag{49}$$

We observe that, if $0 < d < 1$, $D < 0 < m + T$ iff $\bar{B} < B < (1+\theta)/2$. In this case, $m + T > 0$ and $4mD\left(m+T\right) < 0$. If $d > 1$, then $D < 0$ and, thus, $D < 0 < m + T$ iff $B < (1+\theta)/2$. Even in this case, $m + T > 0$ and $4mD\left(m+T\right) < 0$.

Then, in both the cases,

$$\left(\frac{S_3}{T}\right)_- < 0 < \left(\frac{S_3}{T}\right)_+.$$

Clearly, the solution ε_2 of

$$\frac{S_3\left(\varepsilon_2\right)}{T} = \left(\frac{S_3}{T}\right)_- < 0,$$

is not acceptable as Hopf bifurcation value because T and S_3 have opposite sign. Let ε_H be solution of

$$\frac{S_3\left(\varepsilon_2\right)}{T} = \left(\frac{S_3}{T}\right)_+.$$

Replacing (48) in the LHS and (49) in the RHS, we obtain (44). ■

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IMPERFECT MOBILITY OF LABOR ACROSS SECTORS AND FISCAL TRANSMISSION*

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Abstract

Our paper investigates the sectoral effects of government spending shocks and highlights the role of labor mobility. Our VAR evidence for sixteen OECD countries reveals that a shock to government consumption by 1% of GDP increases non-traded value added by 0.7% of GDP and generates a decline in traded value added. The value added share of non-tradables rises by 0.35% of GDP, thus implying that the reallocation of resources accounts for 50% of the sectoral fiscal multiplier. Consistently, our estimates show that the non-traded sector is highly intensive in the government spending shock and experiences a labor inflow. The shift of hours worked toward the non-traded sector is, however, subject to mobility costs which vary across countries. When we explore quantitatively the sectoral effects of a shock to government consumption that is highly intensive in non-traded goods, we find that the model can replicate the magnitude of the rise in the share of non-tradables we document empirically once we allow for both labor mobility and capital installation costs. Financial openness also matters as it further biases the demand shock toward non-tradables. To account for the cross-country dispersion in the responses of sectoral shares we estimate empirically, we have to let the degree of labor mobility vary across countries.

Keywords: Fiscal policy; Labor mobility; Investment; Current account; Non-tradables; Sectoral wages.
JEL Classification: E22; E62; F11; F41; J31.

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1 Introduction

As documented recently, the global financial crisis has led to an output decline in OECD countries which varies along the tradedness of industries. Using sectoral data for Spain, Arellano et al. [2018] find that non-traded production has decreased significantly more than traded production between 2007 and 2013. Evidence by De Ferra [2018] reveals that non-exporting firms experienced the largest drop in sales, hours worked and investment in Italy in 2009-2013. Using U.S. data between 2007 and 2009, Mian and Sufi [2014] find that non-traded employment has been more vulnerable to the recession than employment in traded industries as non-traded firms rely heavily on local demand. To the extent that expansionary fiscal policy targets non-traded industries, a rise in government spending could potentially be an appropriate tool to stabilize output in non-exporting sectors, as emphasized by Schmitt-Grohé and Uribe [2013]. Yet at an empirical and theoretical level, the systematic exploration of how a rise in government spending impacts the non-traded vs. the traded sector is still lacking. In the present paper we address the following question: Do shocks to government consumption affect sectors symmetrically and if not, what are the causes of this asymmetry? We find that shocks to government consumption tend to disproportionately benefit the non-traded sector by producing a labor reallocation toward this sector, and all the more so in countries where workers' costs of switching sectors are lower.

To guide our quantitative analysis, we document VAR evidence on the sectoral effects of a rise in government consumption for sixteen OECD countries. First, a shock to government consumption has a strong expansionary effect on output in the non-traded sector relative to the traded sector. More specifically, we find empirically that a rise in government spending by 1% of GDP increases non-traded value added by 0.7% of GDP on impact and leads to a decline in traded value added. The expansion in the non-traded sector is associated with a rise in the value added share of non-tradables by 0.35% of GDP. Since the latter result indicates that non-traded value added would increase by 0.35% if GDP remained constant, the reallocation of resources toward the non-traded sector thus contributes to 50% of the rise in non-traded value added. The remaining 0.35% of GDP represents the rise in non-traded value added caused by the aggregate fiscal multiplier split across sectors in accordance with their value added share. A necessary condition for the share of non-tradables to increase is that this sector must receive a disproportionate share of the shock to government spending. Our estimates corroborate this hypothesis as we find empirically that government consumption of non-tradables contributes 90% on average to increases in government spending.

For the increase in the share of non-tradables to materialize, productive resources, in particular labor, must be reallocated toward the non-traded sector. The second set of our empirical findings reveals that non-traded hours worked rise by 0.54% of total hours worked, half of this increase being caused by the reallocation of labor. The shift of labor is subject to labor mobility costs, however, since we detect empirically a significant increase in non-traded relative to traded wages. These findings accord well with the evidence documented by Artuç et al. [2010], Dix-Carneiro [2014], Lee and Wolpin [2006] who find substantial barriers of mobility between sectors and furthermore that wages are not equalized across sectors in the short run nor in the long run.

A first way to gauge the role of labor mobility costs for fiscal transmission is to investigate how impact responses of relative sector size vary over time and whether their movements are positively related to labor reallocation following our identified government spending shock. Our estimates reveal that the responses of sectoral shares are reduced over time by about 40% and that this reduction is concomitant and highly correlated with the decline in the rate of workers shifting from one sector to another. When we turn to international differences, the responses of sectoral value added and hours worked shares display a wide cross-country dispersion. Motivated by the cross-country variations in labor mobility costs documented by Artuç et al. [2015], we estimate the elasticity of labor supply across sectors and empirically detect a positive cross-country relationship between the change in relative sector size following a government spending shock and the degree of labor mobility.

To account for our evidence on fiscal transmission, we put forward an open economy version of the neoclassical model with tradables and non-tradables. In calibrating the model to a representative OECD economy, we assume that the non-traded sector receives a share of the rise in government spending which is larger than its relative size, in line with our evidence, so that the government shock is biased toward non-tradables. Our quantitative results show that the model is successful in replicating the sectoral effects of government spending shocks as long as we allow for imperfect mobility of labor (IML henceforth) and capital adjustment costs.[1]

With these two features, the model produces a rise in the share of non-tradables by 0.38% of GDP, close to our empirical findings. If we remove both or either one of these ingredients, the model fails to account quantitatively for our evidence on fiscal transmission, in particular the responses of sectoral output shares which we estimate empirically. Intuitively, if we do not allow for capital adjustment costs, a government spending shock leads to a dramatic fall in investment which offsets the rise in government consumption. As a result, the excess demand in the non-traded goods market is low or even nil. Due to low incentives to shift resources toward the non-traded sector, the open economy experiences a trade balance surplus resulting in the model substantially understating the rise in the share of non-

[1] To generate IML, we consider limited substitutability in hours worked across sectors along the lines of Horvath [2000]. See e.g., Bouakez et al. [2011], Cardi and Restout [2015] who assume that sectoral hours worked are aggregated by means of a CES function in order to account for the evidence related to monetary policy shocks or the long-run effects of productivity shocks biased toward the traded sector.

tradables. Conversely, if we allow for capital adjustment costs, the decline in investment is mitigated, which leads to significant excess demand in the non-traded goods market. However, if we impose perfect mobility of labor across sectors (PML henceforth), high incentives to shift resources toward the non-traded sector result in a large trade balance deficit which leads the model to overstate the rise in the share of non-tradables considerably.

By tilting the demand shock toward non-tradables, financial openness and the tradability of goods are also key dimensions that allow our model with IML to account for the evidence. Shutting down the response of the current account leads the model to understate the rise in the share of non-tradables, the latter increasing by an amount which is twice as small as that estimated empirically. The reason is that when the fiscal stimulus is temporary and the economy has perfect access to world capital markets, households find it optimal to borrow abroad to avoid a large decline in consumption and/or a large increase in labor supply. Since traded goods can be imported and non-traded goods must be produced domestically, access to foreign borrowing further biases the demand shock toward non-tradables.

The final exercise we perform is to investigate whether the model can account for cross-country differences in the responses of sectoral output shares to a fiscal shock. We thus calibrate the model to country-specific data. We find quantitatively that impact responses of sectoral output shares to a government spending shock are sensitive to the degree of labor mobility, as they vary between 0.26% and 0.49% of GDP for non-tradables when we move from the lowest to the highest value of elasticity of labor supply across sectors. In line with the evidence, the cross-country dispersion in the sectoral share responses is the result of international differences in the degree of labor mobility, the rise in the output share of non-tradables being more pronounced in countries with a higher degree of labor mobility.

So far, we have not said much about the sectoral fiscal multiplier which is the result of the change in the sectoral share and the rise in real GDP. Because changes in the sectoral value added and the sectoral share are positively correlated, raising the non-tradable content of the government spending shock or the degree of labor mobility across sectors increases the fiscal multiplier for non-tradables. At an aggregate level, a government spending shock produces a larger fiscal multiplier by targeting the sector that has the highest labor compensation share, i.e., the non-traded sector.[2] By contrast, by mitigating the rise in non-traded wages and thus aggregate wage growth, a higher degree of labor mobility reduces the magnitude of the aggregate fiscal multiplier.

Related Literature. We contribute to the extensive literature investigating fiscal transmission both empirically and theoretically by focusing on the reallocation effect of

[2] Baqaee [2018] provides a decomposition of the contribution of sectors to the aggregate fiscal multiplier and highlights the key role of both the sectoral composition of government purchases and sectoral labor intensity in determining employment effects like us but the mechanism is very different.

government spending shocks. Like Ramey and Shapiro [1998], we emphasize the importance of the composition of government spending in understanding the sectoral effects of a fiscal shock. In contrast to the authors who consider three episodes of expansionary defense spending in the United States driven by foreign policy, we identify exogenous increases in government consumption by assuming that discretionary government spending is subject to certain decision and/or implementation lags, as proposed by Blanchard and Perotti [2002]. Putting aside the advantages and disadvantages inherent to the narrative and SVAR approaches, the identification scheme does matter, as the identified government spending shock can be intensive either in tradables or non-tradables. While the Ramey-Shapiro narrative approach suggests that military shocks, which are heavily concentrated in the manufacturing sector, are intensive in traded goods, our study reveals that government spending shocks, identified on the basis of Blanchard-Perotti assumption, lead to a sharp increase in non-traded relative to traded output.

This finding is in line with estimates documented by Monacelli and Perotti [2008], Benetrix and Lane [2010] which show that an increase in government spending disproportionately benefits the non-traded sector. In contrast to the authors who restrict their attention to sectoral output or labor effects and thus do not investigate the reallocation effects, our paper analyzes and rationalizes the labor composition effect caused by shocks to government consumption like Bredemeier et al. [2019]. Differently, the authors contrast the effects across occupations rather than between sectors.

One additional key finding with respect to the papers mentioned above is that international differences in workers' costs of switching sectors can account for the cross-country dispersion in the responses of sectoral shares, as we uncover a positive cross-country relationship between the degree of labor mobility and the changes in relative sector size. In this regard, our study can be viewed as complementary to the work by Ilzetzki et al. [2013], Born et al. [2013], Brinca et al. [2016] who contrast the effects of fiscal policy on output across a number of country characteristics. In contrast to these papers focusing on the aggregate fiscal multiplier, we explore the size of sectoral fiscal multipliers resulting from the reallocation of resources across sectors.

Finally, our paper also relates to a broad literature which studies fiscal transmission by breaking down aggregate government spending into sub-categories. While Baxter and King [1993] differentiate between government consumption and government investment, we restrict attention to government consumption in accordance with the bulk of the literature investigating fiscal transmission. In contrast to a growing literature exploring the impact on private activity of shocks to government purchases from the private sector and the government sector (the latter essentially consisting of compensation of government employees), respectively, see e.g., Bermperoglou et al. [2017], we focus on the sectoral distribution of an increase in aggregate government spending, the public sector being part of the non-traded sector. Nekarda and Ramey [2011] estimate the effects of a rise in industry-specific government purchases and find that industries with higher concentration and unionization rates experience larger increases in output. Differently, we focus on the asymmetric effects across sectors caused by an increase in government consumption by breaking down sectoral effects into reallocation and aggregate effects.

The remainder of the paper is organized as follows. In section 2, we investigate empirically the sectoral effects of a government spending shock and highlight the role of labor reallocation. In section 3, we develop an open economy version of the neoclassical model with IML. In section 4, we report the results of our numerical simulations and assess the ability of the model to account for the evidence. In section 5, we summarize our main results and present our conclusions. An Online Appendix contains more empirical results and robustness checks, and solves analytically a restricted version of the model to build up intuition on the implications of labor mobility costs.[3]

2 Evidence on Fiscal Transmission across Sectors

In this section, we revisit the time-series evidence on fiscal transmission by differentiating the effects between the traded and non-traded sectors. We first estimate sectoral fiscal multipliers and the responses of sectoral shares to a government spending shock. Then we document evidence which aims to explain the rise in the share of non-tradables along with its variation across time and space. We denote below the level of the variable in upper case, the logarithm in lower case, and the percentage deviation from its initial steady-state by a hat.

2.1 VAR Model and Identification

In order to shed some light on fiscal transmission and guide our quantitative analysis, we estimate a VAR model in panel format on annual data. We use i to index countries and t to index time periods (years). Denoting the vector of endogenous variables by $Z_{i,t}$, the reduced-form VAR reads:

$$Z_{i,t} = \alpha_i + \beta_i t + \sum_{k=1}^{2} A^{-1} B_k Z_{i,t-k} + A^{-1}\epsilon_{i,t}, \tag{1}$$

where k is the number of lags; the specification includes country fixed effects, α_i, and country-specific linear time trends; A is a matrix that describes the contemporaneous relation among the variables collected in vector $Z_{i,t}$, B_k is a matrix of lag specific own- and cross-effects of variables on current observations, and the vector $\epsilon_{i,t}$ contains the structural disturbances which are uncorrelated with each other. In line with the common practice in

[3]A longer version of the paper by Cardi et al. [2018] provides the steps to solve the model laid out in section 3, and proposes additional robustness checks and several theoretical extensions of the model.

the empirical literature estimating the effects of a rise in government spending on annual data, see e.g., Beetsma and Giuliodori [2011], we include two lags in the regression model and use a panel OLS regression to estimate the coefficients $A^{-1}B_k$ and the reduced-form innovations $A^{-1}\epsilon_{i,t}$.

To identify the VAR model and recover the government spending shocks, we need assumptions on the matrix A as the reduced form of the VAR model that we estimate contains fewer parameters than the structural VAR model. Like Blanchard and Perotti [2002], we base the identification scheme on the assumption that discretionary government spending is subject to certain decision and implementation lags that prevent government spending from responding to current output developments. Since there are some delays inherent to the legislative system, this is a natural assumption when using quarterly data. However, this argument may not necessarily be true when using annual data since some adjustment could be possible. To address the potential endogeneity issue, we ran a number of robustness checks which confirm that our identifying strategy is not altered by the use of annual data.[4] An additional obstacle is to identify unexpected fiscal events. We conducted an investigation of the potential presence of anticipation effects by using a dataset constructed by Born, Juessen and Müller [2013] which contains one year-ahead OECD forecasts for government spending. It turns out that differences are moderate when we control for the anticipation effects and that our main results are not affected by the inclusion of forecasts for government spending growth.

2.2 Data Construction

Before presenting the VAR model specification, we briefly discuss the dataset we use. Our sample contains annual observations and consists of a panel of 16 OECD countries. The baseline period is running from 1970 to 2007. Table 1 provides a list of countries and data sources while more details can be found in the Online Appendix A. All quantities are logged, expressed in real terms and scaled by the working age population. Government final consumption expenditure ($G_{i,t}$) in volume is taken from OECD Economic outlook. We describe below how we construct time series at a sectoral level.

We use the EU KLEMS [2011] and OECD STAN [2011] database which provides domestic currency series of value added in current and constant prices, labor compensation and number of hours worked for eleven 1-digit ISIC-rev.3 industries. To split these eleven

[4] To support our identifying assumption, we performed several robustness exercises detailed in Online Appendix E.1 and E.2. Our results accord well with the conclusion reached by Born and Müller [2012] whose test reveals that the assumption that government spending is predetermined within the year cannot be rejected. In particular, we investigate whether our main conclusions hold when adopting a narrative approach which has the advantage of identifying fiscal policy changes that are exogenous to current economic developments. We use narratively-identified government spending shocks from the dataset constructed by Guajardo, Leigh, and Pescatori [2014] whose dataset contains 173 fiscal policy changes for 17 OECD countries over the period 1978-2009. The main conclusions reached in this paper are robust to the identification approach.

industries into traded and non-traded sectors, we follow the classification suggested by De Gregorio et al. [1994] that we updated by following Jensen and Kletzer [2006].[5]

Once industries have been classified as traded or non-traded, denoted by the superscripts T and N, respectively, series for sectoral value added in current (constant) prices are constructed by adding value added in current (constant) prices for all sub-industries k in sector $j = T, N$, i.e., $P^j_{i,t} Y^j_{i,t} = \sum_k P^j_{k,i,t} Y^j_{k,i,t}$ ($\bar{P}^j_i Y^j_{i,t} = \sum_k \bar{P}^j_{k,i} Y^j_{k,i,t}$ where the bar indicates that prices P^j are those at the base year), from which we compute price indices (or sectoral value added deflators), $P^j_{i,t}$. Normalizing base year price indices $\bar{P}^j$ to 1, the relative price of non-tradables, $P_{i,t}$, is defined as the ratio of the non-traded value added deflator to the traded value added deflator (i.e., $P_{i,t} = P^N_{i,t}/P^T_{i,t}$). The same logic applies to constructing series for hours worked ($L^j = \sum_k L^j_{k,i,t}$) and labor compensation in the traded and the non-traded sectors which allow us to construct sectoral wages, $W^j_{i,t}$. The relative wage, $\Omega_{i,t}$, is computed as the ratio of the non-traded wage to the traded wage (i.e., $\Omega_{i,t} = W^N_{i,t}/W^T_{i,t}$). The real consumption wage in sector j, $W^j_{C,i,t}$, is defined as the sectoral nominal wage, $W^j_{i,t}$, divided by the consumption price index, $P_{C,i,t}$. As detailed below, we also construct labor and value added shares, denoted by $\nu^{L,j}_{i,t}$ and $\nu^{Y,j}_{i,t}$.

< Please insert Table 1 about here >

2.3 Sector-Biased Government Spending Shock and Labor Reallocation

Most of the literature investigating the output effects of a government spending shock focuses on the aggregate fiscal multiplier which measures the percentage deviation of real GDP relative to its initial steady-state following a rise in government consumption by 1% of GDP, denoted by $\hat{Y}_R(t)$.[6] In the present paper, we consider an open economy which produces a traded and a non-traded good where the traded good is the numeraire and its price is normalized to 1. Real GDP, $Y_R(t)$, is equal to the sum of traded and non-traded value added at constant prices, i.e., $Y_R(t) = Y^T(t) + PY^N(t)$ where prices at the initial steady-state are those at the base year so that real GDP collapses to nominal GDP, Y, initially. Log-linearizing both sides of the equality in the neighborhood of the initial steady-state leads to $\hat{Y}_R(t) = \nu^{Y,T}\hat{Y}^T(t) + \nu^{Y,N}\hat{Y}^N(t)$ where $\nu^{Y,j} = P^j Y^j/Y$ is the share of sector j in GDP. This expression simply states that following a shock to government consumption by 1% of GDP, the aggregate fiscal multiplier is equal to the sum of sectoral fiscal multipliers expressed in GDP units.

The contribution of each sector j to the aggregate fiscal multiplier will collapse to its

[5] In contrast to De Gregorio et al. [1994] who treat 'Financial intermediation' as non-tradable, we classify this industry as tradable in line with the evidence documented by Jensen and Kletzer [2006] on U.S. data. In Online Appendix D.2, we find that our classification does not drive our results.

[6] It should be mentioned in the interest of clarity that referring to $\hat{Y}_R(t)$ as the fiscal multiplier is an abuse of language as the latter should be computed as the ratio of the present value of the cumulative change in output to the present value of the cumulative change in government consumption. Since we base most of our analysis on impact effects such a simplification does not pose a problem.

value added share $\nu^{Y,j}$ as long as the shock to government consumption is split across sectors in accordance with their share $\nu^{Y,j}$ in GDP. By contrast, if sector j receives a fraction of the rise in government spending which is larger than its value added share, $\nu^{Y,j}$, the shock to government consumption provides incentives to shift productive resources toward this sector. Henceforth, the government spending shock is biased toward sector j which increases its value added share by moving productive resources toward this sector.

To clarify this point, we first break down the sectoral fiscal multiplier into two components. A rise in government spending generates a deviation of sectoral value added relative to its initial steady-state value in percentage denoted by $\hat{Y}^j(t)$. Adding and subtracting the aggregate fiscal multiplier, i.e., $\hat{Y}^j(t) = \hat{Y}_R(t) + \left(\hat{Y}^j(t) - \hat{Y}_R(t)\right)$, and multiplying both sides by $\nu^{Y,j}$ allows us to decompose the sectoral fiscal multiplier as follows:

$$\nu^{Y,j}\hat{Y}^j(t) = \nu^{Y,j}\hat{Y}_R(t) + d\nu^{Y,j}(t), \tag{2}$$

where $d\nu^{Y,j}(t) = \nu^{Y,j}\left(\hat{Y}^j(t) - \hat{Y}_R(t)\right)$ is the change in the value added share of sector j at constant prices in GDP units. The first term on the RHS of eq. (2) (i.e., $\nu^{Y,j}\hat{Y}_R(t)$) captures the rise in sectoral value added if the intensity of sector j in the government spending shock were equal to its value added share, $\nu^{Y,j}$. The second term on the RHS of eq. (2) (i.e., $d\nu^{Y,j}(t)$) states that value added at constant prices of sector j further increases if the value added share of sector j rises. As shown below, for the value added share of sector j, $\nu^{Y,j}(t)$, to increase, the shock to government spending must be biased toward sector j. The same logic applies to sectoral hours worked, except that $d\nu^{L,j}(t)$ measures the differential between the responses of sectoral and total hours worked expressed in total hours worked units, i.e., $d\nu^{L,j}(t) = \alpha^{L,j}\left(\hat{L}^j(t) - \hat{L}(t)\right)$ where $\alpha^{L,j}$ is the labor compensation share of sector j.

Next, we derive a relationship between the change in the value added share of sector j and the biasedness of the shock to government consumption toward good j by using the equality between value added and its final use, i.e., $Y^j(t) = E^j(t) + G^j(t)$ where E^j and G^j stands for private and public demand for good j, respectively. Log-linearizing $Y^j(t) = E^j(t) + G^j(t)$ while keeping private demand fixed leads to $\nu^{Y,j}\hat{Y}^j(t) = P^j dG^j(t)/Y = \omega_{G^j} dG(t)/Y$ where $\omega_{G^j} = P^j G^j/G$ is the share of good j in government consumption. In deriving the last equality, we assume that a constant fraction of government expenditure is spent on good j in line with our evidence which shows that ω_{G^j} is fairly constant over time so that $G(t) = \omega_{G^N} G(t) + \omega_{G^T} G(t)$. Focusing on the non-traded sector, using the fact that $\hat{Y}_R(t) = dG(t)/Y$ because we keep private demand fixed, and subtracting $\nu^{Y,N}\hat{Y}_R(t)$ from both sides of $\nu^{Y,N}\hat{Y}^N(t) = \omega_{G^N} dG(t)/Y$ enables us to relate the change in the value added share of non-tradables to the intensity of the non-traded sector in the government spending shock:

$$d\nu^{Y,N}(t) = \left(\omega_{G^N} - \nu^{Y,N}\right)\left(dG(t)/Y\right). \tag{3}$$

The term $\omega_{G^N} - \nu^{Y,N}$ is a measure of the biasedness of the shock to government consumption toward the non-traded good. Eq. (3) states that when the non-traded sector receives a share ω_{G^N} of the rise in government spending equal to the share of non-tradables in GDP, $\nu^{Y,N}$, the relative size of the non-traded sector remains unchanged, i.e., $d\nu^{Y,N}(t) = 0$. According to eq. (2), under this assumption, the fiscal multiplier of non-tradables, $\nu^{Y,N}\hat{Y}^N(t)$, boils down to the aggregate fiscal multiplier weighted by the non-traded value added share, $\nu^{Y,N}\hat{Y}_R(t)$. By contrast, when the shock to government consumption is biased toward non-tradables, i.e., $\omega_{G^N} > \nu^{Y,N}$, the non-traded sector experiences a demand boom which provides an incentive to shift productive resources toward this sector. As long as mobility costs are not prohibitive, the value added share of non-tradables increases, i.e., $d\nu^{Y,N}(t) > 0$. The lower the labor mobility costs, the more labor is reallocated toward the non-traded sector which amplifies the rise in the value added share of non-tradables. It is worth mentioning that in deriving eq. (3), we shut down the responses of the private sector's demand components. As shown in Online Appendix C where we solve analytically a restricted version of the model and as discussed in section 4 where we solve numerically the full model, the endogenous reaction of the current account to the fiscal shock also matters in determining the response of the sectoral share $\nu^{Y,N}(t)$.

In the sequel, we estimate empirically the change in the sectoral value added at constant prices expressed in GDP units, $\nu^{Y,j}\hat{Y}^j(t)$ (i.e., the LHS term of eq. (2)), and the change in the sectoral value added share, $d\nu^{Y,j}(t)$ (i.e., the second term on the RHS of eq. (2)), following an increase in government consumption by 1% of GDP. Dividing the latter by the former allows us to measure the contribution of the reallocation of productive resources to the sectoral fiscal multiplier. To rationalize the change in the value added share of sector j (see eq. (3)), we estimate the intensity ω_{G^j} of each sector j in the government spending shock. Since the intensity ω_{G^j} varies little between OECD economies, we put forward international differences in labor mobility costs to account for the cross-country dispersion in the responses of sectoral shares to the shock to government consumption we document empirically.

2.4 VAR Specification

In order to investigate the size of sectoral fiscal multipliers, along with the contribution of the reallocation of resources to sectoral fiscal multipliers, we consider three alternative VAR specifications in which the choice of variables is motivated by the variables discussed in the quantitative analysis. To alleviate notations, price indices at the base year are normalized to 1, i.e., $\bar{P}_i^j = 1$, so that (logged) value added at constant prices is reduced to $y_{i,t}^j$ and $y_{i,t}$ stands for (logged) real GDP when this causes no confusion.

- To investigate the magnitude of the sectoral fiscal multiplier (i.e., the LHS term of eq. (2)), we consider a VAR model that includes value added at constant prices in sector j, $y^j_{i,t}$, hours worked in sector j, $l^j_{i,t}$, and the real consumption wage in sector j, $w^j_{C,i,t}$. Our vector of endogenous variables, is given by: $z^j_{i,t} = \left[g_{i,t}, y^j_{i,t}, l^j_{i,t}, w^j_{C,i,t}\right]$ with $j = T, N$.

- To estimate the change in the value added (hours worked) share of sector j (i.e., the second term on the RHS of eq. (2)), we consider a VAR model where we divide sectoral value added at constant prices (sectoral hours worked) by real GDP (total hours worked) in order to filter the change in sectoral output (sectoral hours worked) arising from real GDP (total hours worked) growth, which allows us to isolate the 'pure' reallocation effect and thus gauge the importance of the shift of resources across sectors for the sectoral fiscal multiplier. Our vector of endogenous variables, is given by: $z^{S,j}_{i,t} = \left[g_{i,t}, y^j_{i,t} - y_{i,t}, l^j_{i,t} - l_{i,t}, w^j_{C,i,t}\right]$.
- Finally, to gain further insight into fiscal transmission, we estimate empirically the effects of a government spending shock on the relative price (p) and relative wage (ω), and thus consider a VAR model where we replace sectoral quantities with the ratio of sectoral quantities for both the product and the labor market. Our vector of endogenous variables, is given by: $z^P_{i,t} = \left[g_{i,t}, y^T_{i,t} - y^N_{i,t}, p_{i,t}\right]$ and $z^W_{i,t} = \left[g_{i,t}, l^T_{i,t} - l^N_{i,t}, \omega_{i,t}\right]$.

While in the main text we concentrate on the sectoral effects, in a longer version of the paper, we also document evidence on the aggregate effects of a government spending shock by estimating a VAR model which includes government final consumption expenditure, real GDP, total hours worked, private investment, and the real consumption wage, i.e., $z_{i,t} = [g_{i,t}, y_{i,t}, l_{i,t}, je_{i,t}, w_{C,i,t}]$.[7] We take this model as the baseline to calibrate the government spending shock in the quantitative analysis.[8]

2.5 Sectoral Effects of Government Spending Shocks: VAR Evidence

We generated impulse response functions which summarize the responses of variables to an increase in government spending by 1% of GDP. As displayed in the solid blue line in the left panel of Fig. 1, the response of government consumption is hump-shaped, peaking after one year and then gradually declining; it shows a high level of persistence over time as it is about 8 years before the shock dies out.[9]

Sectoral fiscal multipliers. In Fig. 2, we report results for our three VAR models.[10] The horizontal axis measures time after the shock in years and the vertical axis measures

[7] Aggregate effects of a government spending shock are displayed and discussed in Online Appendix D.1.

[8] Because we consider alternative VAR models, the fact that identified government spending shocks display substantial differences across VAR specifications might be a concern. To address this issue, we ran a number of robustness checks by augmenting each VAR model with the same identified spending shock, ordered first. Results reveal that the discrepancy in the estimated effects is insignificant, see Online Appendix E.3.

[9] The black line with squares in the left panel of Fig. 1 shows the endogenous response of G over the period 1995-2015 as we estimate the responses of G^T and G^N over this period.

[10] For reasons of space, we do not show the responses of real consumption wages which are relegated to Online Appendix D.2. Point estimates at a one-, two-, and four-year horizon are contained in a Table in Online Appendix D.1.

percentage deviations from trend. In each case, the solid line represents the point estimate, while the shaded area indicates the 90% confidence bounds obtained by bootstrap sampling. The first column displays fiscal multipliers for output. We find that a rise in government consumption has a strong expansionary effect on non-traded output which increases significantly on impact by 0.70% of GDP. During the first four years after the shock, the non-traded output multiplier of government spending averages out at about 0.47% of GDP. In contrast, the traded sector displays a negative fiscal multiplier over this period as the government spending shock generates a decline in traded output which remains below trend. Furthermore, as shown in the second column of Fig. 2, higher non-traded output is associated with a sharp increase in hours worked on impact, while the traded sector experiences a gradual decline in hours worked for the first five years.

Sectoral shares. The third column of Fig. 2 enables us to gauge the contribution of the reallocation of inputs, labor especially, to the expansion of the relative size of the non-traded sector. The second row shows that the labor share of tradables declines by 0.27% of total hours worked (see the blue line with squares) while the reverse is true for non-tradables (see the solid black line). Since non-traded hours worked rise by 0.55% of total hours worked, half of this increase is the result of labor reallocation.[11] As shown in the first row of the third column, a fiscal shock lowers the output share of tradables (see the blue line with squares) and substantially increases that of non-tradables (see the solid black line). Henceforth, our evidence shown in Fig. 2 reveals that the government spending shock is biased toward non-tradable goods as it benefits the non-traded sector which experiences a capital and labor inflow. Responses of sectoral shares to a shock to government consumption also enable us to quantify the contribution of the reallocation of resources to the sectoral fiscal multiplier. Quantitatively, since non-traded output rises by 0.7% of GDP while the output share of non-tradables rises by 0.35% of GDP, the shift of resources toward the non-traded sector alone contributes 50% of non-traded output growth.[12]

Relative price of non-tradables. As shown analytically in Online Appendix C.2, all else being equal (i.e., keeping private demand fixed), for the relative price of non-tradables to appreciate, the government spending shock must be biased enough toward non-traded goods, i.e., $\omega_{G^N} > \nu^{Y,N}$. The last column of Fig. 2 supports the conjecture that an aggregate government spending shock triggers a demand shock in favor of non-tradables. More specifically, the relative price of non-tradables (see the solid black line) appreciates

[11] Because we focus on sectoral hours worked, labor reallocation across sectors can occur at the intensive as well as the extensive margin. In Online Appendix D.3, we find that both the rise in hours worked per worker and higher employment contribute to the increase in the labor share of non-tradables while the other way around is true for tradables.

[12] In Online Appendix D.6, we explore empirically which industry drives the responses of sectoral shares following a rise in government spending by 1% of GDP. Our empirical results show that most of the decline in the share of tradables can be attributed to 'Manufacturing' while 'Community Social and Personal Services', 'Construction', and 'Real Estate, Renting, and Business Services' mostly drive the rise in the share of non-tradables.

significantly in the short-run, signaling excess demand in the non-traded goods market, while the ratio of traded output relative to non-traded output decreases substantially (see the blue line with squares).

Relative wage of non-tradables. While the appreciation in the relative price of non-tradables provides incentives for labor to shift away from the traded toward the non-traded sector, our evidence suggests the presence of intersectoral labor mobility costs. As can be seen in the second row of the last column of Fig. 2, the sharp decline in hours worked in the traded relative to the non-traded sector (see the blue line with squares) is associated with a significant increase in non-traded wages relative to traded wages (see the solid black line). The positive response of the relative wage to a government spending shock indicates that workers experience costs of switching sectors.

Relative Size of Countries. Our sample comprises OECD economies which differ greatly across size. Because smaller countries display a higher trade openness and a lower degree of labor mobility due to greater industrial specialization, we perform a split-sample analysis to investigate whether we detect empirically significant differences in the behavior of key variables we focus on in this paper, say sectoral shares, the relative price and the relative wage of non-tradables. In Online Appendix D.7, we provide an empirical analysis for the full set of variables. We split the sample into two groups of countries on the basis of the working age population and run the same VAR model for one sub-sample at a time. The group of large countries includes Australia, Canada, France, Italy, Japan, Spain, the U.K, and the U.S. and the group of small countries includes Austria, Belgium, Denmark, Finland, Ireland, the Netherlands, Norway, Sweden. Empirical results for large countries are shown in the dashed blue line with triangles and those for small countries are displayed in the red line with circles. The solid black line shows baseline results when considering the full sample with the shaded area indicating 90% confidence bounds. While all of the conclusions mentioned above hold, we may notice some differences quantitatively however. As can be seen in the first and the second column of Fig. 3, small countries experience variations of output and labor shares which are less pronounced on impact due probably to a lower degree of labor mobility across sectors. Indeed, Fig. 3(c) reveals that the relative wage of non-tradables increases significantly more in small than in large economies, thus suggesting that labor mobility costs are greater in the former group of countries. While switching costs mitigate labor reallocation, the first two columns of Fig. 3 also show that after four years, the group of small countries experiences greater and more persistent variations in sectoral shares. As shown by Cardi and Müller [2011], more open economies run larger current account deficits following a rise in government spending which should in turn amplify the demand boom for non-tradables. As can be seen in Fig. 3(f), the relative price of non-tradables appreciates more after four years in the group of small countries which provides greater incentives to shift labor toward the non-traded sector, thus explaining the larger responses of sectoral shares in the medium run for these economies.

< Please insert Figures 1-3 about here >

2.6 Intensity of Government Spending Shock in Non-Tradables

We first investigate empirically whether the government spending shock is biased enough toward non-traded goods to increase the relative size of the non-traded sector. In order to quantify the intensity of the government spending shock in non-tradables we split government final consumption expenditure between government consumption on non-tradables, g^N, and tradables, g^T, by using the COFOG database from the OECD which provides a breakdown of government expenditure by function.[13] The sample covers 13 OECD countries over the period 1995-2015, as shown in Table 1. We chose this period as time series for government consumption by function are not available before 1995 for most of the countries in our sample, while the period 1995-2007 would be too short to obtain consistent estimates. Then, we estimate a VAR model in panel format on annual data that includes unanticipated government spending shocks, $\epsilon^G_{i,t}$, ordered first, government consumption spending and sectoral government consumption on non-tradables and tradables. To identify exogenous and unanticipated fiscal shocks, $\epsilon^G_{i,t}$, we estimate the VAR model that includes aggregate variables, i.e., $z_{i,t} = [g_{i,t}, y_{i,t}, l_{i,t}, je_{i,t}, w_{C,i,t}]$, and adopt a Cholesky decomposition. The middle and right panels of Fig. 1 display the response of government consumption of non-tradables and tradables to an exogenous and unanticipated increase in government spending by 1% of GDP, respectively. On impact, government consumption of non-tradables increases by 0.88%. Its contribution to the government spending shock averages 90% and is quite stable over time as it varies from 88% up to 91%.[14] Moreover, we find that the responses of sectoral government consumption to an exogenous fiscal shock are both hump-shaped and seem to mimic the adjustment of government spending shown in Fig. 1(a).

Since $\omega_{G^N} = 90\%$ and the non-tradable content of GDP is 63% in OECD countries (see the last line of the first column of Table 2), the condition under which a shock to government consumption is biased toward non-tradables, as described by inequality (3), is fulfilled. As a result of the high intensity of the non-traded sector in the government spending shock, labor shifts toward the non-traded sector which increases its value added share. We show below that labor reallocation is subject to mobility costs, however, which in turn mitigate the rise in the share of non-tradables.

2.7 Implications of the Degree of Labor Mobility across Sectors

The presence of labor mobility costs preventing wage equalization after a government spending shock squares well with the evidence documented by Artuç et al. [2010], Dix-Carneiro

[13]See Online Appendix A.2 for details about the breakdown of g into g^N and g^T.

[14]See Table 5 in Online Appendix B.2 which displays the mean responses of the two components of government consumption.

[2014], Lee and Wolpin [2006] who find substantial barriers to mobility and observe that wages are not equalized across sectors following either trade liberalization episodes or sector-biased technological change. To assess the importance of IML for fiscal transmission, we investigate below whether the responses of sectoral shares vary across time and space, and whether these variations are positively related to differences in labor mobility.

Labor mobility and sectoral shares across time. A first way to gauge the role of labor mobility costs in determining the adjustment of the relative sector size to a government spending shock is to investigate whether the responses of sectoral shares vary over time and explore their relationship with the extent of labor reallocation across sectors triggered by a rise in government spending. To perform this experiment, we compute the responses of selected variables by using a two-step estimation procedure. We first identify government spending shocks by considering the baseline VAR model that includes aggregate variables, i.e., $z_{i,t} = [g_{i,t}, y_{i,t}, l_{i,t}, je_{i,t}, w_{C,i,t}]$, where government spending is ordered before the other variables. In the second step, we estimate the effects in a rolling 25-year window by using Jordà's [2005] single-equation method.[15] The local projection method amounts to running a series of regressions of each variable of interest on a structural identified shock for each horizon $h = 0, 1, 2, ...$:

$$x^j_{i,t+h} = \alpha^j_{i,h} + \beta^j_{i,h} t + \psi^j_h (L) z_{i,t-1} + \gamma^j_h .\epsilon^G_{i,t} + \eta^j_{i,t+h}, \tag{4}$$

where we include country fixed effects and country-specific linear trends respectively; x^j is the logarithm of the variable of interest of sector j, z is a vector of control variables (i.e., past values of government spending and of the variable of interest), $\psi^j_h (L)$ is a polynomial (of order two) in the lag operator and $\epsilon^G_{i,t}$ is the identified government spending shock. We allow for two lags on the variable of interest and government spending collected in vector z. Since we concentrate on impact effects, horizon h is set to zero in eq. (4). Given that we are primarily interested in the reallocation effects, we estimate the effect of a government spending shock on the labor and the value added share of tradables and non-tradables, i.e., $x^j = \nu^{L,j}, \nu^{Y,j}$ (with $j = T, N$). As can be seen in Fig. 4 which reports impact responses of sectoral shares to the government spending shock (i.e., γ^j_0) in the solid black line for the output share and the blue line with circles for the labor share, the magnitude of changes in relative sector size decreases over time, i.e., γ^N_0 becomes less positive and γ^T_0 less negative.

One obvious candidate to explain a decline in $|\gamma^j_0|$ is an increase in labor mobility

[15] By decoupling the shock identification and the estimate of the responses, the first advantage of Jordà's [2005] projection method is that traded and non-traded variables respond to the same shock. However, our robustness check shows that the shock is identical across all VAR models. The second advantage over the standard VAR approach is that it considerably reduces the number of coefficients and thus is particularly suited when estimating the sectoral effects over overlapping subperiods of fixed length. The third advantage is that it does not impose the dynamic restrictions implicitly embedded in VARs and can accommodate non-linearities in the response function. By imposing fewer restrictions, impulse responses obtained by using the local projection method are rather erratic. Since we contrast empirical with theoretical responses in the quantitative analysis and smooth impulse responses are therefore more appropriate for this exercise, we stick to the VAR methodology, however, for most the empirical analysis undertaken in this paper. That said, both methods lead to very similar, if not identical, results on impact and even at a longer time horizon.

costs. If workers incur higher costs of switching sectors, then a rise in government spending should result in a smaller reallocation of labor between the traded and the non-traded sector. Following Wacziarg and Wallack [2004], we compute the labor reallocation index in year t for country i denoted by $LR_{i,t}$ as the absolute change in sectoral hours worked, $L^j_{i,t}$, resulting from pure shifts of labor across sectors:

$$LR_{i,t}\left(\tau\right) = \frac{\sum_{j=T}^{N} |L^j_{i,t} - L^j_{i,t-\tau}| - \left|\sum_{j=T}^{N} L^j_{i,t} - \sum_{j=T}^{N} L^j_{i,t-\tau}\right|}{0.5\sum_{j=T}^{N}(L^j_{i,t-\tau} + L^j_{i,t})}, \tag{5}$$

where $\tau = 5$. Next, using eq. (4) with $x = LR$, we run a series of regressions of labor reallocation on the structural identified shock to government consumption.

As can be seen in the dotted black line in Fig. 4, the decline in the magnitude of changes in relative sector size is associated with less labor reallocation following a government spending shock, in line with our hypothesis. More specifically, our estimates reveal that, in about fifteen years, the responses of sectoral shares have been reduced over time by about 40% while the shift of labor between sectors has decreased by the same amount as well.[16] Time-varying responses of labor and value added shares are highly correlated with those of labor reallocation, with the correlation coefficient ranging from 0.82 to 0.86. This finding thus suggests that increasing labor mobility costs have contributed to declining effects of fiscal policy on relative sector size over time.

< Please insert Figures 4-5 about here >

Measure of the degree of labor mobility across sectors. We now investigate whether the sectoral effects vary across space. To conduct this study, we explore the cross-country relationship between changes in the relative size of sectors and the magnitude of workers' costs of switching sectors. To measure the degree of labor mobility, we draw on Horvath [2000] and estimate the elasticity of labor supply across sectors for each country i denoted by ϵ_i. Denoting the exogenous weight attached to labor supply in sector $j = T, N$ by ϑ^j_i, the labor supply schedule, which reads as follows $\frac{L^j_{i,t}}{L_{i,t}} = \vartheta^j_i \left(\frac{W^j_{i,t}}{W_{i,t}}\right)^{\epsilon_i}$, states that the share of hours worked in sector j rises by ϵ_i% following a 1% increase in the relative wage. When ϵ takes higher values, workers' mobility costs are lower, which in turn implies a higher degree of labor mobility. In order to estimate consistently the degree of labor mobility between the traded and the non-traded sector, we consider a situation where the labor market clears. Inserting labor demand in sector j, i.e., $W^j_{i,t} = \frac{\theta^j_i P^j_{i,t} Y^j_{i,t}}{L^j_{i,t}}$ where θ^j

[16] While higher mobility costs cause a decline in labor reallocation following a rise in government spending, the rate of workers switching sectors could also decrease as a result of a time-declining intensity of non-tradables in the government spending shock and/or a fall in financial openness. Since the share of government spending in non-tradables is stable over time and financial openness is increasing over the period of estimation, the fall in the LR index can only be attributed to higher labor mobility costs according to our model's predictions. When breaking down the impact response of the wage differential between non-tradables and tradables by skill, our estimates reveal that the skills attached to jobs created in the non-traded sector highly intensive in medium-skilled workers became more sector-specific over time, and this trend has contributed to put upward pressure on labor mobility costs, see Online Appendix F.4. However, we cannot exclude that other factors, such as the extent of capital mobility across sectors and labor demand developments, could also contribute to the decline in the LR index.

is the labor income share, into the supply of labor to sector j, noting that total labor compensation, $W_{i,t}L_{i,t}$, is equal to the sum of labor compensation across sectors, solving for the labor share of sector j and differentiating leads to $\hat{l}^j_{i,t} - \hat{l}_{i,t} = \gamma_i \hat{\beta}^j_{i,t}$ where $\gamma_i = \frac{\epsilon_i}{\epsilon_i+1}$ and $\beta^j_{i,t} = \frac{\theta^j_i P^j_{i,t} Y^j_{i,t}}{\sum_j \theta^j_i P^j_{i,t} Y^j_{i,t}}$.

To estimate γ_i and pin down the value for the elasticity of labor supply across sectors, ϵ_i, we run the regression in panel format on annual data of the percentage change in the labor share of sector j on the percentage change in the relative share of output paid to workers in sector j. The causes of labor market frictions hampering the shift of labor across sectors are diverse. Part of the lack of labor reallocation results from psychological (see e.g., Dix-Carneiro [2014]) and geographical mobility costs (see e.g., Kennan and Walker [2011]). Country fixed effects included in the regression capture these costs which are assumed to be the same for all periods. Differently, parameter ϵ_i we recover by estimating γ_i captures the elasticity of labor supply across sectors with respect to a sectoral wage differential. More specifically, workers accept to join the labor force in sector j as long as the wage differential covers the disutility caused by labor reallocation. As the elasticity of labor supply across sectors takes lower values, workers experience greater disutilities when shifting. The utility loss caused by a shift to a different sector captures barriers to mobility such as sector-specific human capital which may not be perfectly transferable across sectors (see e.g., Lee and Wolpin [2006], Kambourov [2009], Dix-Carneiro [2014]).[17]

Responses of sectoral shares and degree of labor mobility across countries. Once we have estimated the magnitude of workers mobility costs for each country, we then estimate the same model as in eq. (1) but for a single country at a time.[18] In Fig. 5, we plot the impact responses of sectoral labor and sectoral output shares on the vertical axis against our measure of the degree of labor mobility, denoted ϵ, on the horizontal axis. This

[17] Mobility costs captured by the parameter ϵ accord well with the sector-specific skills theory according to which a substantial amount of human capital may be destroyed upon switching industry. We find empirically that our measure of the degree of labor mobility across sectors is positively correlated with the share of young (share of workers aged 15-24 years in total labor force) and low-education workers (share of workers with primary education in total labor force), in line with the evidence documented by Kambourov and Manovskii [2009] which reveals that industry (and occupational) mobility declines with worker's age and education. Intuitively, younger and unskilled workers accumulate relatively less sector-specific human capital, and thus are expected to be more prone to shift from one sector to another. Our results also show that ϵ takes lower values in countries where employment protection legislation (adjusted with the share of permanent workers) is stricter and union density is higher. Drawing on Tang [2012], in countries where labor laws are more protective or where employees are more protected by labor unions, workers expect a more stable relationship with their employers and obtain higher bargaining power vis-a-vis their employers. Thus, they have more incentives to acquire firms specific skills relative to general skills on the job and thus are less prone to change jobs/sectors. Empirical results are contained in Online Appendix F.2.

[18] When estimating the responses of sectoral labor and sectoral output shares to a government spending shock for each country, we omit $w^j_{C,i,t}$ in order to economize some degrees of freedom; the vector of endogenous variables is thus $z^{S,j}_{i,t} = \left[g_{i,t}, \nu^{Y,j}_{i,t}, \nu^{L,j}_{i,t}\right]$. We also estimated the VAR model by including $\omega^j_{C,it}$ and find that the results are similar. We allow for two lags (i.e., $k = 2$ in eq. (1)), as we did for the panel data estimate. It is worth mentioning that Jordà's local projection method gives similar results, except for the cross-country relationship. As shown in Online Appendix D.3, impact responses obtained with VAR and local projection methods are highly correlated, and cross-country relationships between $d\nu^{Y,j}(0)$ and ϵ display the same pattern. However, the slopes of the trend line obtained with the local projection method display substantial differences between tradables and non-tradables which would undermine the quantitative analysis because the slopes by construction should be identical.

exercise may be viewed as tentative as the sectoral effect of a government spending shock varies considerably across countries and there is substantial uncertainty surrounding point estimates given the relatively small number of observations available per country.

The cross-country analysis displayed in Fig. 5 highlights two major findings. First, as shown in the top panels, whether we use labor or output, almost all countries in our sample experience a fall in the relative size of the traded sector as impact responses from the VAR model are below the X-axis. The bottom panels reveal that the reverse is true for the non-traded sector which benefits from the reallocation of inputs. This evidence supports our earlier conjecture according to which a government spending shock is strongly biased toward non-tradables. Second, as can be seen in the top panels of Fig. 5, countries where workers have lower mobility costs experience a larger decline in the share of tradables while the bottom panels show that the relative size of non-tradables increases more in these economies. In sum, our findings reveal that the magnitude of the change in relative sector size following a government spending shock increases with the degree of labor mobility across sectors.

3 Small Open Economy Model with IML

We consider a small open economy populated by a constant number of identical households and firms that have perfect foresight and live forever. The country is small in terms of both world goods and capital markets, and faces a given world interest rate, $r^{\star}$.[19] One sector produces a traded good denoted by the superscript T which can be exported at no cost, invested and consumed domestically. A second sector produces a non-traded good denoted by the superscript N which can be consumed domestically or invested. The traded good is chosen as the numeraire. Time is continuous and indexed by t.

3.1 Households

At each instant the representative household consumes traded and non-traded goods denoted by C^T and C^N, respectively, which are aggregated by means of a CES function:

$$C(t) = \left[\varphi^{\frac{1}{\phi}} \left(C^T(t)\right)^{\frac{\phi-1}{\phi}} + (1-\varphi)^{\frac{1}{\phi}} \left(C^N(t)\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{6}$$

where $0 < \varphi < 1$ is the weight of the traded good in the overall consumption bundle and ϕ corresponds to the elasticity of substitution between traded goods and non-traded goods.

The representative household supplies labor L^T and L^N in the traded and non-traded sectors, respectively. In line with our empirical findings which reveal that labor reallocation

[19] The price of the traded good is determined on the world market and exogenously given for the small open economy. In the empirical analysis, we control for the size of countries as we divide quantities by the working age population. However, countries such as the U.S. are large enough on world goods market to influence the price of its export goods. As shown in Online Appendix H, all results obtained in the main text are robust both qualitatively and quantitatively to the assumption of exogenous terms of trade.

is driven by the rise in both employment and hours per worker, we do not make a distinction between the extensive and intensive margin. To rationalize the rise in the non-traded relative to traded wages, we assume that workers experience a utility loss when shifting hours worked from one sector to another. More specifically, in the lines of Horvath [2000], we consider that hours worked in the traded and the non-traded sectors are imperfect substitutes and aggregated by means of a CES function:

$$L(t) = \left[\vartheta^{-1/\epsilon}\left(L^T(t)\right)^{\frac{\epsilon+1}{\epsilon}} + (1-\vartheta)^{-1/\epsilon}\left(L^N(t)\right)^{\frac{\epsilon+1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon+1}}, \tag{7}$$

and $0 < \vartheta < 1$ parametrizes the weight attached to the supply of hours worked in the traded sector and ϵ is the degree of substitutability in hours worked across sectors. The advantage of our modelling of IML is threefold. First, the formulation (7) lends itself easily to the estimation of the deep parameter ϵ for each country of our sample and thus serves our purpose which is to assess quantitatively the ability of the neoclassical model to account for our evidence. In this regard, the CES form (7) gives rise to a first-order condition which relates the labor flow in sector j to the sectoral wage differential as in Artuç et al. [2010] who specify a dynamic equilibrium model of costly labor adjustment. Second, the case of PML is nested under the assumption that ϵ tends towards infinity which makes our results directly comparable with those obtained in the special case where workers no longer experience switching costs. Finally, the assumption of limited substitutability of labor supply across sectors generates IML without deviating from the tractable representative agent framework which allows us to derive analytical results in Online Appendix C.

The representative agent is endowed with one unit of time, she/he supplies a fraction $L(t)$ as labor, and consumes the remainder $l(t) \equiv 1 - L(t)$ as leisure. At any instant of time, households derive utility from their consumption and experience disutility from working. Assuming that the felicity function is additively separable in consumption and labor, the representative household maximizes the following objective function:

$$U = \int_0^{\infty} \left\{ \ln C(t) - \frac{L(t)^{1+\frac{1}{\sigma_L}}}{1+\frac{1}{\sigma_L}} \right\} e^{-\beta t} \mathrm{d}t, \tag{8}$$

where β is the discount rate and $\sigma_L > 0$ is the Frisch elasticity of labor supply.

Factor income is derived by supplying labor $L(t)$ at a wage rate $W(t)$, and capital $K(t)$ at a rental rate $R(t)$. In addition, households accumulate internationally traded bonds, $B(t)$, that yield net interest rate earnings of $r^{\star}B(t)$. Denoting lump-sum taxes by $T(t)$, households' flow budget constraint states that real disposable income (on the RHS) can be saved by accumulating traded bonds, consumed, $P_C(t)C(t)$, or invested, $P_J(t)J(t)$:

$$\dot{B}(t) + P_C(t)C(t) + P_J(t)J(t) = r^{\star}B(t) + R(t)K(t) + W(t)L(t) - T(t), \tag{9}$$

where $P_C(P(t))$ and $P_J(P(t))$ are consumption and the investment price index, respectively, which are a function of the relative price of non-traded goods, $P(t)$. The aggregate wage index, $W(t) = W\left(W^T(t), W^N(t)\right)$, associated with the labor index (7) is:

$$W(t) = \left[\vartheta \left(W^T(t)\right)^{\epsilon+1} + (1-\vartheta)\left(W^N(t)\right)^{\epsilon+1}\right]^{\frac{1}{\epsilon+1}}, \tag{10}$$

where $W^T(t)$ and $W^N(t)$ are wages paid in the traded and the non-traded sectors. The investment good is produced (costlessly) using traded good and non-traded good inputs according to a constant returns to scale function which is assumed to take a Cobb-Douglas form (in accordance with estimates documented by Bems [2008] for OECD countries):

$$J(t) = \left(\frac{J^N(t)}{\alpha_J}\right)^{\alpha_J}\left(\frac{J^T(t)}{1-\alpha_J}\right)^{1-\alpha_J}, \tag{11}$$

where α_J and $1-\alpha_J$ are the shares of investment expenditure on non-tradables and tradables, respectively. Installation of new investment goods involves increasing and convex costs, assumed quadratic, of net investment. Thus, total investment $J(t)$ differs from effectively installed new capital, $I(t)$:

$$J(t) = I(t) + \frac{\kappa}{2}\left(\frac{I(t)}{K(t)} - \delta_K\right)^2 K(t), \tag{12}$$

where the parameter $\kappa > 0$ governs the magnitude of adjustment costs to capital accumulation, and $0 \leq \delta_K < 1$ is a fixed depreciation rate. Net investment gives rise to capital accumulation according to the dynamic equation:

$$\dot{K}(t) = I(t) - \delta_K K(t). \tag{13}$$

Households choose consumption, worked hours and investment in physical capital by maximizing lifetime utility (8) subject to (9) and (13) together with (12). Denoting by λ and Q' the co-state variables associated with (9) and (13), the first-order conditions characterizing the representative household's optimal plans are:

$$C(t) = \left(P_C(t)\lambda(t)\right)^{-1}, \tag{14a}$$

$$L(t) = \left(W(t)\lambda(t)\right)^{\sigma_L}, \tag{14b}$$

$$\frac{I(t)}{K(t)} = \frac{1}{\kappa}\left(\frac{Q(t)}{P_J(t)} - 1\right) + \delta_K, \tag{14c}$$

$$\dot{\lambda}(t) = \lambda(t)\left(\beta - r^\star\right), \tag{14d}$$

$$\dot{Q}(t) = \left(r^\star + \delta_K\right)Q(t) - \left\{R(t) + P_J(t)\frac{\kappa}{2}\left(\frac{I(t)}{K(t)} - \delta_K\right)\left(\frac{I(t)}{K(t)} + \delta_K\right)\right\}, \tag{14e}$$

and the transversality conditions $\lim_{t\to\infty} \lambda B(t) e^{-\beta t} = 0$, $\lim_{t\to\infty} Q(t)K(t)e^{-\beta t} = 0$. To derive (14c), we used the fact that $Q(t) = Q'(t)/\lambda$ which is the shadow value of capital in terms of foreign assets. In an open economy model with a representative agent who has perfect foresight, a constant rate of time preference and perfect access to world capital markets, we impose $\beta = r^\star$ in order to generate an interior solution. Setting $\beta = r^\star$ into (14d) yields $\lambda = \bar{\lambda}$. Eq. (14c) states that investment is an increasing function of Tobin's q, which is defined as the shadow value to the firm of installed capital, $Q(t)$, divided by its replacement cost, $P_J(t)$. For the sake of clarity, we drop the time argument below provided this causes no confusion.

Applying Shephard's lemma (or the envelope theorem) to consumption expenditure yields the following demand for the traded and non-traded good, respectively:

$$C^T = \varphi \left(1/P_C\right)^{-\phi} C, \qquad C^N = (1-\varphi)\left(P/P_C\right)^{-\phi} C. \tag{15}$$

Denoting the share of non-traded goods in consumption expenditure by α_C, expenditure in non-tradables and tradables is given by $PC^N = \alpha_C P_C C$ and $C^T = (1-\alpha_C) P_C C$.

Applying the same logic for labor, given the aggregate wage index (10), we can derive the allocation of aggregate labor supply to the traded and non-traded sectors:

$$L^T = \vartheta \left(W^T/W\right)^{\epsilon} L, \qquad L^N = (1-\vartheta)\left(W^N/W\right)^{\epsilon} L, \tag{16}$$

where ϵ is the elasticity of labor supply across sectors. As ϵ takes higher values, more labor shifts from one sector to another and thus the degree of labor mobility increases. Denoting by α_L the labor compensation share of non-tradables, labor income from supplying hours worked in the non-traded and the traded sectors are $W^N L^N = \alpha_L W L$ and $W^T L^T = (1-\alpha_L) W L$.

3.2 Firms

Each sector consists of a large number of identical firms which use labor, L^j, and physical capital, K^j, according to a constant returns to scale technology:

$$Y^j = Z^j \left(L^j\right)^{\theta^j} \left(K^j\right)^{1-\theta^j}, \tag{17}$$

where Z^j represents the TFP index which is introduced for calibration purposes only and θ^j corresponds to the share of labor income in the value added of sector j. Firms lease capital from households and hire workers. They face two cost components: a capital rental cost equal to R, and wage rates in the traded and non-traded sectors equal to W^T and W^N, respectively. Both sectors are assumed to be perfectly competitive and thus choose capital and labor by taking prices as given. Since capital can move freely between the two sectors, the value of marginal products in the traded and non-traded sectors equalizes while costly labor mobility implies a wage differential across sectors:

$$Z^T \left(1-\theta^T\right) \left(k^T\right)^{-\theta^T} = P Z^N \left(1-\theta^N\right) \left(k^N\right)^{-\theta^N} \equiv R, \tag{18a}$$

$$Z^T \theta^T \left(k^T\right)^{1-\theta^T} \equiv W^T, \tag{18b}$$

$$P Z^N \theta^N \left(k^N\right)^{1-\theta^N} \equiv W^N, \tag{18c}$$

where $k^j \equiv K^j/L^j$ denotes the capital-labor ratio for sector $j = T, N$.

Aggregating over the two sectors gives us the resource constraint for capital:

$$K^T + K^N = K. \tag{19}$$

3.3 Government

The final agent in the economy is the government. Total government spending, G, goes on goods, G^N, produced by non-traded firms and goods, G^T, produced by traded firms. Both components of government spending are determined exogenously. The government finances public spending by raising lump-sum taxes, T. As a result, Ricardian equivalence obtains and the time path of taxes is irrelevant for the real allocation. We may thus assume without loss of generality that government budget is balanced at each instant:[20]

$$G = G^T + PG^N = T. \tag{20}$$

3.4 Model Closure and Equilibrium

To fully describe equilibrium, we first impose the market clearing condition for non-tradables:

$$Y^N(t) = C^N(t) + J^N(t) + G^N(t). \tag{21}$$

Equality between non-traded output and its demand counterpart is achieved through adjustments to the relative price of non-tradables, $P(t)$.

Regarding the allocation of government consumption to good $j = T, N$, we consider a rise in G which is split between non-tradables and tradables in accordance with their respective shares in government expenditure, i.e.,

$$dG(t) = \omega_{G^N} dG(t) + \omega_{G^T} dG(t). \tag{22}$$

where ω_{G^j} is the share of good j in government consumption which is assumed to be constant over time in line with our evidence. In order to account for the non-monotonic pattern of the dynamic adjustment of $G(t)$ (see Fig. 1(a)), we assume that the deviation of government spending relative to its initial value as a percentage of initial GDP is:

$$\left(G(t) - \tilde{G}\right)/\tilde{Y} = e^{-\xi t} - (1-g)\,e^{-\chi t}, \tag{23}$$

where we denote the steady-state value with a tilde; $g > 0$ parametrizes the magnitude of the exogenous fiscal shock, $\xi > 0$ and $\chi > 0$ parametrize the degree of persistence of the fiscal shock; as ξ and χ take higher values, government spending returns to its initial level more rapidly. More specifically, eq. (23) allows us to generate an inverted U pattern for the endogenous response of $G(t)$: if $\chi > \xi$, we have $\dot{G}(t) > 0$ following the exogenous fiscal shock and then $G(t)$ declines after reaching a peak at some time t.

After inserting appropriate first-order conditions into the non-traded good market clearing condition (21) and the no arbitrage condition (14e), it can be shown that the adjustment of the open economy towards the steady-state is described by a dynamic system

[20] In a longer version of the paper, we allow for distortionary labor taxation and consider a rise in government spending which is debt-financed. Quantitative results displayed in Online Appendix H show that the sectoral impact of fiscal policy is similar to that obtained when assuming a balanced-budget government spending shock.

which comprises two equations that form a separate subsystem in $K(t)$ and $Q(t)$, i.e., $\dot{K}(t) \equiv \Upsilon\left(K(t), Q(t), G(t)\right)$ and $\dot{Q}(t) \equiv \Sigma\left(K(t), Q(t), G(t)\right)$. Linearizing these equations in the neighborhood of the steady-state and using (22) leads to a system of first-order linear differential equations which can be solved by applying standard methods and making use of (23) (see Buiter [1984] who presents the continuous time adaptation of the method of Blanchard and Kahn [1980]):

$$K(t) - \tilde{K} = X_1(t) + X_2(t), \quad Q(t) - \tilde{Q} = \omega_2^1 X_1(t) + \omega_2^2 X_2(t), \tag{24}$$

where we denote the negative eigenvalue by ν_1, the positive eigenvalue by ν_2, and ω_2^i is the element of the eigenvector associated with the eigenvalue ν_i (with $i = 1, 2$) and $X_1(t)$ and $X_2(t)$ are solutions which characterize the trajectory of $K(t)$ and $Q(t)$:[21]

$$X_1(t) = e^{\nu_1 t}\left[\left(K_0 - \tilde{K}\right) + \Gamma_2\left(1 - \Theta_2\right) - \Gamma_1\left(1 - \Theta_1\right)\right] + \Gamma_1\left(e^{-\xi t} - \Theta_1 e^{-\chi t}\right), \tag{25a}$$

$$X_2(t) = -\Gamma_2\left(e^{-\xi t} - \Theta_2 e^{-\chi t}\right), \tag{25b}$$

where K_0 is initial stock of physical capital.

Using the fact that $R(t)K(t) + W(t)L(t) = Y^T(t) + P(t)Y^N(t)$ and inserting the market clearing condition for non-tradables (21) into (9) gives the current account equation:

$$\dot{B}(t) = r^\star B(t) + Y^T(t) - C^T(t) - G^T(t) - J^T(t). \tag{26}$$

Substituting appropriate short-run solutions, eq. (26) can be written as a function of state and control variables, i.e., $\dot{B}(t) \equiv r^\star B(t) + \Xi\left(K(t), Q(t), G(t)\right)$. Linearizing around the steady state, substituting the solutions for $K(t)$ and $Q(t)$ given by (24), solving and invoking the transversality condition leads to the intertemporal solvency condition:[22]

$$\tilde{B} - B_0 = -\frac{\omega_B^1}{\nu_1 - r^\star} + \frac{\omega_B^2}{\xi + r^\star}, \tag{27}$$

where B_0 is the initial stock of traded bonds. The assumption $\beta = r^\star$ implies that temporary policies have permanent effects. In this regard, eq. (27) determines the steady-state change in the net foreign asset position following a temporary fiscal expansion.

4 Quantitative Analysis

In this section, we analyze the effects of a temporary and unanticipated rise in government consumption quantitatively. For this purpose we solve the model numerically.[23] We begin

[21] The coefficients of the Jacobian matrix are partial derivatives evaluated at the steady-state, e.g., $\Upsilon_X = \frac{\partial \Upsilon}{\partial X}$ with $X = K, Q$, and the direct effects of an exogenous change in government spending on K and Q are described by $\Upsilon_G = \frac{\partial \Upsilon}{\partial G}$ and $\Sigma_G = \frac{\partial \Sigma}{\partial G}$, also evaluated at the steady-state. The terms on the RHS of eq. (25) are functions of parameters and read as $\Gamma_i = -\frac{\Phi_i \tilde{Y}}{\nu_1 - \nu_2}\frac{1}{(\nu_i + \xi)}$, $\Phi_1 = \left(\Upsilon_K - \nu_2\right)\Upsilon_G + \Upsilon_Q \Sigma_G$, $\Phi_2 = \left(\Upsilon_K - \nu_1\right)\Upsilon_G + \Upsilon_Q \Sigma_G$, and $\Theta_i = (1 - g)\frac{\nu_i + \xi}{\nu_i + \chi}$ (with $i = 1, 2$).

[22] The terms in the RHS of eq. (27) are functions of parameters. The first term reads as $\omega_B^1 = \left[\Xi_K + \Xi_Q \omega_2^1\right]\left[\left(K_0 - \tilde{K}\right) + \Gamma_2\left(1 - \Theta_2\right) - \Gamma_1\left(1 - \Theta_1\right)\right]$, with $\Xi_K = \frac{\partial \Xi}{\partial K}$, $\Xi_Q = \frac{\partial \Xi}{\partial Q}$, $\Xi_G = \frac{\partial \Xi}{\partial G}$ evaluated at the steady-state. The second term reads as $\omega_B^2 = \Xi_G \tilde{Y}\left(1 - \Theta'\right) + \left[\Xi_K + \Xi_Q \omega_2^1\right]\Gamma_1\left(1 - \Theta_1'\right) - \left[\Xi_K + \Xi_Q \omega_2^2\right]\Gamma_2\left(1 - \Theta_2'\right)$ where $\Theta' = (1 - g)\frac{r^\star + \xi}{r^\star + \chi}$, and $\Theta_i' = \Theta_i \frac{r^\star + \xi}{r^\star + \chi}$ (with $i = 1, 2$).

[23] Technically, the assumption $\beta = r^\star$ requires the joint determination of the transition and the steady state since the constancy of the marginal utility of wealth implies that the intertemporal solvency condition

by discussing the parameter values before turning to the short-term consequences of higher government consumption.

4.1 Calibration

To calibrate our model, we estimated a set of parameters so that the initial steady state is consistent with the key empirical properties of a representative OECD economy. As summarized in Table 1, our sample covers the sixteen OECD economies in our dataset and our reference period for the calibration is running from 1990 to 2007. The choice of this period was dictated by data availability for all the countries in the sample. Table 2 summarizes our estimates of the non-tradable content of GDP, employment, consumption, gross fixed capital formation, government spending, labor compensation, and gives the share of government spending on traded and non-traded goods in their respective sectoral output, the shares of labor income in output in both sectors, for all countries in our sample. Moreover, columns 12-14 of Table 2 display investment expenditure and government spending as a percentage of GDP together with the labor income share, respectively, for the whole economy. To capture the key properties of a typical OECD economy, chosen as the baseline scenario, we take unweighted average values, as shown in the last line of Table 2. As summarized in Table 3, some of the parameter values can be taken directly from the data, but others like φ, ϑ, δ_K together with initial conditions (B_0, K_0) need to be calibrated endogenously to fit a set of aggregate and sectoral ratios. We choose the model period to be one year and therefore set the world interest rate, $r^\star$, which is equal to the subjective time discount rate, β, to 4%.

< Please insert Tables 2-3 about here >

The degree of labor mobility captured by ϵ is set to 0.75, in line with the average of our estimates shown in the last line of column 16 of Table 2. Excluding the estimates for Denmark and Norway which are not statistically significant at 10% over 1970-2007, estimated values of ϵ range from a low of 0.22 for the Netherlands to a high of 1.39 for the U.S. and 1.64 for Spain. To explore the implications of the degree of labor mobility for sectoral effects, we allow for ϵ to vary between 0.22 and 1.64 .

Building on our panel data estimates, the elasticity of substitution ϕ between traded and non-traded goods is set to 0.77 in the baseline calibration since this value corresponds to the average of estimates shown in the last line of column 15 of Table 2.[24] The weight of consumption in non-tradables $1-\varphi$ is set to 0.51 to target a non-tradable content in total consumption expenditure, α_C, of 53%, in line with the average of our estimates shown in the last line of column 2. In our baseline parametrization, we set the intertemporal elasticity of substitution for labor supply σ_L to 0.4, in line with evidence reported by Fiorito and

(27) depends on eigenvalues and eigenvectors' elements, see e.g., Turnovsky [1997].

[24]The average value is calculated by excluding estimates for Italy which are negative.

Zanella [2012]. The weight of labor supply to the non-traded sector, $1-\vartheta$, is set to 0.68 to target a share of the non-tradable sector in total hours worked of 67%, in line with the average of our estimates shown in the last line of column 5 of Table 2.

We now describe the calibration of production-side parameters. We assume that physical capital depreciates at a rate δ_K of 6% to target an investment-to-GDP ratio of 21% (see column 12 of Table 2). Labor income shares in the traded (θ^T) and non-traded sectors (θ^N) are set to 0.58 and 0.68, respectively, which correspond roughly to the averages for countries with $k^T > k^N$ (see columns 9 and 10 of Table 2). Such values give an aggregate labor income share of 64% (see the last line of column 14 of Table 2). In line with our evidence shown in the last column of Table 2, we assume that traded firms are 28 percent more productive than non-traded firms; hence we set Z^T and Z^N to 1.28 and 1 respectively. We set the share of investment expenditure on non-tradable goods, α_J, to 64%, in accordance with the evidence shown in column 3 of Table 2. We choose the value of parameter κ so that the elasticity of I/K with respect to Tobin's q, i.e., Q/P_J, is equal to the value implied by estimates in Eberly, Rebelo, and Vincent [2008]. The resulting value of κ is equal to 17.

As shown in column 4 of Table 2, the non-tradable content of government spending, ω_{G^N}, averages 90%. We set government consumption on non-traded goods, G^N, and traded goods, G^T, so as to yield a non-tradable share of government spending, ω_{G^N}, of 90%, and government spending as a share of GDP to 20%.

We choose initial conditions for B_0 and K_0 so that trade is initially balanced. Since net exports are nil and $P_J I/Y = 21\%$ and $G/Y = 20\%$, the accounting identity according to which GDP is equal to the sum of the final uses of goods and services, leads to a consumption-to-GDP ratio of $P_C C/Y = 59\%$. It is worthwhile mentioning that the non-tradable content of GDP is determined endogenously by the non-tradable content of consumption, α_C, investment, α_J, and government expenditure, ω_{G^N}, along with the consumption-to-GDP ratio, ω_C, and the investment-to-GDP ratio, ω_J. More precisely, dividing the non-traded good market clearing condition (21) by Y leads to the non-tradable content of GDP:

$$PY^N/Y = \omega_C \alpha_C + \omega_J \alpha_J + \omega_{G^N} \omega_G = 63\%, \tag{28}$$

where $\omega_C = 59\%$, $\alpha_C = 53\%$, $\omega_J = 21\%$, $\alpha_J = 64\%$, $\omega_{G^N} = 90\%$, and $\omega_G = 20\%$. According to (28), the ratios we target are consistent with a non-tradable content of GDP of 63% found in the data (see the last line of column 1 of Table 2).

In order to capture the endogenous response of government spending to an exogenous fiscal shock, we assume that the dynamic adjustment of government consumption is governed by eq. (23). In the quantitative analysis, we set $g = 0.01$ so that government consumption increases by 1% of initial GDP. To calibrate ξ and χ that parametrize the shape of the dynamic adjustment of government consumption along with its persistence, we proceed as follows.[25] Because $G(t)$ peaks after one year, we have $dG(1)/Y = \left[e^{-\xi} - (1-g)\, e^{-\chi}\right] = g' > g$ with $g' = 0.011265$ and $\dot{G}(1)/Y = -\left[\xi e^{-\xi} - \chi\,(1-g)\, e^{-\chi}\right] = 0$. Solving the system gives us $\xi = 0.408$ and $\chi = 0.415$. Left-multiplying eq. (23) by ω_{G^j} (with $j = N, T$) gives the dynamic adjustment of sectoral government consumption to an exogenous fiscal shock:

$$\omega_{G^j}\left(G(t) - \tilde{G}\right)/Y = \omega_{G^j}\left[e^{-\xi t} - (1-g)\, e^{-\chi t}\right], \tag{29}$$

where ω_{G^j} is the fraction of government consumption in good j. To determine (29), we assume that the parameters that govern the persistence and shape of the response of sectoral government consumption are identical across sectors, while the sectoral intensity of the government spending shock is constant over time and thus corresponds to the share of government final consumption expenditure on good j, in line with the VAR evidence documented in subsection 2.6.[26]

As the baseline scenario, we take the model with IML and capital adjustments costs and we set $\epsilon = 0.75$ and $\kappa = 17$. We also conduct a sensitivity analysis with respect to these two parameters by setting alternatively ϵ to 0.22 and 1.64 and κ to 0.

4.2 Results

In this subsection, we analyze in detail the role of IML in shaping the dynamics of the open economy in response to a government spending shock. Our primary objective is to explain how workers' costs of switching sectors change the model's predictions in a way that makes them consistent with our empirical findings on fiscal policy transmission, especially the responses of sectoral value added shares.

Table 4 shows the simulated impact effects of an exogenous and unanticipated increase in government consumption by 1% of GDP while column 1 shows impact responses from our VAR model for comparison purposes. Column 2 shows results for the baseline model which we contrast with those obtained when we impose PML (i.e., we set $\epsilon \to \infty$) and abstract from capital installation costs (i.e., we set $\kappa = 0$), as displayed in column 7. Other columns give the results for the alternative scenarios discussed below. While in Table 4, we restrict our attention to impact responses, in Fig. 6 and 7 we show the dynamic adjustment to an unanticipated increase in government consumption by 1% of GDP. Figures display the model predictions together with the respective VAR evidence. In each panel, the solid blue line displays the point estimate of the VAR model, with the shaded area indicating the 90% confidence bounds; the thick solid black line with squares shows theoretical responses from the baseline model.

[25]Our calibration of the government consumption shock is based on estimates of the first VAR model $z_{i,t} = [g_{i,t}, y_{i,t}, l_{i,t}, je_{i,t}, w_{C,i,t}]$.

[26]The mapping between the non-tradable content of the government spending shock and the non-tradable content of government spending will be useful when we calibrate the model to country-specific data since the number of observations per country for sectoral government consumption is too small to estimate empirically the contribution of G^N to the identified government spending shock for each economy.

As shown in the left panel of Fig. 6, the endogenous response of government spending to an exogenous fiscal shock that we generate theoretically by specifying the law of motion (23) reproduces very well the dynamic adjustment from the VAR model, as the black line with squares and the blue line cannot be differentiated. The right panel of Fig. 6 contrasts empirical responses of sectoral government consumption to an exogenous fiscal shock with theoretical responses derived from eq. (29) by setting ω_{G^N} and ω_{G^T} to 0.9 and 0.1, respectively. The upper and lower lines show the responses of G^N and G^T, respectively. Overall, the theoretical responses perform well in reproducing the evidence and thus the assumptions underlying the dynamic equation (29) which governs the adjustment of G^j are consistent with data.

< Please insert Table 4 and Figures 6-7 about here >

Aggregate effects. We need to start with the whole picture since aggregate and sectoral effects are strongly intertwined. The rise in total hours worked and in real GDP determine the size of sectoral fiscal multipliers if the reallocation of resources were absent (see the first term on the RHS of eq. (2)). Differently, adjustments in investment and the current account determine the size of the reallocation effects by influencing excess demand in goods markets (see the second term on the RHS of eq. (2)).

Impact effects of a government spending shock on GDP, its demand components and labor market variables are shown in panels A and B of Table 4.[27] By producing a negative wealth effect, a balanced-budget government spending shock leads agents to supply more labor, which in turn increases real GDP. As shown in panel A, whether we impose PML (columns 7-8) or assume IML (column 2), both models understate the rise in total hours worked and in real GDP. Because labor mobility costs put upward pressure on the aggregate wage, the positive response of L and the size of the aggregate fiscal multiplier are amplified with IML which makes the model closer to the evidence.

A model imposing PML overstates the current account deficit or predicts a current account surplus depending on whether capital adjustment costs are included or not (see columns 8 and 7). On the contrary, the baseline model (see column 2) is able to produce a decline in investment and the current account on impact which accords well with our VAR estimates. Intuitively, following a temporary government spending shock, households lower their savings in order to avoid a large decrease in their consumption and/or mitigate the rise in their labor supply. Lower savings results in a decline in investment or the current account or both. With IML, capital shifts toward the non-traded sector which lowers k^T and increases the return on domestic capital. As a result, the fall in investment is mitigated and a current account deficit appears. Capital adjustment costs further moderate the decline in

[27] For reasons of space, the empirical and theoretical responses of GDP, its demand components and labor market variables are contrasted in Online Appendix G.1. It is worthwhile mentioning that the simulated responses from the baseline model lie within the confidence interval along the transitional adjustment for all aggregate variables, with the exception of the real consumption wage.

investment and amplify the current account deficit (see column 2 where $\kappa = 17$) compared with a model imposing $\kappa = 0$ (see column 6).

We turn to the sectoral and reallocation effects. Panels C and D of Table 4 show impact responses of labor and product market variables, respectively. In Fig. 7, we report the model predictions together with the VAR evidence of sectoral variables. In Fig. 7, we also contrast the responses from the benchmark setup with those from a model imposing $\epsilon \to \infty$ and $\kappa = 0$, as displayed in the dotted black line.

PML. Focusing first on impact responses, column 7 of Table 4 shows that a model imposing PML can generate qualitatively a rise in the share of non-tradables but substantially understates its magnitude. More specifically, the model predicts a rise in the value added share of non-tradables by 0.24%, a value below what is estimated empirically (i.e., 0.35%). Because the model also understates the increase in real GDP, it produces a rise in Y^N by 0.28% which is far below the estimated value (0.70%). The inability of the model to account for the reallocation and distributional effects across sectors of a rise in government spending we document empirically lies in the combined effect of the absence of capital adjustment and labor mobility costs. The dramatic fall in investment caused by the absence of capital adjustment costs mitigates the excess demand for non-traded goods and thus the incentives for reallocating productive resources toward the non-traded sector. The absence of mobility costs leads labor to move instantaneously toward the non-traded sector to eliminate the excess demand in the non-traded goods market which further mitigates the incentives to shift capital toward this sector by leaving the relative price of non-tradables unchanged, in contradiction with our evidence. The relative wage of non-tradables is also unchanged because sectoral wages increase by the same amount. To assess the respective role of labor mobility and capital adjustment costs, we analyze below two restricted versions of the model where one of the two features is, respectively, shutdown.

PML and capital installation costs. Column 8 of Table 4 shows the predictions of a model imposing PML but allowing for capital installation costs. By mitigating the decline in investment, capital installation costs amplify the excess demand for non-tradables. However, without labor mobility costs, high incentives to shift productive resources toward the non-traded sector now lead the model to overstate the rise in the labor and value added share of non-tradables (0.74% and 0.76%, resp.) which are about three and two times larger, respectively, what is estimated empirically. Intuitively, workers are willing to shift their whole time to the sector that pays the highest wage. As a result, sectoral labor and thus sectoral output become unrealistically sensitive to a change in the relative price, the latter appreciating by 0.02% instead of 1.06% in the data.

IML and capital installation costs. In contrast, as displayed in column 2, the ability of the model with capital adjustment costs to account for the reallocation and distributional effects across sectors of a rise in government spending improves, as long as we allow for IML. To begin with, the baseline model can account for the rise in the relative wage. Intuitively, non-traded firms are encouraged to produce and thus to hire more to meet additional demand. As workers experience intersectoral mobility costs, non-traded firms must pay higher wages to attract workers which raises the relative wage, Ω, by 1.44%.

Because labor shifts toward the non-traded sector, the baseline model predicts a rise in hours worked in non-tradables by 0.44%, which accords well with the evidence shown in column 1. Labor reallocation pushes up non-traded output by 0.50%, the response being almost double that obtained with PML (see column 7). Intuitively, labor mobility costs put upward pressure on the aggregate wage which amplifies the rise in labor supply and thus further raises output in the non-traded sector since it is relatively more labor intensive.

As long as there is a difficulty in reallocating labor across sectors, excess demand shows up in the non-traded goods market. As a result, the price of non-traded goods relative to traded goods appreciates by 0.88%, as shown in the fourth line of panel D. The appreciation in P triggers a reallocation of capital and labor toward the non-traded sector which raises its output share by 0.38% of GDP, a value close to our estimates.

IML and no capital installation costs. To emphasize the importance of capital installation costs, column 6 reports impact responses from a model assuming IML while setting $\kappa = 0$. As investment is crowded out by a larger amount than if capital were subject to adjustment costs, the excess demand in the non-traded goods market is lower so that P appreciates less, resulting in smaller shifts of labor and capital toward the non-traded sector. As a result, the model generates a rise in the labor and value added share of non-tradables by 0.17% and 0.27% which are below the values we estimate empirically (0.27% and 0.35%, resp.).

Financial Openness. As shown analytically in Online Appendix C.2, in addition to IML, financial openness and the tradability of goods also matter in determining the responses of sectoral shares. A way to gauge the importance of access to foreign borrowing for sectoral effects of fiscal policy is to decompose the change in the share of non-tradables in demand components:

$$d\nu^{Y,N}(0) = \underbrace{\left(\omega_{G^N} - \nu^{Y,N}\right) g}_{=+0.27} + \underbrace{\omega_C \left[\alpha_C \hat{C}^N(0) - \nu^{Y,N} \hat{C}(0)\right]}_{=-0.06} + \underbrace{\omega_J \left[\alpha_J \hat{J}^N(0) - \nu^{Y,N} \hat{J}(0)\right]}_{=-0.04} \underbrace{-\nu^{Y,N} dCA(0)/Y}_{=+0.21},$$

where $g = dG(0)/Y$ normalized to 1% of GDP on impact. The figures below each demand component add up to 0.38% of GDP. When we abstract from general equilibrium effects, i.e., when the responses of private sector's demand components are shut down (see eq. (3)), $d\nu^{Y,N}(0)$ collapses to the first term on the RHS of the above equation which indicates that the relative intensity of non-tradables in the government spending shock causes the share of non-tradables in real GDP to increase by 0.27% of GDP. The second and the third term on the RHS reveal that changes in consumption and investment following a government spending shock do not favor the non-traded sector since higher prices for non-tradables tilt consumption and investment toward traded goods. However, these relative price effects are more than offset by the impact of the current account deficit on expenditure on non-tradables, as captured by the last term on the RHS of the above equation which tilts the demand shock toward non-tradables. If the current account were unresponsive to the government spending shock, the share of non-tradables would rise by 0.17% of GDP only, an amount which is half what is estimated empirically. Conversely, the ability of the open economy to borrow abroad increases the share by non-tradables from 0.17% to 0.38%.[28]

Intensity of non-tradables in the government spending shock. To further emphasize the importance of general equilibrium effects, we set $\omega_{G^N} = \nu^{Y,N}$ in column 3 of Table 4. If private demand components were unresponsive, labor reallocation should be absent because the rise in government spending is split between sectors in accordance with their relative size. However, panel C indicates that labor shifts toward the non-traded sector whose output share increases by 0.25% of GDP (see panel D). While higher non-traded prices tilt the demand shock toward traded goods which lowers the share of non-tradables by -0.05% of GDP, the current account deficit by 0.48% of GDP increases the share of non-tradables by 0.30% of GDP.

Effect of higher labor mobility. As we move from column 4 to column 5 of Table 4, the utility loss resulting from the shift from one sector to another is reduced. As shown analytically in Online Appendix C, a rise in ϵ exerts two opposite effects on sectoral output shares: while workers are more willing to shift across sectors, P appreciates less, which mitigates the incentive for labor reallocation. We find numerically that raising ϵ from 0.22 to 1.64 amplifies the rise in the output share of non-tradables from 0.26% to 0.49% of GDP, in accordance with our evidence documented in section 2.7. Thus, the former effect more than offsets the latter.

Sectoral share/sectoral multiplier/aggregate multiplier. As shown in eq. (2), the sectoral fiscal multiplier is equal to the fraction of the aggregate fiscal multiplier received by the sector plus the change in the sectoral share. Across all scenarios, the change in sectoral value added is positively correlated with the change in the sectoral share. When ϵ is increased from 0.22 (column 4) to 1.64 (column 5), the share of non-tradables almost doubles while the fiscal multiplier for non-traded output increases from 0.41 to 0.59. The reason why the rise in non-traded value added does not double lies in the fact that as ϵ is increased, non-traded wages and thus W increase less which mitigates the rise in L. On

[28]Conversely, capital inflows exert a negative impact on the output share of tradables since foreign borrowing leads households to import traded goods, thus producing a trade balance deficit. Since less resources are necessary to produce traded goods domestically, inputs are reallocated toward the non-traded sector.

the contrary, raising ω_{G^N} increases both the non-traded and total output multipliers of government spending (i.e., we move from column 3 to column 2). Intuitively, by targeting the sector that has the highest labor compensation share, IML puts upward pressure on wages in this sector which in turn increases W and amplifies the response of labor supply to the government spending shock.

Dynamics. Turning to the adjustment of sectoral variables following a government spending shock as shown by the solid black line with squares in Fig. 7, the dynamics of P and Ω are captured fairly well by the baseline model. As G falls and is restored to its initial level, excess demand in the non-traded goods market is reduced, which depreciates P along the transitional path, as shown in Fig. 7(a). Decreasing prices of non-tradables relative to tradables encourage non-traded firms to reduce hours worked and thus to lower output, in line with the evidence in Fig. 7(h) and 7(g). Because W^N falls relative to W^T during the transitional adjustment, as shown in Fig. 7(b), labor is reallocated toward the traded sector, which recovers gradually, while both hours worked and output remain below their initial levels for almost ten years. As shown in Fig. 7(e) and 7(d), the model tends to somewhat understate the contraction of L^T and Y^T in the medium run.

Conversely, as displayed by the dotted black line, the performance of the model declines when imposing $\epsilon \to \infty$ and setting $\kappa = 0$; in this special case, the model predicts a flat temporal path for Ω and P, which is in conflict with the evidence; while it understates the responses of sectoral output shares on impact, the model overstates their changes along the transitional path. The reason is that the capital stock falls sharply in the short-run and then recovers rapidly after two years, resulting in sharp changes in the relative size of sectors due to the Rybczynski effect.[29]

Taking stock. Overall, the baseline model with IML and capital adjustment costs captures well the sectoral effects of an exogenous increase in government spending but is subject to some caveats. As shown in Fig. 7(d), the model tends to somewhat overstate the decline in traded output over the first two years and understate its contraction afterwards. While the theoretical response of non-traded output lies within the confidence bounds of the point estimate, as can be seen in Fig. 7(g), the model still overstates the rise in Y^N relative to its trend after two years. We conducted several robustness checks with respect to the value of parameters we set and by relaxing several assumptions of our model and found that similar results obtain. Motivated by the rise in aggregate TFP following a rise in government spending documented by Jørgensen and Ravn [2018], we have investigated whether sectoral TFPs, Z^j, respond to a government spending shock. According to our empirical results, traded TFP increases above trend over the first two years and then declines which could explain the difficulty to reproduce well the dynamics for Y^T when keeping Z^T fixed. On the

[29] In Online Appendix G.3, we contrast the dynamic adjustment from the baseline model with the responses from the restricted model where one of the two features is shut down.

contrary, Z^N falls significantly after one year and remains below trend which may provide a rationale for the (moderate) discrepancy between empirical and theoretical responses for Y^N when assuming an exogenous non-traded TFP. Empirical results can be found in Online Appendix D.5 and we leave further analysis of these issues for future research.

4.3 Cross-Country Differences: Taking the Model to Data

We have shown above that the performance of the neoclassical model in replicating the evidence related to sectoral effects of a government spending shock improves as long as we allow for IML and capital adjustment costs. We now move a step further and assess the ability of the model to generate a similar cross-country relationship between the degree of labor mobility and changes in the relative size of sectors to that in the data.

To compute the impact responses of sectoral output shares to a government spending shock numerically, we calibrate our model to match the key characteristics of the OECD economies in our sample summarized in Table 2. While we explore the sectoral effects of a rise in G by 1% of GDP for each country in our sample, to be consistent with the calibration to a representative OECD economy described in section 4.1, we assume that the increase in public purchases is split between non-tradables and tradables in accordance with their respective shares in government spending (see column 4 of Table 2). Since the goal of our exercise below is to compare the rise in the share of non-tradables across countries when we allow for international differences in the degree of labor mobility across sectors, we exclude Australia and Ireland from our quantitative exercise as these two economies experience a fall the share of non-tradables and/or a rise in the share of tradables.[30]

< Please insert Figures 8-9 about here >

To explore the cross-country relationship quantitatively, we first plot in Fig. 8 the simulated responses of sectoral output shares on the vertical axis against the degree of labor mobility captured by the parameter ϵ on the horizontal axis. Impact changes in non-traded output relative to real GDP range from 0.26% of GDP for the Netherlands to 0.49% of GDP for Spain. Fig. 8(a) and 8(b) also show that these differences in the responses of sectoral output shares are positively correlated with the measure of the degree of labor mobility across sectors. This result thus reveals that the sectoral impact of fiscal policy increases with the degree of labor mobility, which accords with our evidence. Quantitatively, as we move along the trend line shown in Fig. 8(a), our model predicts that a country with a low degree of labor mobility, as captured by a value of ϵ of 0.15, will experience a decline in the output share of tradables of 0.3% of GDP, while a country with a higher degree of

[30] We find empirically that the output share of non-tradables does not increase on impact in Australia and Ireland. This result is puzzling since ω_{G^N} averages 88% and 90% for Australia and Ireland and thus the government spending shock should be biased toward non-tradables. Motivated by the evidence documented by Jørgensen and Ravn [2018], in Online Appendix D.5, we explore the responses of sectoral TFP to a government spending shock for these two countries and find that their movements overturn the positive impact of the government spending shock on the output share of non-tradables.

labor mobility, as captured by a value of ϵ of 1.5, will face a fall by 0.45% of GDP, a decline which is 50% larger.

In Fig. 9, we contrast the cross-country relationship from the calibrated baseline model shown by the solid black line with circles with the cross-country relationship from the VAR model shown by the solid blue line. When we calibrate our model to cross-country data, we obtain a correlation between the responses of sectoral output shares and the measure of the degree of labor mobility of -0.11 for tradables (t−stat = -5.90) and 0.11 for non-tradables (t−stat = 5.90). While it tends to understate the changes in the relative size of sectors since the cross-country relationship is higher for tradables and lower for non-tradables, the model is able to generate a cross-country relationship between the responses of sectoral output shares and the degree of labor mobility which is quite similar to that in the data.

5 Conclusion

This paper contributes to the literature related to the effects of a government spending shock both empirically and theoretically. From an empirical point of view, we provide new evidence on the sectoral effects of a shock to government consumption. Using a panel of 16 OECD countries over the period 1970-2007 and adopting a SVAR approach, our estimates reveal that the non-traded sector is very intensive in government spending shocks which trigger a shift of resources toward this sector. More precisely, our evidence reveals that the non-tradable content of the government spending shock averages 90%, while the reallocation of inputs alone contributes to 50% of non-traded output growth on impact. While the shift of labor is responsible for half of the increase in non-traded hours worked, our evidence points to the presence of labor mobility costs, as we detect empirically a significant increase in non-traded wages relative to traded wages. The degree of labor mobility across sectors appears empirically to be a key determinant of the response of the share of non-tradables to a government spending shock, which varies across time and space. Our estimates show that time-declining responses of sectoral shares are highly correlated with lower intersectoral reallocation of labor over time following a rise in government spending. Turning to international differences, we find that the relative size of the non-traded sector increases more in economies where the degree of labor mobility across sectors is higher.

To rationalize our evidence, we develop a two-sector open economy model with two key features. First, we allow the non-traded sector to be highly intensive in the government spending shock in line with our empirical findings while financial openness further biases the demand shock toward toward non-tradables. Second, as in Horvath [2000], agents cannot costlessly reallocate hours worked from one sector to another. Because mobility is costly in utility terms, workers demand higher wages in order to compensate for their cost of switching sectors. Calibrating the model to a representative OECD economy and considering a rise in government consumption biased toward non-tradables, we find quantitatively that the model can account for the panel VAR evidence, in particular the changes in relative sector size, as long as we allow for adjustment costs to physical capital accumulation along with IML. The former feature mitigates the decline in investment and thus guarantees that the excess demand and therefore incentives to shift resources toward the non-traded sector are high enough. By reducing the elasticity of labor supply across sectors, the latter feature hampers the reallocation of labor and thus allows the model to match the changes in relative sector size quantitatively. In contrast, the restricted version of the model where one of the two features is shut down fails to account for the evidence.

When we calibrate our baseline model to each OECD economy in our sample, our numerical results reveal that international differences in the degree of labor mobility generate a wide dispersion in the responses of sectoral output shares as changes in the relative size of sectors are fifty percent stronger in countries with the highest degree of labor mobility than in economies with the lowest labor mobility. Importantly, our model reproduces pretty well the cross-country relationship between the degree of labor mobility and the responses of sectoral output shares that we estimate empirically.

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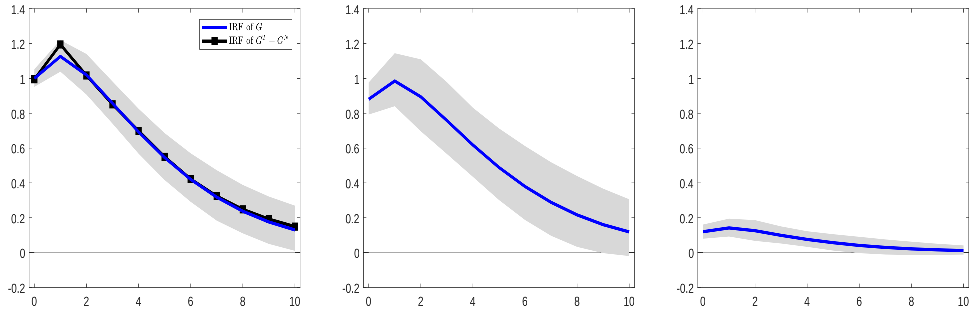
Table 1: Sample Range for Empirical and Numerical Analysis

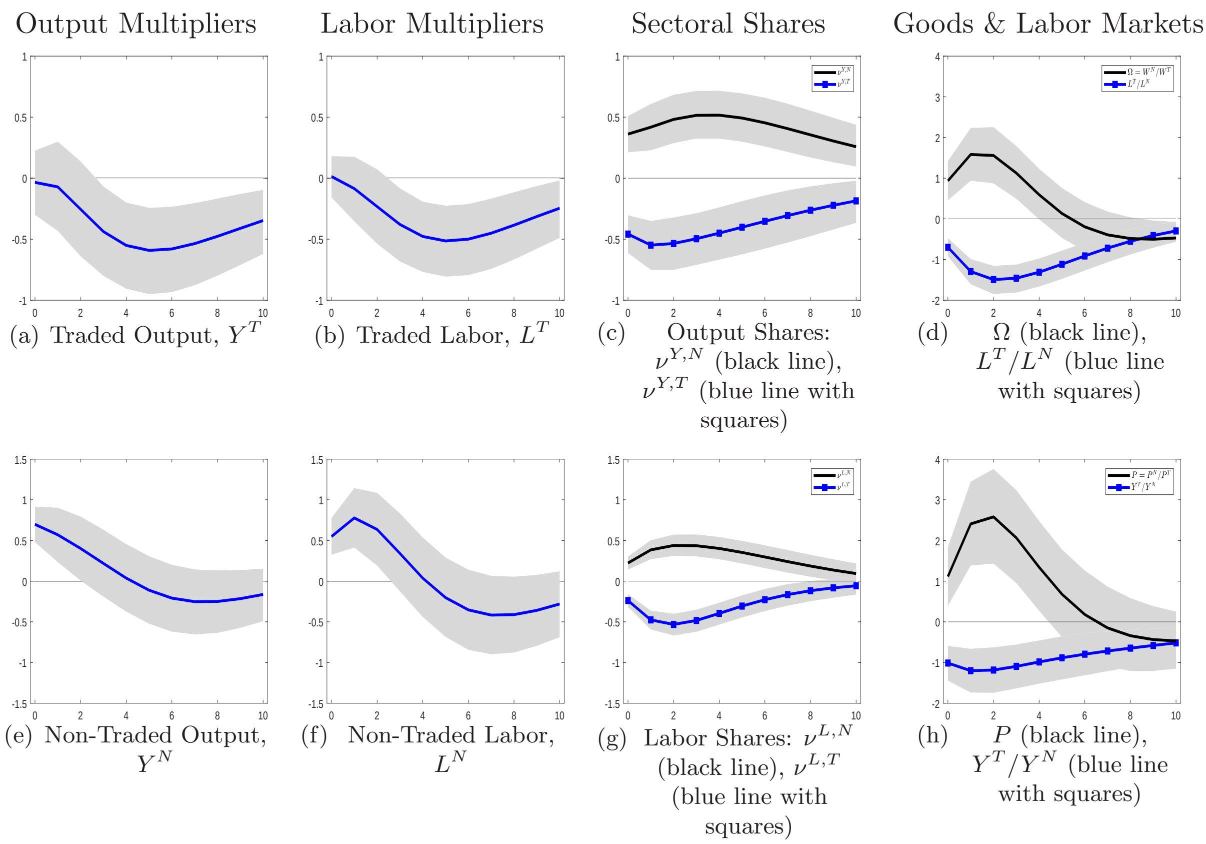
Country		Sectoral Effects		Sectoral Decomposition of G		Model Calibration
		Period	Obs.	Period	Obs.	Period
Australia	(AUS)	1970 - 2007	38	–	–	1990 - 2007
Austria	(AUT)	1970 - 2007	38	1995 - 2015	21	1990 - 2007
Belgium	(BEL)	1970 - 2007	38	1995 - 2015	21	1990 - 2007
Canada	(CAN)	1970 - 2007	38	–	–	1990 - 2007
Denmark	(DNK)	1970 - 2007	38	1995 - 2015	21	1990 - 2007
Spain	(ESP)	1970 - 2007	38	1995 - 2015	21	1990 - 2007
Finland	(FIN)	1970 - 2007	38	1995 - 2015	21	1990 - 2007
France	(FRA)	1970 - 2007	38	1995 - 2015	21	1990 - 2007
Great Britain	(GBR)	1970 - 2007	38	1995 - 2015	21	1990 - 2007
Ireland	(IRL)	1970 - 2007	38	1995 - 2015	21	1990 - 2007
Italy	(ITA)	1970 - 2007	38	1995 - 2015	21	1990 - 2007
Japan	(JPN)	1974 - 2007	34	–	–	1990 - 2007
Netherlands	(NLD)	1970 - 2007	38	1995 - 2015	21	1990 - 2007
Norway	(NOR)	1970 - 2007	38	1995 - 2015	21	1990 - 2007
Sweden	(SWE)	1970 - 2007	38	1995 - 2015	21	1990 - 2007
United States	(USA)	1970 - 2007	38	1995 - 2015	21	1990 - 2007
Total number of obs.			604		273	
Main data sources		OECD Economic Outlook EU KLEMS & OECD STAN		OECD Economic Outlook OECD COFOG		OECD Economic Outlook EU KLEMS & OECD STAN

Notes: The column 'period' gives the first and last observation available. Obs. refers to the number of observations available for each country. Data to construct time series for sectoral government consumption expenditure are available for all the countries in our sample except Canada. In efforts to have a balanced panel and time series of a reasonable length, Australia (1998-2015) and Japan (2005-2015) are removed from the sample, which leaves us with 13 OECD countries over the period 1995-2015.



(a) Government Final Consumption Expenditure, G (b) Gov. Exp. on Non-Tradables, G^N (c) Gov. Exp. on Tradables, G^T

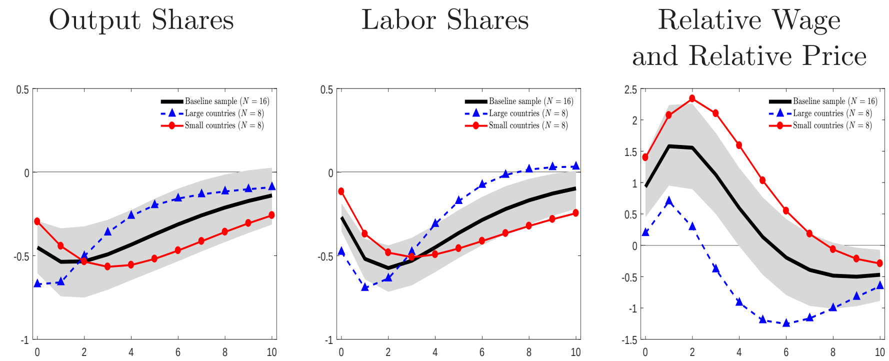
Figure 1: Effects of Unanticipated Government Spending Shock on Government Final Consumption Expenditure and its Non-Tradable and Tradable Components. Notes: Exogenous increase of government consumption by 1% of GDP. The government spending shock is identified by estimating a VAR model that includes real government final consumption expenditure, GDP (constant prices), total hours worked, private fixed investment, and the real consumption wage. The baseline response of government final consumption expenditure is displayed by the solid blue line in the left panel with shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; sample: 16 OECD countries, 1970-2007, annual data. The responses of (logged) government final consumption expenditure on non-tradables (i.e., g^N) and tradables (i.e., g^T) to the identified government spending shock (in the baseline VAR model) are displayed by solid blue lines in panels (b) and (c) with shaded area indicating the 90 percent confidence bounds; sample: 13 OECD countries, 1995-2015, annual data. The black line with squares in the left panel displays the dynamic response of government final consumption expenditure which has been computed by summing mean responses of government consumption expenditure on non-tradables and tradables.



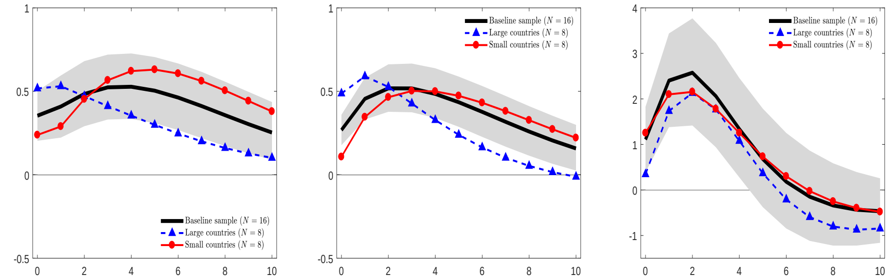
(a) Traded Output, Y^T (b) Traded Labor, L^T (c) Output Shares: $\nu^{Y,N}$ (black line), $\nu^{Y,T}$ (blue line with squares) (d) Ω (black line), L^T/L^N (blue line with squares)

(e) Non-Traded Output, Y^N (f) Non-Traded Labor, L^N (g) Labor Shares: $\nu^{L,N}$ (black line), $\nu^{L,T}$ (blue line with squares) (h) P (black line), Y^T/Y^N (blue line with squares)

Figure 2: Sectoral Effects of Unanticipated Government Spending Shock. Notes: Exogenous increase of government consumption by 1% of GDP. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in output units (sectoral output, sectoral output shares), percentage deviation from trend in labor units (sectoral labor, sectoral labor shares), deviations from trend (ratio of traded value added to non-traded value added, ratio of hours worked of tradables to hours worked of non-tradables), and percentage deviation from trend (relative price, relative wage). Blue and black lines display point estimates. Solid black lines show responses of $\nu^{Y,N}$ in Fig. 2(c), $\nu^{L,N}$ in Fig. 2(g), Ω, in Fig. 2(d), P in Fig. 2(h); Blue lines with squares show responses of $\nu^{Y,T}$ in Fig. 2(c), $\nu^{L,T}$ in Fig. 2(g), L^T/L^N, in Fig. 2(d), Y^T/Y^N in Fig. 2(h). Shaded areas: bootstrapped 90% confidence intervals; sample: 16 OECD countries, 1970-2007, annual data.

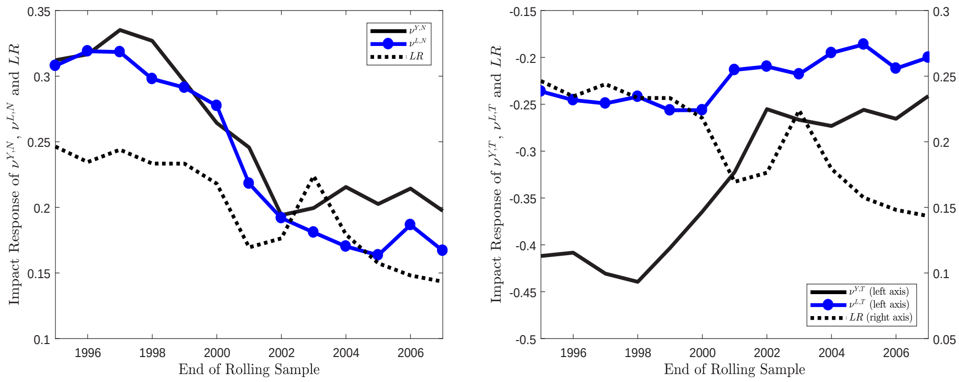


(a) Output Share of Tradables, $\nu^{Y,T}$ (b) Labor Share of Tradables, $\nu^{L,T}$ (c) Relative Wage, $\Omega = W^N/W^T$



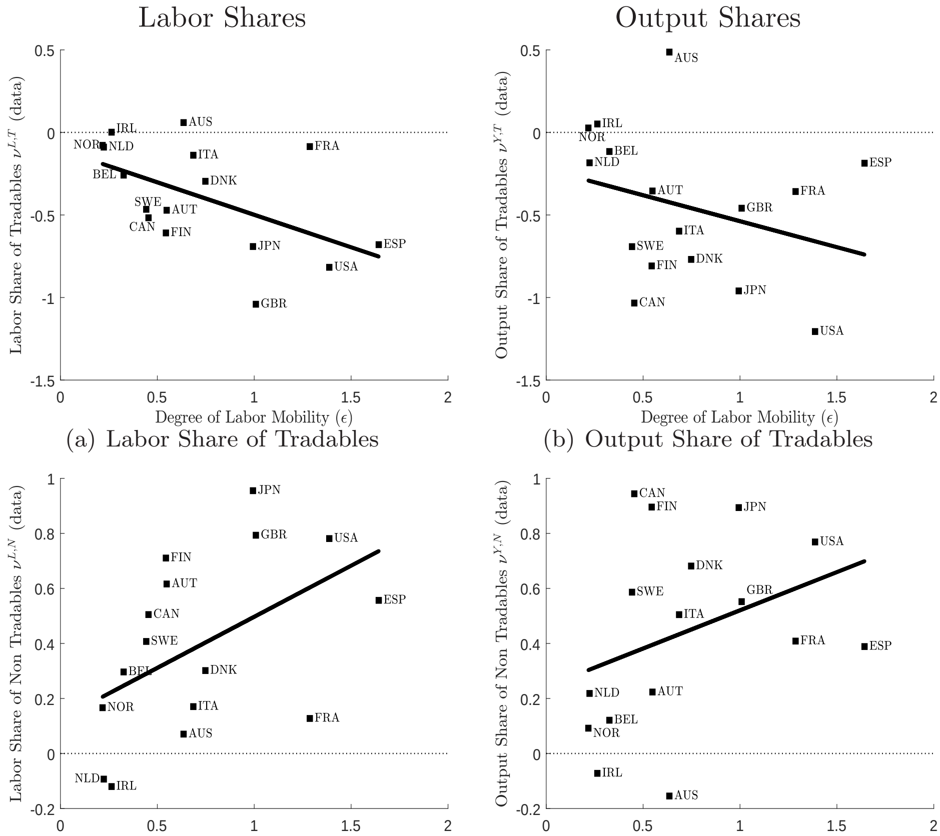
(d) Output Share of Non Tradables, $\nu^{Y,N}$ (e) Labor Share of Non Tradables, $\nu^{L,N}$ (f) Relative Price, $P = P^N/P^T$

Figure 3: Sectoral Effects of Unanticipated Government Spending Shock across Countries' Size. Notes: Exogenous increase of government consumption by 1% of GDP. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in output units (sectoral output shares), percentage deviation from trend in total hours worked units (sectoral hours worked shares) and percentage deviation from trend (relative price and relative wage of non-tradables). Results for the baseline (all countries) are displayed by the solid black line with the shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling. We split the sample into two groups of countries on the basis of the working age population and run the same VAR model for one sub-sample at a time. The dashed blue line with triangles (red line with circles resp.) shows results for the group of large countries (small countries resp.). Sample: 16 OECD countries, 1970-2007, annual data.



(a) Non-Traded Goods Share, ν^N (b) Traded Goods Share, ν^T

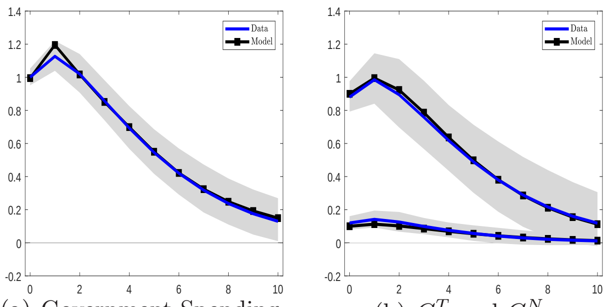
Figure 4: Plot of Impact Responses of Sectoral Shares to a Government Spending Shock in Rolling Sub-Samples against Impact Response of Intersectoral Labor Reallocation. Notes: Exogenous increase of government consumption by 1% of GDP. The government spending shock is identified by estimating a VAR model that includes real government final consumption expenditure, GDP (constant prices), total hours worked, private fixed investment, and the real consumption wage. We adopt the local projection method for estimating impulse responses of the sectoral shares (i.e., $\nu^{Y,j}, \nu^{L,j}$) and the labor reallocation index (i.e., LR) to identified government spending shock; as we restrict attention to impact effects, we run the regression of each variable of interest on the structural shock, setting $h = 0$ into eq. (4). To explore empirically time-varying effects of government spending shocks, we estimate impact effects on rolling 25-year window. The time-varying impact response of the value added (labor) share of sector j is shown in the solid black line (blue line with circles) while the time-varying impact response of intersectoral labor reallocation is displayed in the dotted black line; sample: 16 OECD countries, 1970-2007, annual data.



(a) Labor Share of Tradables (b) Output Share of Tradables

(c) Labor Share of Non-Tradables (d) Output Share of Non-Tradables

Figure 5: Effect of Government Spending Shocks on Sectoral Shares against the Degree of Labor Mobility across Sectors. Notes: Figure 5 plots impact responses of sectoral labor and sectoral output shares to a government spending shock. Impact responses shown in the vertical axis are obtained by running a VAR model for each country and are expressed in percentage point. Horizontal axis displays the elasticity of labor supply across sectors, ϵ, which captures the degree of labor mobility across sectors; panel data estimates for ϵ are taken from column 16 of Table 2.



(a) Government Spending (b) G^T and G^N

Figure 6: Effects of Unanticipated Government Spending Shock on Government Final Consumption Expenditure and Its Two Components: Empirical vs. Theoretical Impulse Response Functions. Notes: The baseline response of government final consumption expenditure is displayed by the solid blue line in the left panel with shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; sample: 16 OECD countries, 1970-2007, annual data. The responses of government final consumption expenditure on non-tradables (i.e., g^N) and tradables (i.e., g^T) to the identified government spending shock (in the baseline VAR model) are displayed by solid blue lines in the right panel; sample: 13 OECD countries, 1995-2015, annual data. Theoretical responses of government final consumption expenditure, g, along with of those of its two components, g^N and g^T, are displayed by solid black lines with squares in the left and the right panel, respectively.

Table 2: Data to Calibrate the Two-Sector Model (1990-2007)

Countries	Non tradable Share						G^j/Y^j		LIS		Product.	Aggregate ratios			Elasticities	
	Output	Consump.	Inv.	Gov. Spending	Labor	α_L.	G^T/Y^T	G^N/Y^N	θ^T	θ^N	Z^T/Z^N	I/Y	G/Y	LIS	ϕ	ϵ
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
AUS	0.63	0.56	n.a.	0.88	0.68	0.67	0.05	0.25	0.55	0.66	1.30	0.25	0.18	0.62	0.27	0.64
AUT	0.64	0.52	0.62	0.89	0.64	0.64	0.06	0.26	0.65	0.66	1.05	0.24	0.19	0.65	0.99	0.55
BEL	0.65	0.53	n.a.	0.89	0.68	0.66	0.07	0.30	0.65	0.67	1.28	0.21	0.22	0.66	0.80	0.33
CAN	0.63	0.54	0.67	n.a.	0.69	0.67	n.a.	n.a.	0.53	0.63	1.32	0.20	0.20	0.59	0.39	0.45
DNK	0.66	0.54	0.60	0.93	0.68	0.68	0.05	0.36	0.63	0.70	1.17	0.20	0.26	0.68	2.07	0.15
ESP	0.64	0.54	0.72	0.91	0.66	0.67	0.04	0.25	0.60	0.66	1.18	0.25	0.18	0.64	0.78	1.64
FIN	0.58	0.53	0.68	0.89	0.63	0.63	0.06	0.35	0.59	0.73	1.47	0.20	0.22	0.67	1.07	0.54
FRA	0.70	0.51	0.69	0.93	0.69	0.68	0.05	0.31	0.70	0.64	1.05	0.19	0.23	0.66	0.94	1.29
GBR	0.64	0.52	0.58	0.94	0.70	0.65	0.03	0.29	0.70	0.73	1.54	0.17	0.20	0.72	0.48	1.01
IRL	0.52	0.52	0.69	0.90	0.62	0.62	0.03	0.29	0.46	0.69	1.83	0.22	0.17	0.58	0.37	0.26
ITA	0.64	0.46	0.57	0.92	0.63	0.62	0.04	0.28	0.71	0.64	1.00	0.21	0.19	0.67	-	0.69
JPN	0.63	0.57	0.63	0.86	0.64	0.65	0.06	0.22	0.57	0.63	0.96	0.26	0.16	0.61	0.65	0.99
NLD	0.65	0.53	0.63	0.90	0.70	0.69	0.07	0.32	0.60	0.70	1.38	0.21	0.23	0.67	0.71	0.22
NOR	0.54	0.49	0.67	0.91	0.66	0.67	0.04	0.36	0.38	0.65	1.44	0.22	0.21	0.52	0.98	0.22
SWE	0.64	0.56	0.55	0.94	0.68	0.67	0.05	0.39	0.63	0.71	1.42	0.18	0.27	0.68	0.36	0.44
USA	0.69	0.63	0.64	0.88	0.73	0.69	0.06	0.20	0.61	0.63	1.12	0.19	0.16	0.62	0.67	1.39
Mean	0.63	0.53	0.64	0.90	0.67	0.66	0.05	0.30	0.60	0.67	1.28	0.21	0.20	0.64	0.77	0.75

Notes: The last line of Table 2 shows the mean of ratios and the mean of parameter values across countries. We take sixteen OECD countries' unweighted averages since in our empirical analysis, quantities are divided by the working age population and thus all countries receive the same weight. α_L refers to the labor compensation share of non-tradables. 'LIS' refers to the labor income share. The ratio G^j/Y^j is the share of government spending on good j in output of sector j; θ^j is the share of labor income in value added at current prices of sector $j = T, N$; Z^T/Z^N corresponds to the ratio of productivity of tradables to productivity of non-tradables. I/Y is the investment-to-GDP ratio and G/Y is government spending as a share of GDP. Because estimates of ϵ for Denmark and Norway over 1970-2007 are not statistically significant at a standard threshold, we exclude these two countries from the calculation of the average value for ϵ. When running the regression (34) over two sub-periods, we find a value for ϵ of 0.217 for Norway over 1970-1989. Since this value is statistically significant, we set ϵ to 0.217 for Norway when we calibrate the model to country-specific data.

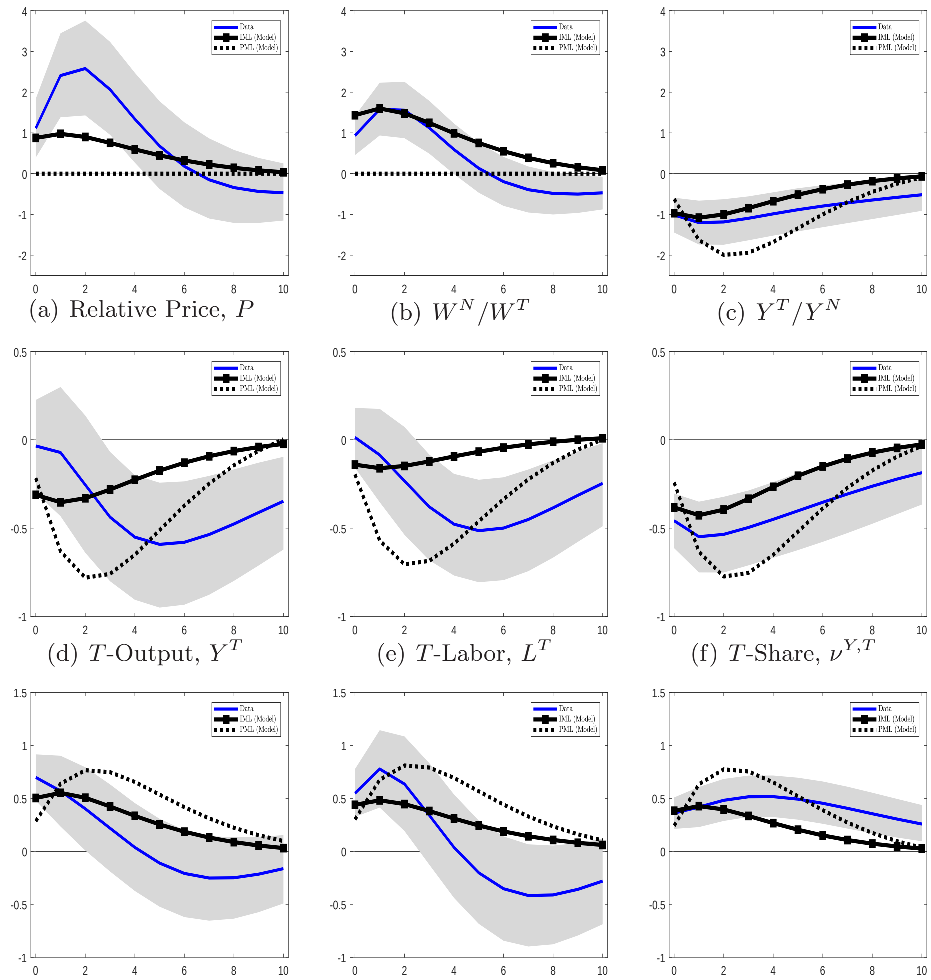
Table 3: Baseline Parameters (Representative OECD Economy)

Definition	Value		Reference
	OECD	Sensitivity	
Period of time	year	year	data frequency
A.Preferences			
Subjective time discount rate, β	4%	4%	equal to the world interest rate
Frisch elasticity of labor supply, σ_L	0.4	0.4	Fiorito and Zanella [2012]
Elasticity of labor supply across sectors, ϵ	0.75	0.2-1.6	our estimates (EU KLEMS [2011] and OECD STAN databases)
Elasticity of substitution between C^T and C^N, ϕ	0.77	0.77	our estimates (KLEMS [2011], OECD Economic Outlook)
Elasticity of substitution between J^T and J^N, ϕ_J	1	1	Bems [2008]
B.Non tradable share			
Weight of consumption in non-traded goods, $1-\varphi$	0.51	0.51	set to target $\alpha_C = 53\%$ (United Nations [2011])
Weight of labor supply to the non-traded sector, $1-\vartheta$	0.68	0.68	set to target $L^N/L = 67\%$ (KLEMS [2011])
Weight of non traded investment, $1-\varphi_J$	0.64	0.64	set to target $\alpha_J = 64\%$ (OECD Input-Output database [2012])
Non Tradable content of government expenditure, ω_{G^N}	0.90	0.63	our estimates (COFOG, OECD [2017])
Labor income share in the non-traded sector, θ^N	0.68	0.68	our estimates (EU KLEMS [2011] and OECD STAN databases)
Labor income share in the traded sector, θ^T	0.58	0.58	our estimates (EU KLEMS [2011] and OECD STAN databases)
Productivity of tradables relative to non-tradables Z^T/Z^N	1.28	1.28	our estimates (KLEMS [2011])
C.GDP demand components			
Physical capital depreciation rate, δ_K	6%	6%	set to target $\omega_J = 21\%$ (Source: OECD Economic Outlook Database)
Parameter governing capital adjustment cost, κ	17	0	set to match the elasticity I/K to Tobin's q (Eberly et al. [2008])
Government spending as a ratio of GDP, ω_G	20%	20%	our estimates (Source: OECD Economic Outlook Database)
D.Government spending shock			
Exogenous fiscal shock, g	0.01	0.01	To generate $dG(0)/Y = 1\%$
Persistence and shape of endogenous response of G, ξ	0.408675	0.408675	set to match $dG(1) = g'$ and $\dot{G}(1) = 0$
Persistence and shape of endogenous response of G, χ	0.415722	0.415722	set to match $dG(1) = g'$ and $\dot{G}(1) = 0$

Table 4: Impact Responses of Aggregate and Sectoral Variables to a Rise in Government Consumption (in %)

	Data	Imperfect Mobility					Perfect Mobility	
		Bench	N-Gov	Mobility		No Adj. Cost.	No Adj. Cost.	With Adj. Cost
		($\epsilon = 0.75$)	$\omega_{G^N} = 0.63$	($\epsilon = 0.22$)	($\epsilon = 1.64$)	($\kappa = 0$)	($\kappa = 0$)	($\kappa = 17$)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
A.GDP & Components								
Real GDP, $dY_R(0)$	0.51	0.19	0.15	0.22	0.16	0.15	0.07	0.09
Investment, $dI(0)$	-0.01	-0.13	-0.09	-0.17	-0.08	-0.41	-0.84	0.04
Current account, $dCA(0)$	-0.30	-0.34	-0.48	-0.22	-0.46	-0.12	0.06	-0.75
B.Labor & Real Wage								
Labor, $dL(0)$	0.53	0.30	0.23	0.34	0.25	0.24	0.11	0.15
Real consumption wage, $d\left(W/P_C\right)(0)$	0.48	0.07	0.03	0.08	0.06	0.05	0.00	0.07
C.Sectoral Labor								
Traded labor, $dL^T(0)$	0.01	-0.14	-0.08	0.02	-0.29	-0.09	-0.20	-0.68
Non-traded labor, $dL^N(0)$	0.54	0.44	0.31	0.32	0.55	0.33	0.30	0.83
Relative labor, $d\left(L^T/L^N\right)(0)$	-0.71	-0.52	-0.33	-0.19	-0.86	-0.36	-0.53	-1.86
Relative wage, $d\Omega(0)$	0.93	1.44	0.91	1.87	1.03	1.02	0.00	0.00
Labor share of tradables, $d\nu^{L,T}(0)$	-0.27	-0.24	-0.15	-0.09	-0.38	-0.17	-0.23	-0.74
Labor share of non-tradables, $d\nu^{L,N}(0)$	0.27	0.24	0.15	0.09	0.38	0.17	0.23	0.74
D.Sectoral Output								
Traded output, $dY^T(0)$	-0.03	-0.31	-0.19	-0.19	-0.43	-0.21	-0.22	-0.72
Non-traded output, $dY^N(0)$	0.70	0.50	0.34	0.41	0.59	0.37	0.28	0.82
Relative output, $d\left(Y^T/Y^N\right)(0)$	-1.03	-0.97	-0.62	-0.64	-1.30	-0.64	-0.62	-3.16
Relative price, $dP(0)$	1.06	0.88	0.55	1.13	0.64	0.62	0.00	0.02
Output share of tradables, $d\nu^{Y,T}(0)$	-0.45	-0.38	-0.25	-0.26	-0.49	-0.27	-0.24	-0.76
Output share of non-tradables, $d\nu^{Y,N}(0)$	0.35	0.38	0.25	0.26	0.49	0.27	0.24	0.76

Notes: Effects of an unanticipated and temporary exogenous rise in government consumption by 1% of GDP. Panels A,B,C,D show the initial deviation in percentage relative to steady-state for aggregate and sectoral variables. Market product (aggregate and sectoral) quantities are expressed in percent of initial GDP while labor market (aggregate and sectoral) quantities are expressed in percent of initial total hours worked. To explore the aggregate effects empirically, we consider a VAR model that includes in the baseline specification (log) government consumption, $g_{i,t}$, GDP, $y_{i,t}$, total hours worked, $l_{i,t}$, private fixed investment, $je_{i,t}$, and the real consumption wage denoted by $w_{C,i,t}$. Our vector of endogenous variables, is given by: $z_{i,t} = \left[g_{i,t}, y_{i,t}, l_{i,t}, je_{i,t}, w_{C,i,t}\right]$. In the second specification we replace private investment with the current account expressed in percentage of GDP, $ca_{i,t}$; ϵ measures the degree of substitutability in hours worked across sectors and captures the degree of labor mobility; κ governs the magnitude of adjustment costs to capital accumulation. In our baseline calibration (labelled 'Bench'), we set $\epsilon = 0.75$ and $\kappa = 17$.

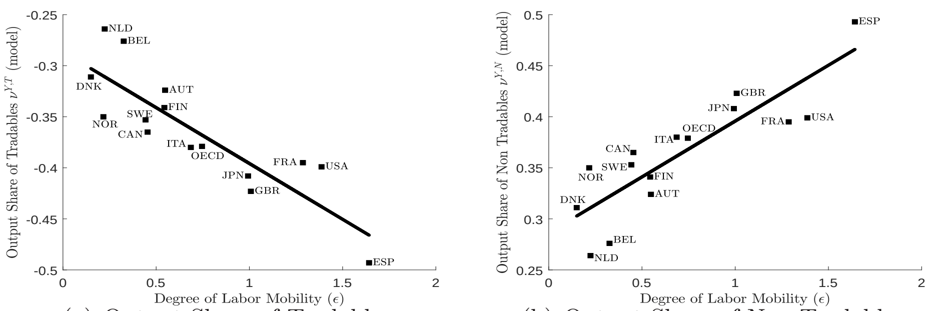


(a) Relative Price, P (b) W^N/W^T (c) Y^T/Y^N

(d) T-Output, Y^T (e) T-Labor, L^T (f) T-Share, $\nu^{Y,T}$

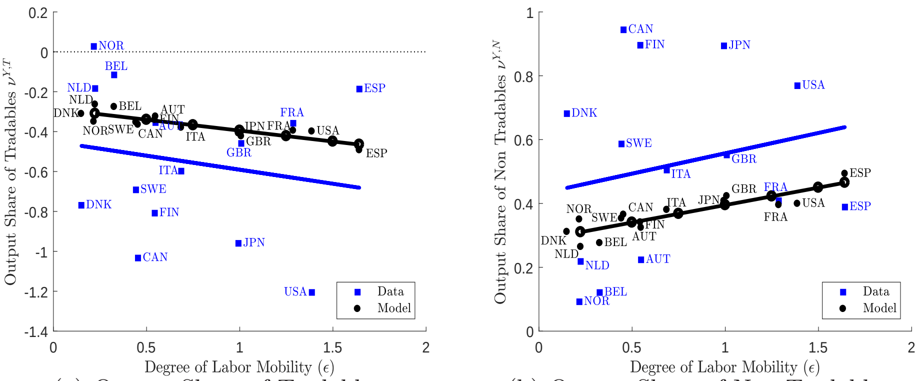
(g) N-Output, Y^N (h) N-Labor, L^N (i) N-Share, $\nu^{Y,N}$

Figure 7: Theoretical vs. Empirical Responses Following Unanticipated Government Spending Shock. Notes: Solid blue line displays point estimate of VAR with shaded areas indicating 90% confidence bounds; the thick solid black line with squares displays model predictions in the baseline scenario with IML ($\epsilon = 0.75$) and capital installation costs ($\kappa = 17$) while the dotted black line shows predictions of the model imposing PML ($\epsilon \to \infty$) and abstracting from capital adjustment costs ($\kappa = 0$).



(a) Output Share of Tradables (b) Output Share of Non-Tradables

Figure 8: Cross-Country Relationship between the Responses of Sectoral Output Shares to Government Spending shock and the Degree of Labor Mobility across Sectors. Notes: Horizontal axes display panel data estimates of the elasticity of labor supply across sectors, ϵ, taken from the last column of Table 2, which captures the degree of labor mobility across sectors. Vertical axes report simulated impact responses from the baseline model with IML and adjustments costs to capital accumulation.



(a) Output Share of Tradables (b) Output Share of Non-Tradables

Figure 9: Cross-Country Relationship between Sectoral Output Responses and ϵ: Model vs. Data. Notes: Horizontal axes display panel data estimates of the elasticity of labor supply across sectors, ϵ, taken from the last column of Table 2, which captures the degree of labor mobility across sectors. Vertical axes report simulated responses from the baseline model (black circles) and impact responses from the VAR model (blue squares). The solid blue line shows the cross-country relationship from VAR estimates while the solid black line with circles displays the cross-country relationship from numerical estimates.

Monetary and fiscal policy interactions with central bank transparency and public investment

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Abstract: In this paper, we study how the interactions between central bank transparency and fiscal policy affect macroeconomic performance and volatility, in a framework where productivity-enhancing public investment could improve future growth potential. We analyze the effects of central bank's opacity (lack of transparency) according to the marginal effect of public investment by considering the Stackelberg equilibrium where the government is the first mover and the central bank the follower. We show that the optimal choice of tax rate and public investment, when the public investment is highly productivity-enhancing, eliminates the effects of distortionary taxation and fully counterbalance both the direct and the fiscal-disciplining effects of opacity, on the level and variability of inflation and output gap. In the case where the public investment is not sufficiently productivity-enhancing, opacity could still have some disciplining effects as in the benchmark model, which ignores the effects of public investment.

Keywords: Distortionary taxes, output distortions, productivity-enhancing public investment, central bank transparency (opacity), fiscal disciplining effect.

JEL classification numbers: E52, E58, E62, E63, H21, H30.

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1. Introduction

Over the past two decades, an increasing number of central banks have become more transparent about their objectives, procedures, rationales, models and data. This has stimulated an intensive ongoing research about the effects of central bank transparency.[1] Most economists agree that openness and communication with the public are crucial for the effectiveness of monetary policy, because they allow the private sector to improve expectations and hence to make better-informed decisions (Blinder, 1998). Counterexamples have been provided, with addition of distortions, where information disclosure reduces the ability of central banks to strategically use their private information, and therefore, greater transparency may not lead to welfare improvement (e.g., Sorensen (1991), Faust and Svensson (2001), Jensen (2002), Grüner (2002), Morris and Shin (2002)).[2] In effect, according to the second best theory, the removal of one distortion may not always lead to a more efficient allocation when other distortions are present.

Typical models on monetary policy transparency usually consider two players, the monetary authority and the private sector. Departing from this approach, several authors introduce monetary and fiscal policy interactions.[3] In a framework where the government sets a distortionary tax rate, it was shown that uncertainty (or opacity) about the "political" preference parameter of the central bank, i.e. the relative weight assigned to inflation and output gap targets, could reduce average inflation as well as inflation and output variability (Hughes Hallett and Viegi (2003), Ciccarone *et al*. (2007), Hefeker and Zimmer (2010)). Higher distortionary taxes

[1] Pioneered by Cukierman and Metzler (1986), transparency issue has been examined both theoretically and empirically by Nolan and Schaling (1998), Faust and Svensson (2001), Chortareas et al. (2002), Eijffinger and Geraats (2006), Demertzis and Hughes Hallet (2007), among others. See Geraats (2002) and Eijffinger and van der Cruijsen (2010) for a survey of the literature.

[2] See Dincer and Eichengreen (2007) for a short survey about these models including distortions.

[3] Some researchers study the relationship between central bank transparency and the institutional design (Walsh, 2003; Hughes Hallett and Weymark, 2005; Hughes Hallett and Libich, 2006, 2009; Geraats, 2007).

necessary for financing higher public expenditures will induce lower output gap and higher unemployment. Thus, central bank increases the inflation rate and workers claim higher nominal wages. In terms of macroeconomic volatility, less central bank political transparency has a disciplining effect on the fiscal authority, which could dominate the direct effect of opacity when the government cares less about the public expenditures, and the central bank is quite populist whilst the initial degree of central bank opacity is sufficiently high.[4]

However, the aforementioned studies do not distinguish the different components of public expenditures by separating public consumption (e.g. public sector wages and current public spending on goods) from public investment (e.g., infrastructure, health and education). A substantial theoretical and empirical research has been directed towards identifying the components of public expenditure that have significant effects on economic growth (Barro (1990)). The introduction of both public capital (infrastructures) and public services (education) as inputs in the production of final goods, theoretical models suggested that public investment generates higher growth in the long run through raising private sector productivity (e.g. Futagami et al. (1993), Cashin (1995), Glomm and Ravikumar (1997), Ghosh and Roy (2004), Hassler *et al*. (2007), Klein *et al*. (2008), Azzimonti *et al*. (2009)). In addition, empirical studies confirm the positive impact of public investment on productivity and output (e.g. Aschauer (1989), Morrison and Schwartz (1996), Pereira (2000), and Mittnik and Neuman (2001)).

Usually, the frameworks used in theoretical studies on public investment ignore the effects due to monetary and fiscal interactions. Cavalcanti Ferreira (1999) examines the interaction between public investment and inflation tax and has found that the distortionary effect of

[4] The term "political transparency" used here corresponds to the information disclosure about the weights assigned by the central bank to the output gap and inflation stabilisation. Five motives for central bank transparency (i.e. political transparency, economic transparency, procedural transparency, policy transparency and operational transparency) are defined in Geraats (2002).

inflation tax is compensated by the productive effect of public expenditures. Ismihan and Ozkan (2004) consider the relationship between central bank independence and productivity-enhancing public investment, and argue that although central bank independence delivers lower inflation in the short term, it may reduce the scope for productivity-enhancing public investment and so harm future growth potential. Ismihan and Ozkan (2007) extend the previous model by taking into account the issues of public debt, and have found that, under alternative fiscal rules (balanced-budget rule, capital borrowing rule), the contribution of public investment to future output plays a key role in determining its effects on macroeconomic performance.

The distinction between public consumption and public investment could allow us to introduce in the literature of central bank transparency the effects of public investment on the aggregate supply. These effects could correct the distortionary effects of taxation and therefore interact with central bank transparency. For this purpose, we re-examine in this paper the interaction between central bank political transparency and fiscal policies in a two-period model, similar to Ismihan and Ozkan (2004), where the public investment is productivity-enhancing and could compensate, partially or totally, the distortions generated by the taxes on revenue. The aim of the paper is to investigate to what extent the disciplining effect of opacity could be generalized to a framework where the government has more than one policy instrument.

The paper is organized as follows. The next section presents the model. Section 3 presents the benchmark equilibrium where there is no productivity-enhancing public investment. Section 4 examines how the inclusion of public investment affects the effects of opacity according to the marginal effect of public investment on the aggregate supply. The last section summarizes our findings.

2. The model

The two-period model of discretionary policy making is similar to the one presented by Ismihan and Ozkan (2004). To model the effects of distortionary taxes and public investment on the supply, we consider a representative competitive firm, which chooses labor to maximize profits by taking price (or inflation rate π_t), wages (hence expected inflation π_t^e), and tax rate (τ_t) on the total revenue of the firm in period t as given, subject to a production technology with productivity enhanced by public investment in the previous period (g_{t-1}^i). The normalized output-supply function is:

$$x_t = \pi_t - \pi_t^e - \tau_t + \psi g_{t-1}^i, \qquad t = 1,2; \qquad (1)$$

where x_t (in log terms) represents the normalized output (or output gap). Equation (1) captures the effects of supply-side fiscal policies on the aggregate supply of output, with the effect of distortionary taxes being clearly distinguished from that of public investment.[5]

The public expenditures are composed by public sector consumption ($g_t^c > 0$) and investment ($g_t^i \geq 0$), both expressed as percentages of the output. The public investment consists of productivity-enhancing expenditures on infrastructure, health, education etc. However, as its favorable consequences indirectly affect the consumers' utility, this type of expenditure is not taken into account in the policy maker's utility function. On the contrary, public consumption made up of public sector wages, current public spending on goods and other government spending is assumed to yield immediate utility to the government. The fiscal authority's loss function is

[5] The variable τ allows covering a whole range of structural reforms. In effect, τ could also represent non-wage costs associated with social security (or job protection legislation), the pressures caused by tax or wage competition on a regional basis or the more general effects of supply-side deregulation (Demertzis *et al*., 2004).

$$L_0^G = \tfrac{1}{2}\mathrm{E}_0 \sum_{t=1}^{2} \beta_G^{t-1} [\delta_1 \pi_t^2 + x_t^2 + \delta_2 (g_t^c - \overline{g}_t^c)^2], \tag{2}$$

where E_0 is an operator of mathematical expectations, β_G the government's discount factor, δ_1 and δ_2 the weights assigned to the stabilization of inflation and public consumption respectively, while the output-gap stabilization is assigned a weight equal to unity.

The government's objectives are the stabilization of the inflation rate and the output gap around zero, and of the public consumption around its target $\overline{g}_t^c$. The government minimizes the above two-period loss function subject to the following budget constraint:

$$g_t^i + g_t^c = \tau_t, \qquad \text{with} \quad t = 1,2. \tag{3}$$

Equation (3) is a simple form of the budget constraint since public debt and seigniorage revenue are not taken into account. Even though g_t^i enhances the productivity in the future, it is implemented and financed in the current period.

The government delegates the conduct of the monetary policy to the central bank while it retains control of its fiscal instruments. The central bank sets its policy in order to minimize the loss function

$$L_0^{CB} = \tfrac{1}{2}\mathrm{E}_0 \sum_{t=1}^{2} \beta_{CB}^{t-1} [(\mu - \varepsilon)\pi_t^2 + (1+\varepsilon) x_t^2], \quad \mu > 0, \tag{4}$$

where β_{CB} is the central bank's discount factor. The parameter μ is the expected relative weight that the central bank assigns to the inflation target and it could be equal or different from δ_1. It is therefore an indicator of central bank *conservatism* (larger μ values) versus *liberalism* or *populism*. According to the literature, we assume that the central bank can fully neutralize the effects of policy shocks (including public spending) or exogenous demand shocks affecting the goods market through appropriate setting of its policy instrument π.

The weights assigned by the central bank to the inflation and output-gap targets are more or less predictable by the government and private sector, meaning that ε is a stochastic variable. The fact that ε is associated to both inflation and output objectives is adopted for avoiding the arbitrary effects of central bank preference uncertainty on average monetary policy (Beetsma and Jensen, 2003). The distribution of ε is characterized by $\mathrm{E}(\varepsilon)=0$, $\mathrm{var}(\varepsilon)=\mathrm{E}(\varepsilon^2)=\sigma_\varepsilon^2$ and $\varepsilon \in [-1,\mu]$. Variance σ_ε^2 represents the degree of opacity about central bank preferences. When $\sigma_\varepsilon^2 = 0$, the central bank is completely predictable and hence, completely transparent. As the random variable ε is taking values in a compact set and has an expectation equal to zero, Ciccarone *et al.*, (2007) have proved that σ_ε^2 has an upper bound so that $\sigma_\varepsilon^2 \in [0,\mu]$.

The timing of the game is the following. First, the private sector forms inflation expectations, then, the government sets the tax rate and public investment, and finally the central bank chooses the inflation rate. The private sector composed of atomistic agents plays a Nash game against the central bank. The government, as Stackelberg leader, plays a Stackelberg game against the central bank. The game is solved by backward induction.

3. The benchmark equilibrium without public investment

First, we consider a benchmark case where the public investment has no supply-side effect. Therefore, it is optimal for the government to set its level at zero. This benchmark case is drawn directly from Hefeker and Zimmer (2010). It is different from Ciccarone *et al.* (2007) who also introduce distortions in the labor market through the wage determination by an all-encompassing monopoly union, as well as from Hughes Hallett and Viegi (2003) who consider a Nash game between the fiscal and monetary authorities, both concerned by distortionary taxes.

Equations (1) and (3) are rewritten as:

$$x_t = \pi_t - \pi_t^e - \tau_t, \tag{5}$$

$$g_t^c = \tau_t. \tag{6}$$

The central bank minimizes the loss function (4) subject to (5). Its reaction function is:

$$\pi_t = \frac{(1+\varepsilon)(\pi_t^e + \tau_t)}{1+\mu}. \tag{7}$$

Equations (5)-(7) allow us to express the output gap as:

$$x_t = \frac{-(\mu-\varepsilon)(\pi_t^e + \tau_t)}{1+\mu}. \tag{8}$$

The government has only one instrument to choose between the tax rate and public consumption due to the budget constraint (6). Setting its fiscal policy, the government cannot predict (7)-(8) with precision due to imperfect disclosure of information about the central bank preferences. Substituting g_t^c, π_t and x_t given by (6)-(8), the government's constrained minimization problem is rewritten, after rearranging the terms, as an unconstrained minimization problem:

$$\min_{\tau_t} L_0^G = \tfrac{1}{2} \mathrm{E}_0 \sum_{t=1}^{2} \beta_G^{t-1} \left\{ \tfrac{(\varepsilon-\mu)^2 + \delta_1 (1+\varepsilon)^2}{(1+\mu)^2} (\pi_t^e + \tau_t)^2 + \delta_2 (\tau_t - \overline{g}_t^c)^2 \right\}. \tag{9}$$

Using the second-order Taylor approximation to obtain $\Theta = \mathrm{E}[\frac{(\varepsilon-\mu)^2 + \delta_1 (1+\varepsilon)^2}{(1+\mu)^2}] \approx \frac{\mu^2 + \delta_1}{(1+\mu)^2} + \frac{(1+\delta_1)}{(1+\mu)^2} \sigma_\varepsilon^2$, the government's loss function is rewritten as

$$L_0^G \cong \tfrac{1}{2} \sum_{t=1}^{2} \beta_G^{t-1} [\Theta (\pi_t^e + \tau_t)^2 + \delta_2 (\tau_t - \overline{g}_t^c)^2]. \tag{10}$$

Proposition 1. *For given expected inflation and tax rate, an increase in central bank's opacity generally induces higher social welfare loss.*

Proof. Deriving (10) with respect to σ_ε^2 yields $\frac{\partial L_0^G}{\partial \sigma_\varepsilon^2} \cong \frac{1}{2}\sum_{t=1}^{2} \beta_G^{t-1}[\frac{1+\delta_1}{(1+\mu)^2}(\pi_t^e + \tau_t)^2] > 0$ if $\pi_t^e + \tau_t \neq 0$. ■

As the government has an objective of public consumption, τ_t cannot be fixed in a way to completely neutralize the effects of central bank's opacity in the social loss function. If the government sets $\tau_t = -\pi_t^e$ to neutralize the effects of opacity on the social loss function, it will suffer from high marginal cost due to insufficient public consumption. Hence, the optimal level of the tax rate depends on the degree of opacity. From the first-order condition of the government's minimization problem we obtain:

$$\tau_t = \frac{\delta_2 \bar{g}_t^c - \Theta \pi_t^e}{\Theta + \delta_2} = \frac{\delta_2 (\mu+1)^2 \bar{g}_t^c - [(\mu^2 + \delta_1) + (1+\delta_1)\sigma_\varepsilon^2]\pi_t^e}{\mu^2 + \delta_1 + (1+\delta_1)\sigma_\varepsilon^2 + \delta_2(1+\mu)^2}. \tag{11}$$

Substituting τ_t given by (11) into (7) and imposing rational expectations yields:

$$\pi_t^e = \frac{\delta_2 \bar{g}_t^c}{\delta_2 \mu + \Theta(1+\mu)} = \frac{\delta_2 (1+\mu)\bar{g}_t^c}{\delta_2 \mu(1+\mu) + \mu^2 + \delta_1 + (1+\delta_1)\sigma_\varepsilon^2}. \tag{12}$$

Substituting π_t^e given by (12) into (11) and taking account of (6) lead to:

$$\tau_t = g_t^c = \frac{\delta_2 \mu \bar{g}_t^c}{\delta_2 \mu + \Theta(1+\mu)} = \frac{\delta_2 \mu(1+\mu)\bar{g}_t^c}{\delta_2 \mu(1+\mu) + \mu^2 + \delta_1 + (1+\delta_1)\sigma_\varepsilon^2}. \tag{13}$$

Using (12)-(13) into (7)-(8) and the budget constraint (6) yields:

$$\pi_t = \frac{(1+\varepsilon)\delta_2 \bar{g}_t^c}{\delta_2 \mu + \Theta(1+\mu)} = \frac{(1+\varepsilon)\delta_2(1+\mu)\bar{g}_t^c}{\delta_2 \mu(1+\mu) + \mu^2 + \delta_1 + (1+\delta_1)\sigma_\varepsilon^2}, \tag{14}$$

$$x_t = \frac{(\varepsilon - \mu)\delta_2 \bar{g}_t^c}{\delta_2 \mu + \Theta(1+\mu)} = \frac{(\varepsilon-\mu)(1+\mu)\delta_2 \bar{g}_t^c}{\delta_2 \mu(1+\mu) + \mu^2 + \delta_1 + (1+\delta_1)\sigma_\varepsilon^2}, \tag{15}$$

$$g_t^c - \bar{g}_t^c = \frac{-\Theta(1+\mu)\bar{g}_t^c}{\delta_2 \mu + \Theta(1+\mu)} = \frac{-[\mu^2 + \delta_1 + (1+\delta_1)\sigma_\varepsilon^2]\bar{g}_t^c}{\delta_2 \mu(1+\mu) + \mu^2 + \delta_1 + (1+\delta_1)\sigma_\varepsilon^2}. \tag{16}$$

Calculating the variance of π_t and x_t results to:

$$\mathrm{var}(\pi_t) = \mathrm{var}(x_t) = \frac{(\delta_2 \overline{g}_t^c)^2 \sigma_\varepsilon^2}{[\delta_2 \mu + \Theta(1+\mu)]^2} = \frac{[\delta_2 (1+\mu) \overline{g}_t^c]^2 \sigma_\varepsilon^2}{[\delta_2 \mu (1+\mu) + \mu^2 + \delta_1 + (1+\delta_1)\sigma_\varepsilon^2]^2} . \qquad (17)$$

From (13)-(17), we observe that the denominator increases as the degree of opacity σ_ε^2, while the numerator of (16) decreases as σ_ε^2 and the numerator of (17) is increases as σ_ε^2. It follows that τ_t, g_t^c, π_t and x_t are all decreasing in σ_ε^2. On the other hand, $\mathrm{var}(\pi_t)$ and $\mathrm{var}(x_t)$ could be both increasing or decreasing in σ_ε^2, as shown by the results of Hefeker and Zimmer (2010) that we reformulate in the following proposition.

Proposition 2. *An increase in central bank's opacity reduces the tax rate, inflation and output distortions but increases deviations of public consumption from its target level. It reduces the variability of inflation and output gap if the initial degree of opacity is sufficiently high and vice versa.*

Proof. Deriving τ_t, π_t, x_t and $g_t^c - \overline{g}_t^c$ given by (13)-(16) with respect to σ_ε^2, leads to the first part of Proposition 2. Deriving $\mathrm{var}(\pi_t)$ and $\mathrm{var}(x_t)$ given by (17) with respect to σ_ε^2, yields:

$$\frac{\partial\, \mathrm{var}(\pi_t)}{\partial \sigma_\varepsilon^2} = \frac{\partial\, \mathrm{var}(x_t)}{\partial \sigma_\varepsilon^2} = \frac{[\delta_2 \mu (1+\mu) + \mu^2 + \delta_1 - (1+\delta_1)\sigma_\varepsilon^2][\delta_2 (1+\mu) \overline{g}_t^c]^2}{[\delta_2 \mu (1+\mu) + \mu^2 + \delta_1 + (1+\delta_1)\sigma_\varepsilon^2]^3} .$$

It follows that $\frac{\partial\, \mathrm{var}(\pi_t)}{\partial \sigma_\varepsilon^2} = \frac{\partial\, \mathrm{var}(x_t)}{\partial \sigma_\varepsilon^2} > 0$ if $\sigma_\varepsilon^2 < \frac{\delta_2 \mu (1+\mu) + \mu^2 + \delta_1}{1+\delta_1}$ and *vice versa*. ■

Distortions introduced by taxes used to finance public expenditures imply higher current and expected inflation rates. Brainard's (1967) conservatism principle implies that the government is incited to adopt a less aggressive fiscal policy ("disciplining effect") because the perceived marginal costs associated with higher taxes are higher under central bank opacity. This stance of fiscal policy leads to lower output gap and inflation rate at the cost of larger deviation of public consumption from its target level. In terms of macroeconomic volatility, opacity triggers two opposing effects. The first corresponds to the direct effect of opacity on the variability of inflation and output gap for a given tax rate (or given level of distortions). The second refers to the disciplining effect, since uncertainty about the central bank preference leads to greater fiscal discipline, contributing to the reduction of inflation and output volatility. The disciplining effect is more likely to dominate the direct effect of opacity if the central bank is less averse to inflation (smaller μ) and the government is less concerned with the public consumption deviations (smaller δ_2).

Using the property $\sigma_\varepsilon^2 \in [0,\mu]$, shown by Ciccarone *et al*. (2007), we extend the previous results in the following proposition.

Proposition 3. *If the government assign a sufficiently high weight to the public consumption, i.e.* $\delta_2 > \frac{(1+\delta_1)\mu-(\mu^2+\delta_1)}{\mu(1+\mu)}$, *the disciplining effect of central bank's opacity will always be dominated by the direct effect of opacity on the variability of inflation and output gap and vice versa.*

Proof. We obtain $\frac{\partial \operatorname{var}(\pi_t)}{\partial \sigma_\varepsilon^2} = \frac{\partial \operatorname{var}(x_t)}{\partial \sigma_\varepsilon^2} < 0$, $\forall \sigma_\varepsilon^2 > \frac{(\mu^2+\delta_1)+\delta_2\mu(1+\mu)}{(1+\delta_1)}$. According to Ciccarone *et al*. (2007), there exists an upper bound on σ_ε^2 so that $\sigma_\varepsilon^2 \in [0,\mu]$. Thus, the previous lower bound on σ_ε^2 is valid only when $\frac{(\mu^2+\delta_1)+\delta_2\mu(1+\mu)}{(1+\delta_1)} < \mu$. This leads to $\delta_2 < \frac{(1+\delta_1)\mu-(\mu^2+\delta_1)}{\mu(1+\mu)}$. If $\delta_2 > \frac{(1+\delta_1)\mu-(\mu^2+\delta_1)}{\mu(1+\mu)}$, the only possible case is that we have always $\sigma_\varepsilon^2 < \frac{(\mu^2+\delta_1)+\delta_2\mu(1+\mu)}{(1+\delta_1)}$. In this case, the direct effect of opacity will always dominate the disciplining effect. ■

In the following, we examine the validity of the previous results in the case where the public investment is productivity-enhancing.

4. Effects of productivity-enhancing public investment

Consider that the public investment is productivity-enhancing. However, according to the marginal effect of such investment, the government might be incited to implement positive, zero or even negative public investment in period 1 or/and 2. Even though negative public investments, such as privatization of infrastructure and education institutions, are possible in practice, they cannot be captured in the present model. That is because such disinvestments are considered to generate a negative effect on the productivity while the privatization suggests a transfer of property but not an inversion of effects of such investments on the productivity. Thus, we assume that negative public investments are not allowed. This implies that we must introduce two supplementary constraints for the government, i.e. $g_1^i \geq 0$ and $g_2^i \geq 0$.

Minimizing the central bank's loss function (4) subject to the economic constraint (1) yields the central bank's reaction function:

$$\pi_t = \frac{(1+\varepsilon)(\pi_t^e + \tau_t - \psi g_{t-1}^i)}{1+\mu}, \quad \text{with} \quad t = 1,2. \tag{18}$$

Using (1)-(3) and (18), we rewrite the government's loss function as:

$$L_0^G = \tfrac{1}{2}\{\Theta(\pi_1^e + \tau_1 - \psi g_0^i)^2 + \delta_2(\tau_1 - g_1^i - \overline{g}_1^c)^2 + \beta_G[\Theta(\pi_2^e + \tau_2 - \psi g_1^i)^2 + \delta_2(\tau_2 - g_2^i - \overline{g}_2^c)^2]\}. \tag{19}$$

Proposition 4. *For given τ_t, g_t^c and g_t^i, if $\pi_t^e + \tau_t - \psi g_{t-1}^i \neq 0$, an increase in central bank's opacity induces a higher social welfare loss.*

Proof. Deriving the loss function given in (19) with respect to σ_ε^2 and using the definition of Θ, we obtain: $\frac{\partial L_0^G}{\partial \sigma_\varepsilon^2} = \frac{(1+\delta_1)}{2(1+\mu)^2}[(\pi_1^e + \tau_1 - \psi g_0^i)^2 + \beta_G(\pi_2^e + \tau_2 - \psi g_1^i)^2] > 0$ if $\pi_t^e + \tau_t - \psi g_{t-1}^i \neq 0$. ■

Opacity has negative effects on the social welfare. In the absence of productivity-enhancing public investment, the government has incentive to reduce the tax rate but at the risk of increasing the deviation of public consumption from its target level. In the case of productivity-enhancing public investment, when positive interior solutions exist for public investment in two periods, the effects of past public investment allow a complete compensation of the distortions introduced by the taxes. Thus, the government is enabled to set a tax rate to ensure that the objective of public consumption is realized. Since the distortions disappear, the central bank has no incentive to set an inflation rate higher than zero. In contrast, the distortions will only be partially compensated when such interior solutions do not exist. In the following we consider the case where positive interior solutions exist for public investment and two cases of corner solutions.

4.1. The case where positive interior solutions exist for public investment

This is the case where the public investment is sufficiently productivity-enhancing, such that public investments are set optimally by the government at a strictly positive level in two periods. The first-order conditions of the minimization problem (19) are:

$$\frac{\partial L_t^G}{\partial \tau_1} = \Theta(\pi_1^e + \tau_1 - \psi g_0^i) + \delta_2(\tau_1 - g_1^i - \bar{g}_1^c) = 0\,, \tag{20}$$

$$\frac{\partial L_t^G}{\partial g_1^i} = -\delta_2(\tau_1 - g_1^i - \bar{g}_1^c) - \beta_G \psi \Theta(\pi_2^e + \tau_2 - \psi g_1^i) = 0\,, \tag{21}$$

$$\frac{\partial L_t^G}{\partial \tau_2} = \beta_G \Theta(\pi_2^e + \tau_2 - \psi g_1^i) + \beta_G \delta_2(\tau_2 - g_2^i - \bar{g}_2^c) = 0\,, \tag{22}$$

$$\frac{\partial L_t^G}{\partial g_2^i} = -\beta_G \delta_2(\tau_2 - g_2^i - \bar{g}_2^c) = 0\,. \tag{23}$$

Solving (20)-(23) gives the government's reaction functions:

$$\tau_1 = -\pi_1^e + \psi g_0^i, \tag{24}$$

$$g_1^i = -\overline{g}_1^c - \pi_1^e + \psi g_0^i, \tag{25}$$

$$\tau_2 = \psi^2 g_0^i - \psi \overline{g}_1^c - \pi_2^e - \psi \pi_1^e, \tag{26}$$

$$g_2^i = \psi^2 g_0^i - \psi \overline{g}_1^c - \overline{g}_2^c - \pi_2^e - \psi \pi_1^e. \tag{27}$$

To determine the expected inflation rates, we substitute τ_1, g_1^i and τ_2 respectively, given by (24)-(26) into (18). Imposing rational expectations yields:

$$\pi_1^e = \pi_2^e = 0. \tag{28}$$

Using the results given by (28) into (24)-(27) leads to the equilibrium solutions

$$\tau_1 = \psi g_0^i, \tag{29}$$

$$g_1^i = \psi g_0^i - \overline{g}_1^c, \tag{30}$$

$$\tau_2 = \psi^2 g_0^i - \psi \overline{g}_1^c, \tag{31}$$

$$g_2^i = \psi^2 g_0^i - \psi \overline{g}_1^c - \overline{g}_2^c. \tag{32}$$

From (30) and (32), we deduce the minimal value of ψ for ensuring that the optimal public investment is strictly positive in two periods, as follows:

$$\psi > \frac{\overline{g}_1^c \pm \sqrt{\overline{g}_1^{c2} + 4 g_0^i \overline{g}_2^c}}{2 g_0^i}.$$

Under this condition, we have simultaneously $g_1^i > 0$ and $g_2^i > 0$.

Using (29)-(32) into (3), we get the public consumptions:

$$g_t^c = \overline{g}_t^c, \qquad \text{with} \quad t = 1,2. \tag{33}$$

Compared to the benchmark solution (13), the solutions of tax rate and public consumption given by (29), (31) and (33), are extremely simple. They depend only on the initial public investment, the marginal effect of public investment and the targets of public consumption.

Proposition 5. *If the public investment is sufficiently productivity-enhancing, i.e.* $\psi > \frac{\bar{g}_1^c \pm \sqrt{\bar{g}_1^{c2} + 4 g_0^i \bar{g}_2^c}}{2 g_0^i}$*, the government will optimally set the tax rate and public investment such as to neutralize the effects of central bank preferences and hence the effects of opacity on its decisions.*

Proof. It follows straightforward from (29)-(33). ■

We remark that the government's decisions given by (29)-(33) are not dependent on central bank preferences. The central bank's "type" (more or less conservative) has neither effect on the tax rate and public investment nor on their variability. Thus, the degree of transparency has no impact on these decisions. The introduction of sufficiently productivity-enhancing public investment incites the government to increase the tax rate to finance higher investment in period 1, but not necessarily in period 2. In effect, the government can collect more taxes, given the higher productivity in period 2. But, as the benefits of public investment in period 2 will be attributed to the next government, the government has no incentive to increase public investment in this period. However, the government is not urged to set the public investment in period 2 at zero, since the tax rate which neutralizes the distortions could generate more tax revenue than what is optimal to spend on the public consumption. The current government is elected on a mandate which implies that it should not set a too high public consumption to avoid the deterioration of the social welfare.

We notice that the tax rate and public investment in the two periods do not depend on the preferences of fiscal authorities. In effect, when the government, whatever are the government preferences, sets separately the tax rate and public investment, it must ensure that the optimal choices allow concealing the effects of these two policy instruments on production and hence inflation.

Using the results given by (28)-(31) into (1) and (18), we obtain:

$$\pi_1 = \pi_2 = 0\,, \tag{34}$$

$$x_1 = x_2 = 0\,. \tag{35}$$

The above equilibrium solutions show that inflation and output-gap targets of the central bank are always realized.

Proposition 6. *If the public investment is sufficiently productivity-enhancing, i.e.* $\psi > \frac{\bar{g}_1^c \pm \sqrt{\bar{g}_1^{c2} + 4 g_0^i \bar{g}_2^c}}{2 g_0^i}$*, the optimal choice of tax rate and public investment by the government allows the neutralization of the effects of central bank preferences and hence the effects of opacity on the level and variability of inflation and output gap.*

Proof. It follows directly from the solutions given by (34)-(35). ■

In contrast to the existing literature on the interaction between fiscal policies and central bank transparency, the degree of political transparency in the present case is irrelevant for the economic equilibrium and macroeconomic stabilization. This is because the government, which has two free policy instruments, is able to conceal the distortionary effects of taxes collected to finance the public expenditures through the optimal choice of tax rate and public investment. Then, the central bank has no motivation to set an inflation rate higher than the target inflation, which is zero. This is rationally expected by the wage setters, thus leading to the elimination of the output distortions.

Our findings imply that the government could generally neutralize the effects of opacity when positive interior solutions exist for tax rates and public investments. There is neither a case against, nor a case for more opacity of the central bank. Meanwhile, in contrast to the benchmark case, the central bank has no incentive to be more opaque since the disciplining effects of opacity have disappeared.

4.2. The cases of corner solutions for public investment

We now consider two cases of corner solutions. In the first case, the public investment is insufficiently productivity-enhancing such that the constraints $g_1^i \geq 0$ and $g_2^i \geq 0$ are both binding. In the second case, it is quite productivity-enhancing such that only the second constraint is binding.

Case 1. Public investments are set to zero in two periods

This is the case where $\psi < \frac{\overline{g}_1^c}{g_0^i}$ (or $\psi g_0^i - \overline{g}_1^c < 0$), i.e. the marginal effect of the past investment on the current productivity is smaller than the ratio of public consumption target in period 1 over public investment in period 0. Because the condition $\psi g_0^i - \overline{g}_1^c < 0$ implies that $g_1^i = -\overline{g}_1^c + \psi g_0^i < 0$ and $g_2^i = \psi^2 g_0^i - \psi \overline{g}_1^c - \overline{g}_2^c = \psi g_1^i - \overline{g}_2^c < 0$, the interior solutions of g_1^i and g_2^i are both negative. Taking into account the constraints $g_1^i, g_2^i \geq 0$, the government sets $g_1^i = g_2^i = 0$. This leads to $\frac{\partial L_t^G}{\partial g_1^i} > 0$ and $\frac{\partial L_t^G}{\partial g_2^i} > 0$, i.e. a decrease in g_1^i and g_2^i will improve the social welfare. Using $g_1^i = g_2^i = 0$ into the first-order conditions (20) and (22), we obtain:

$$\tau_1 = \frac{\Theta(\psi g_0^i - \pi_1^e) + \delta_2 \bar{g}_1^c}{\delta_2 + \Theta}, \tag{36}$$

$$\tau_2 = \frac{\delta_2 \bar{g}_2^c - \Theta \pi_2^e}{\Theta + \delta_2}. \tag{37}$$

Using (36)-(37) in (18) and taking mathematical expectations of the resulting equations yield:

$$\pi_1^e = \frac{\delta_2(\bar{g}_1^c - \psi g_0^i)}{\mu\delta_2 + \Theta(1+\mu)} > 0, \tag{38}$$

$$\pi_2^e = \frac{\delta_2 \bar{g}_2^c}{\mu\delta_2 + \Theta(1+\mu)} > 0. \tag{39}$$

Using (38)-(39) into (36)-(37) and taking account of (3) and the definition of Θ, results to:

$$\tau_1 = g_1^c = \frac{\psi(1+\mu)\Theta g_0^i + \mu\delta_2 \bar{g}_1^c}{\mu\delta_2 + \Theta(1+\mu)} = \frac{\psi(1+\mu)[\mu^2 + \delta_1 + (1+\delta_1)\sigma_\varepsilon^2] g_0^i + \mu\delta_2(1+\mu)\bar{g}_1^c}{\mu\delta_2(1+\mu) + (\mu^2 + \delta_1) + (1+\delta_1)\sigma_\varepsilon^2}, \tag{40}$$

$$\tau_2 = g_2^c = \frac{\mu\delta_2 \bar{g}_2^c}{\mu\delta_2 + \Theta(1+\mu)} = \frac{\mu\delta_2(1+\mu)\bar{g}_2^c}{\mu\delta_2(1+\mu) + \mu^2 + \delta_1 + (1+\delta_1)\sigma_\varepsilon^2}. \tag{41}$$

Using (1), (3), (18), (38)-(41), $g_1^i = g_2^i = 0$, and the definition of Θ, we obtain:

$$\pi_1 = \frac{(1+\varepsilon)\delta_2(\bar{g}_1^c - \psi g_0^i)}{\mu\delta_2 + \Theta(1+\mu)} = \frac{(1+\varepsilon)\delta_2(1+\mu)(\bar{g}_1^c - \psi g_0^i)}{\mu\delta_2(1+\mu) + \mu^2 + \delta_1 + (1+\delta_1)\sigma_\varepsilon^2}, \tag{42}$$

$$x_1 = \frac{(\varepsilon-\mu)\delta_2(\bar{g}_1^c - \psi g_0^i)}{\mu\delta_2 + \Theta(1+\mu)} = \frac{(\varepsilon-\mu)\delta_2(1+\mu)(\bar{g}_1^c - \psi g_0^i)}{\mu\delta_2(1+\mu) + \mu^2 + \delta_1 + (1+\delta_1)\sigma_\varepsilon^2}, \tag{43}$$

$$\pi_2 = \frac{(1+\varepsilon)\delta_2 \bar{g}_2^c}{\delta_2\mu + \Theta(1+\mu)} = \frac{(1+\varepsilon)\delta_2(1+\mu)\bar{g}_2^c}{\mu\delta_2(1+\mu) + \mu^2 + \delta_1 + (1+\delta_1)\sigma_\varepsilon^2}, \tag{44}$$

$$x_2 = \frac{(\varepsilon-\mu)\delta_2 \bar{g}_2^c}{\delta_2\mu + \Theta(1+\mu)} = \frac{(\varepsilon-\mu)\delta_2(1+\mu)\bar{g}_2^c}{\mu\delta_2(1+\mu) + \mu^2 + \delta_1 + (1+\delta_1)\sigma_\varepsilon^2}. \tag{45}$$

The equilibrium solutions given by (40)-(45) allow us to examine how the economy will behave under central bank opacity when the public investment is insufficiently productivity-enhancing.

Proposition 7. *If the public investment is insufficiently productivity-enhancing in the sense that* $\psi < \frac{\bar{g}_1^c}{g_0^i}$*, the public investments in the two periods are set to zero. Compared to the benchmark case, the tax rate and public consumption are higher and the inflation rate and output distortions lower in period 1, and their equilibrium values are the same in period 2.*

Proof. It follows straightforwardly from comparing (40)-(45) with (13)-(16). ■

In the present case, even though the government has no incentive to implement a positive public investment in periods 1 and 2, the effects of public investment in period 0 allow the government to increase the tax rate and public consumption in period 1 while reducing distortions. Therefore, the inflation rate and output distortions are both lower in period 1. In period 2, as the effects of past investment disappear, the government will behave exactly like in the benchmark case.

Proposition 8a. *If the public investment is insufficiently productivity-enhancing in the sense that* $\psi < \frac{\bar{g}_1^c}{g_0^i}$*, the tax rate and public consumption in period 1 react positively to an increase in opacity if* $\frac{\bar{g}_1^c}{(1+\mu)g_0^i} < \psi < \frac{\bar{g}_1^c}{g_0^i}$*, and negatively if* $\psi < \frac{\bar{g}_1^c}{(1+\mu)g_0^i}$*. The inflation rate and output distortions in period 1 are negatively affected by an increase in opacity independently of* ψ*. In period 2, all these variables are negatively related to the degree of opacity independently of* ψ*.*

Proof. It follows straightforwardly from deriving (40)-(45) with respect to σ_ε^2. ■

The productivity-enhancing effect of public investment in period 0 enables the government to increase the tax rate and hence public consumption in period 1. Thus, the disciplining effect of opacity in the tax rate and the effect of public investment allow reducing the inflation rate and output distortions. In period 2, since the effect of past public investment disappears, all these variables will behave as in the benchmark case.

Using (42)-(45), the variances of π_t and x_t are calculated as:

$$\mathrm{var}(\pi_1)=\mathrm{var}(x_1)=\frac{[\delta_2(1+\mu)(\bar{g}_1^c-\psi g_0^i)]^2\sigma_\varepsilon^2}{[\mu\delta_2(1+\mu)+\mu^2+\delta_1+(1+\delta_1)\sigma_\varepsilon^2]^2}, \quad (46)$$

$$\mathrm{var}(\pi_2)=\mathrm{var}(x_2)=\frac{[\delta_2(1+\mu)\bar{g}_t^c]^2\sigma_\varepsilon^2}{[\delta_2\mu(1+\mu)+\mu^2+\delta_1+(1+\delta_1)\sigma_\varepsilon^2]^2}. \quad (47)$$

We notice that (47) is the same than (17).

Proposition 8b. *If the public investment is insufficiently productivity-enhancing in the sense that* $\psi<\frac{\bar{g}_1^c}{g_0^i}$, *an increase in opacity has similar but smaller effects on the variability of inflation and output gap in period 1, and identical effects in period 2 compared to the benchmark case.*

Proof. Deriving (46)-(47) with respect to σ_ε^2 yields:

$$\frac{\partial\,\mathrm{var}(\pi_1)}{\partial\sigma_\varepsilon^2}=\frac{\partial\,\mathrm{var}(x_1)}{\partial\sigma_\varepsilon^2}=\frac{[\delta_2(1+\mu)(\bar{g}_1^c-\psi g_0^i)]^2[\mu\delta_2(1+\mu)+\mu^2+\delta_1-(1+\delta_1)\sigma_\varepsilon^2]}{[\mu\delta_2(1+\mu)+\mu^2+\delta_1+(1+\delta_1)\sigma_\varepsilon^2]^3},$$

$$\frac{\partial\,\mathrm{var}(\pi_2)}{\partial\sigma_\varepsilon^2}=\frac{\partial\,\mathrm{var}(x_2)}{\partial\sigma_\varepsilon^2}=\frac{[\delta_2(1+\mu)\bar{g}_t^c]^2[\delta_2\mu(1+\mu)+\mu^2+\delta_1-\sigma_\varepsilon^2(1+\delta_1)]}{[\delta_2\mu(1+\mu)+\mu^2+\delta_1+(1+\delta_1)\sigma_\varepsilon^2]^3}.$$

The above derivatives are positive if $\sigma_\varepsilon^2<\frac{\mu\delta_2(1+\mu)+\mu^2+\delta_1}{1+\delta_1}$ and *vice versa*. According to the poof of Proposition 3, if $\delta_2>\frac{(1+\delta_1)\mu-(\mu^2+\delta_1)}{\mu(1+\mu)}$, the only possible case is that these derivatives are positive due to the upper bound on the initial degree of opacity, i.e. $\sigma_\varepsilon^2\le\mu$. ■

These results are explained by the fact that the past investment weakens the distortionary effects of the taxes in period 1 without modifying the mechanism through which the effects of opacity are transmitted to the economy. The disciplining effect of opacity dominates the direct effect of opacity on macroeconomic volatility only if the initial degree of opacity is sufficiently high and the weight assigned by the government to the public consumption sufficiently low. The conditions imposed on these parameters are exactly the same as in the benchmark case.

Case 2. Public investment is set to zero only in period 2

This corresponds to the case where the marginal effect of public investment on the productivity is at an intermediate level such that $\frac{\bar{g}_1^c}{g_0^i} < \psi < \frac{\bar{g}_1^c + \sqrt{(\bar{g}_1^c)^2 + 4 g_0^i \bar{g}_2^c}}{2 g_0^i}$. This is equivalent to have simultaneously $\psi g_0^i - \bar{g}_1^c > 0$ and $\psi^2 g_0^i - \psi \bar{g}_1^c - \bar{g}_2^c < 0$. Thus, the interior solution of public investment in period 1 is positive, i.e. $g_1^i = \psi g_0^i - \bar{g}_1^c > 0$ and that in period 2 is negative, i.e. $g_2^i = \psi^2 g_0^i - \psi \bar{g}_1^c - \bar{g}_2^c = \psi g_1^i - \bar{g}_2^c < 0$. Setting $g_2^i = 0$ implies that $\frac{\partial L_t^G}{\partial g_2^i} > 0$, i.e. a decrease in g_2^i under zero will improve the social welfare. Using $g_2^i = 0$ and the first-order conditions (20)-(22), we obtain:

$$\tau_1 = \frac{-(\delta_2 + \Theta + \beta_G \psi^2 \Theta)(\pi_1^e - \psi g_0^i) + \beta_G \psi [\delta_2 \pi_2^e + \psi \delta_2 \bar{g}_1^c + \delta_2 \bar{g}_2^c]}{(\Theta + \delta_2)(1 + \beta_G \psi^2)}, \tag{48}$$

$$g_1^i = \frac{\beta_G \psi (\pi_2^e + \bar{g}_2^c) - (\pi_1^e - \psi g_0^i) - \bar{g}_1^c}{(1 + \beta_G \psi^2)}, \tag{49}$$

$$\tau_2 = \frac{[(\Theta + \delta_2)\beta_G \psi^2 + \delta_2]\bar{g}_2^c + \Theta \psi (\psi g_0^i - \pi_1^e) - \Theta \pi_2^e - \Theta \psi \bar{g}_1^c}{(\Theta + \delta_2)(1 + \beta_G \psi^2)}. \tag{50}$$

Substituting τ_1, g_1^i and τ_2 respectively given by (48)-(50) into (18), we obtain:

$$\pi_1^e = \frac{\beta_G \delta_2 \psi (-\psi^2 g_0^i + \psi \bar{g}_1^c + \bar{g}_2^c)}{(1 + \beta_G \psi^2)[\delta_2 \mu + \Theta(1 + \mu)]}, \tag{51}$$

$$\pi_2^e = \frac{\delta_2(-\psi^2 g_0^i + \psi\overline{g}_1^c + \overline{g}_2^c)}{(1+\beta_G\psi^2)[\delta_2\mu + \Theta(1+\mu)]}. \tag{52}$$

Since $\psi^2 g_0^i - \psi\overline{g}_1^c - \overline{g}_2^c < 0$, we have $\pi_1^e, \pi_2^e > 0$.

Substituting the above solutions of π_1^e and π_2^e into (48)-(50) yields:

$$g_1^i = \frac{\beta_G\psi\overline{g}_2^c + \psi g_0^i - \overline{g}_1^c}{1+\beta_G\psi^2}, \tag{53}$$

$$\tau_1 = \frac{\psi\Theta(1+\mu)(1+\beta_G\psi^2)g_0^i + \mu\delta_2\psi g_0^i + \beta_G\delta_2\psi\mu(\psi\overline{g}_1^c + \overline{g}_2^c)}{(1+\beta_G\psi^2)[\delta_2\mu + \Theta(1+\mu)]}, \tag{54}$$

$$\tau_2 = \frac{[\delta_2\mu(1+\beta_G\psi^2) + \beta_G\psi^2\Theta(1+\mu)]\overline{g}_2^c + \Theta\psi(1+\mu)(\psi g_0^i - \overline{g}_1^c)}{(1+\beta_G\psi^2)[\delta_2\mu + \Theta(1+\mu)]}. \tag{55}$$

Using (3), (53)-(54) and $g_2^i = 0$, the public consumption in periods 1 and 2 is:

$$g_1^c = \frac{[\delta_2\mu(1+\beta_G\psi^2) + \Theta(1+\mu)]\overline{g}_1^c + \beta_G\psi\Theta(1+\mu)(\psi^2 g_0^i - \overline{g}_2^c)}{(1+\beta_G\psi^2)[\delta_2\mu + \Theta(1+\mu)]}, \tag{56}$$

$$g_2^c = \frac{[\delta_2\mu(1+\beta_G\psi^2) + \beta_G\psi^2\Theta(1+\mu)]\overline{g}_2^c + \Theta\psi(1+\mu)(\psi g_0^i - \overline{g}_1^c)}{(1+\beta_G\psi^2)[\delta_2\mu + \Theta(1+\mu)]}. \tag{57}$$

Finally, using (1), (18), (51)-(53) and (55), we get the inflation rate and output gap in periods 1 and 2:

$$\pi_1 = \frac{(1+\varepsilon)\beta_G\delta_2\psi(-\psi^2 g_0^i + \psi\overline{g}_1^c + \overline{g}_2^c)}{(1+\beta_G\psi^2)[\delta_2\mu + \Theta(1+\mu)]}, \tag{58}$$

$$x_1 = \frac{(\varepsilon-\mu)\beta_G\delta_2\psi(-\psi^2 g_0^i + \psi\overline{g}_1^c + \overline{g}_2^c)}{(1+\beta_G\psi^2)[\delta_2\mu + \Theta(1+\mu)]}, \tag{59}$$

$$\pi_2 = \frac{(1+\varepsilon)\delta_2(-\psi^2 g_0^i + \psi\overline{g}_1^c + \overline{g}_2^c)}{(1+\beta_G\psi^2)[\delta_2\mu + \Theta(1+\mu)]}, \tag{60}$$

$$x_2 = \frac{(\varepsilon-\mu)\delta_2(-\psi^2 g_0^i + \psi\overline{g}_1^c + \overline{g}_2^c)}{(1+\beta_G\psi^2)[\delta_2\mu + \Theta(1+\mu)]}. \tag{61}$$

Using (58)-(61), the variances of π_t and x_t are calculated as:

$$\mathrm{var}(\pi_1) = \mathrm{var}(x_1) = \frac{[\beta_G \delta_2 \psi(-\psi^2 g_0^i + \psi \bar{g}_1^c + \bar{g}_2^c)]^2 \sigma_\varepsilon^2}{\{(1+\beta_G \psi^2)[\delta_2 \mu + \Theta(1+\mu)]\}^2}, \tag{63}$$

$$\mathrm{var}(\pi_2) = \mathrm{var}(x_2) = \frac{[\delta_2(-\psi^2 g_0^i + \psi \bar{g}_1^c + \bar{g}_2^c)]^2 \sigma_\varepsilon^2}{\{(1+\beta_G \psi^2)[\delta_2 \mu + \Theta(1+\mu)]\}^2}. \tag{64}$$

In the following, we compare the equilibrium solutions given by (53)-(61) with these obtained in the first case of the corner solutions (40)-(45) and with the benchmark solutions (13)-(15). Furthermore, we compare the macroeconomic volatility obtained in the present case with these observed in the benchmark solution (17) and in the first case of the corner solutions (46)-(47).

Proposition 9a. *If the public investment is relatively productivity-enhancing in the sense that* $\frac{\bar{g}_1^c}{g_0^i} < \psi < \frac{\bar{g}_1^c + \sqrt{(\bar{g}_1^c)^2 + 4 g_0^i \bar{g}_2^c}}{2 g_0^i}$*, the optimal level of public investment is positive in period 1 and zero in period 2. Compared to the benchmark case, the tax rate and public consumption are higher in two periods, the inflation rate and output distortions are lower (higher) in period 1 if* $\psi^2 g_0^i - \psi \bar{g}_1^c < \bar{g}_2^c < \frac{\bar{g}_1^c}{\beta_G \psi} + \psi^2 g_0^i$ *(if* $\bar{g}_2^c > \frac{\bar{g}_1^c}{\beta_G \psi} + \psi^2 g_0^i$*) while they are always lower in period 2.*

Proof. See Appendix A, part I. ■

In the second case of the corner solutions, a positive public investment is implemented in period 1 but not in period 2. Compared to the benchmark case, the government can increase the tax rate and public consumption in periods 1 and 2 while reducing distortions due to the effects of public investment in periods 0 and 1. Therefore, the inflation rate and output distortions are both lower in period 1 if the public consumption target of period 2 is not too higher. In effect, if the latter is too high, the intertemporal trade-off will incite the government to increase the tax rate in the way that it can invest more in period 1, leading to higher inflation rate and output distortions in this period. In period 2, as the public investment in period 1 has a positive effect on the production in period 2, the government reduces output distortions and this incites the central bank to reduce the inflation rate.

Proposition 9b. *Compared to the case where* $\psi < \frac{\bar{g}_1^c}{g_0^i}$, *the tax rate is higher in two periods. The public consumption is higher, and the inflation rate and output distortions lower in period 1 only if the target of public consumption in period 2 is not too high. The public consumption is higher, and the inflation rate and output distortions lower in the period 2.*

Proof. See Appendix A, part II. ■

The second case of the corner solutions is intermediate between the first case (where the government does not invest in periods 1 and 2) and the case of the interior solutions (where the government has incentive to invest in both periods). The productivity-enhancing effect of past investment urges the government to increase the public consumption in period 1, but this effect could be dominated by the effect of intertemporal trade-off. More precisely, if the public consumption target of period 2 is too high, the government will lower the public consumption in period 1 to implement a higher level of public investment allowing it to recover more fiscal revenue in the period 2.

Proposition 10a. *If the public investment is relatively productivity-enhancing in the sense that* $\frac{\bar{g}_1^c}{g_0^i} < \psi < \frac{\bar{g}_1^c + \sqrt{(\bar{g}_1^c)^2 + 4 g_0^i \bar{g}_2^c}}{2 g_0^i}$ *such that the public investment is set to zero only in period 2, the public investment in period 1 is not affected by central bank opacity, while the tax rate, public consumption, inflation rate and output distortions in two periods are negatively affected by an increase in opacity.*

Proof. It follows straightforwardly from deriving (53)-(61) with respect to σ_ε^2, taking into account that $\Theta \approx \frac{\mu^2+\delta_1}{(1+\mu)^2} + \frac{(1+\delta_1)}{(1+\mu)^2}\sigma_\varepsilon^2$ and $\psi^2 g_0^i - \psi \bar{g}_1^c - \bar{g}_2^c < 0$. ■

In the second case of the corner solutions, as in the benchmark case, the disciplining effect of opacity on the tax rate allows the reduction of the output distortions and hence of the inflation rate. However, public investments are independent of central bank preferences and hence of central bank opacity. This is because the public investment allows the reduction of the output distortions, and the government has to trade-off between its current consumption and current investment, something that affects the future public consumption. Therefore, the choice of public investment depends only on the parameter representing the marginal effect of public investment, on the supply function and the parameters characterizing the government preferences.

Proposition 10b. *If the public investment is relatively productivity-enhancing in the sense that* $\frac{\bar{g}_1^c}{g_0^i} < \psi < \frac{\bar{g}_1^c + \sqrt{(\bar{g}_1^c)^2 + 4 g_0^i \bar{g}_2^c}}{2 g_0^i}$*, an increase in opacity has similar but smaller effects on the variability of inflation and output gap in period 1* (*except when the public consumption target in period 2 is too high, i.e.* $\bar{g}_2^c > \frac{\bar{g}_1^c}{\beta_G \psi} + \psi^2 g_0^i$) *and identical effects in period 2, compared to the benchmark.*

Proof. See Appendix B. ■

As discussed above, the public investments in periods 0 and 1 attenuate the distortionary effects of the taxes in periods 1 and 2 but do not modify the mechanism through which the effects of opacity are transmitted to the economy. As in the benchmark case, an increase in opacity could reduce the macroeconomic volatility only when the direct effect of opacity is dominated by the fiscal disciplining effect of opacity. This is possible only when the initial degree of opacity is sufficiently high and the weight assigned by the government to the public consumption sufficiently low, with the conditions imposed on these parameters being the same as in the benchmark case.

Our findings suggest that when the public investment is highly productivity-enhancing, the government will have another free policy instrument that can be used efficiently to neutralize the distortionary effects of taxes necessary for financing public expenditures. In this case, central bank opacity has no effect on the macroeconomic performance and volatility. However, as shown by the corner solutions, when the public investment is not sufficiently productivity-enhancing, the government cannot use it to completely counterbalance the distortionary effects of taxes. Therefore, the level of output distortions and the effects of opacity in the macroeconomic performance and volatility will situate between these found in the benchmark case and these in the case where the public investment is highly productivity-enhancing.

The benchmark case suggests that an increase in opacity improves the macroeconomic performance by reducing the tax rate, and hence the inflation rate and output distortions through the fiscal disciplining effect. It could reduce the macroeconomic volatility when the direct effect of opacity is dominated by the fiscal disciplining effect, i.e. if the initial degree of opacity is sufficiently high and the weight assigned by the government to the target of public consumption low enough. Under these conditions, there is clearly a case for central bank opacity. If the weight assigned by the government to the target of public consumption is high enough, then there is a trade-off between macroeconomic performance and volatility, because an increase in opacity induces lower inflation rate and output distortions but higher macroeconomic volatility. The trade-off is cancelled if the public investment is highly productivity-enhancing, since the government could neutralize the distortionary effects of the taxes. However, when the public investment is insufficiently productivity-enhancing, the implications of the benchmark case are still valid even though the effects of opacity on the macroeconomic performance and volatility could be weakened by the productivity-enhancing effects of public investment in the past and/or in period 1.

Our previous results are obtained by assuming a Stackelberg game, a budget constraint excluding debt-financing and a particular timing sequence concerning the effect of productivity-enhancing public investment. A robust check of our results would need to consider the implications of alternative assumptions about these points. In the following, without giving full algebraic developments, we just provide some intuitions about how our findings could be affected if these alternative assumptions are adopted.

Regarding the timing of the fiscal policy innovations, it is to notice that in equation (1) which models the link between fiscal aggregates and the output gap, distortionary taxes enter such equation contemporaneously, i.e. an immediate impact on the business cycle is allowed. In contrast, fiscal expenditures through productivity-enhancing public investment exert their positive impact on the business cycle with a one-period lag. Such time discrepancy can be explained by the fact that the achievement of such investment may take a delay and the government pays the contractors of public investment before its achievement under a fiscal rule which asks each government to use current fiscal revenue to finance current public investment even though the later has positive effect on next period revenue.

However, one might think of fiscal expenditures planned and implemented in advance on the basis of an expected amount of revenues collected later on. Under this interpretation, it would call for the debt-financing of public investment in order to share the burden of its cost over time, leading to the presence of real public debt in the economy. Therefore, one possible extension of the present model is to consider, following Ismihan and Ozkan (2007), a government budget constraint which creates the link between the fiscal and monetary policies:

$$g_t^i + g_t^c + (1 + r_{t-1})d_{t-1} = k\pi_t + \tau_t + d_t \,, \tag{65}$$

where k is the real holdings of base money as share of output, d_{t-1} denotes the amount of single-period indexed public debt issued (as a ratio of output) in period $t-1$ and to be re-paid in period t, r_{t-1} represents the rate of interest at which it is borrowed, d_t is the new debt issue in period t. Such an extension implies that we have to modify the budget constraint in the benchmark case to include as well the seigneuriage revenue:

$$g_t^i + g_t^c = k\pi_t + \tau_t \,. \tag{66}$$

Taking into account of public debt or/and seigniorage revenue complicate considerably the algebraic analysis. Consider first the benchmark Stackelberg equilibrium with the budget constraint (66). The seigeneuriage revenue is an alternative source of financing which can substitute the tax revenues since, for a given public expenditures, a higher seigeneuriage revenue will allow the government to reduce the distortionary tax rate. Therefore, the central bank has incentive to let inflation rate be higher in order to reduce the distortions induced by distortionary taxes. On the other hand, by increasing the tax rate, the government could induce a higher inflation in order to boost total fiscal revenue. In this framework, the inclusion of seigneuriage revenue could decrease the disciplining effect of central bank opacity. The final effects of central bank opacity will be ambiguous and depend on the structural parameters of the model.

In the case where the policymaker has access to borrowing from the public in order to finance public investment, it has two supplementary (intertemporal) instruments at its disposal, public investment and public debt. The first can be utilized to improve future output prospects and the second to spread the cost of financing public spending over time. In effect, the fiscal authority's optimization now requires balancing the intertemporal consequences of both g_1^i and d_1 in addition to equalizing the marginal welfare losses from different sources of taxation (π_t and τ_t). In opposite to an increase in public investment, a rise in the first period's public borrowing has a favourable effect on macroeconomic performance (higher inflation rate, output gap and current spending gap) but an unfavourable effect on the second period's one. In effect, the presence of sufficiently productivity-enhancing public investment opportunities, i.e. when ψ is higher enough, enables the policymaker to finance popularity-enhancing public consumption in both periods with the help of intertemporal instruments (g_1^i and d_1) without hampering output and inflation performance (Ismihan and Ozkan, 2007). In this framework, central bank opacity will have negative effects on social welfare given the choice of public debt, public investment and tax rate. As we have argued before, since the government has more than one instrument at its disposal, it could generally neutralize the effect of central bank opacity on the Stackelberg equilibrium if the public investment is sufficiently productivity-enhancing. In the other cases, the effects of central bank opacity will persist but will be less than at the benchmark equilibrium.

An alternative assumption about the timing of the fiscal policy innovations, i.e. the current public investment has productivity-enhancing effects on current supply, could justify better the nonexistence of public debt in the budget constraint given by equation (3). If government expenditures entered equation (1) contemporaneously, without giving the detailed algebra which will be quite simple to do, we conjecture that the general results of the model will not be significantly modified. There will not be any intertemporal but just intratemporal arbitrage between public investment, tax rate and public consumption. As the government will be able to neutralize the effects of distortions induced by the taxes when the public investment is sufficiently productivity-enhancing, the effects of central bank opacity which act through the economic distortions induced by taxes will again disappear. In the other case, the effects of central bank opacity will be identical to these at the benchmark equilibrium.

Our main findings and previous discussions are based on the assumption that the government is the Stackelberg leader and the central bank the Stackelberg follower. This corresponds to the case where the government sets its fiscal policy once a year, say at the beginning of the period, and the central bank makes monetary policy decisions on numerous occasions during that year. However, it is possible that important policy decisions also occur contemporaneously. One would like to understand how the results would change if the assumption on the timing of the strategic game is modified by allowing the government and the central bank to move simultaneously in a Nash game. The basic difference in terms of results will appear in the benchmark model's Nash equilibrium. Central bank opacity is likely to induce higher inflation expectations and hence higher inflation rate. The reason is that, in the Nash game, the government does not make any commitment as in the Stackelberg game. The central bank will doubt if opacity has any fiscal disciplining effects and will tend to consider that the fiscal authority will not restrain its public consumption and taxes. As a result, the fiscal authority will have incentive to restrict as less as possible its taxes and public consumption. At the equilibrium, the fiscal disciplining effect of central bank opacity would be present only if the government attributes a too high relative weight to the public consumption. The direct effect of central bank opacity will dominate the fiscal disciplining effect of opacity if the latter exists. Central bank opacity will always induce higher inflation rate and lower output gap in the presence of distortionary taxes, leading to higher inflation and output volatility. Whatever is the fiscal (un)disciplining effect in the Nash equilibrium, the introduction of productivity-enhancing public investment will give the government a supplementary policy instrument to fully neutralize the direct and indirect effects of central bank opacity if the marginal productivity of public investment is sufficiently high. In the other cases, central bank opacity could still have undesirable effects on the macroeconomic performance.

5. Conclusion

In a two-period model where productivity-enhancing public investment could improve future growth potential, we have examined the interaction between central bank transparency and fiscal policy and the resulting effects on macroeconomic performance and volatility. In the framework of the Stackelberg equilibrium, where the government is the first mover and the central bank the follower, we have shown that the effects of central bank's opacity (or lack of transparency) depend on the marginal effect of public investment.

In the benchmark case (without productivity-enhancing public investment), central bank's opacity reduces the inflation rate, tax rate, public consumption and output distortions when the direct effect of opacity is dominated by the fiscal disciplining effect of opacity. The latter condition is verified when the weight assigned to the public consumption is low enough, the central bank is quite populist, and the initial degree of opacity is high enough. We have demonstrated that the government's optimal choice of tax rate and public investment, when the public investment is highly productivity-enhancing, eliminate the effects of distortionary taxation and fully counterbalance both the direct and the fiscal-disciplining effects of opacity at the level and variability of inflation and output gap.

However, in the intermediate cases, where the public investment is insufficiently or relatively productivity-enhancing, the effects of opacity would be between these predicted by the benchmark model. Even though the effects of opacity on the macroeconomic performance and volatility could be weakened by the productivity-enhancing effects of public investment, the implications of the benchmark case, regarding the effects of opacity, will be valid again.

Finally, the present study can be extended into different directions by considering, for example, a Nash game structure, a budget constraint including seigneuriage revenue and public debt used to finance the public investment, and/or the contemporary effect of public investment. Some of these extensions could affect significantly the benchmark equilibrium and/or the transmission mechanism of monetary and fiscal policy in the full model. However, we conjecture that our findings concerning the neutralization of the effects of central bank opacity when the public investment is sufficiently productivity-enhancing are robust to these alternative assumptions.

Appendix A: Proof of Propositions 9a and 9b

Denote the solutions in the benchmark case with a super index "*b*", the first corner solutions with a super index "*fc*" and the second-case corner solutions with a super index "*sc*". The parameter ψ is also indexed so that we have in the first-case of corner solutions $\frac{\bar{g}_1^c}{g_0^i} < \psi^{sc} < \frac{\bar{g}_1^c + \sqrt{(\bar{g}_1^c)^2 + 4g_0^i \bar{g}_2^c}}{2g_0^i}$, which can be decomposed as $\psi^{sc} g_0^i - \bar{g}_1^c > 0$ and $\psi^{sc2} g_0^i - \psi^{sc} \bar{g}_1^c - \bar{g}_2^c < 0$, where ψ^{sc2} stands for $(\psi^{sc})^2$, and we have in the second case of corner solutions, $\psi^{fc} g_0^i - \bar{g}_1^c < 0$. Furthermore, we have $\psi^{sc} - \psi^{fc} > 0$.

Part I: Second case of corner solutions versus the benchmark case

Comparing the second-case corner solutions (54)-(61) with the benchmark solutions (13)-(16), and using the condition $\frac{\bar{g}_1^c}{g_0^i} < \psi^{sc} < \frac{\bar{g}_1^c + \sqrt{(\bar{g}_1^c)^2 + 4g_0^i \bar{g}_2^c}}{2g_0^i}$, we obtain:

$$\tau_1^{sc} - \tau_1^{b} = \frac{\psi^{sc}\Theta(1+\mu)(1+\beta_G\psi^{sc2})g_0^i + \mu\delta_2(\psi^{sc}g_0^i - \overline{g}_1^c) + \beta_G\delta_2\psi^{sc}\mu\overline{g}_2^c}{(1+\beta_G\psi^{sc2})[\delta_2\mu + \Theta(1+\mu)]} > 0,$$

$$g_1^{\mathrm{csc}} - g_1^{cb} = \frac{\Theta(1+\mu)\overline{g}_1^c + \beta_G\psi^{sc}\Theta(1+\mu)(\psi^{sc2}g_0^i - \overline{g}_2^c)}{(1+\beta_G\psi^{sc2})[\delta_2\mu + \Theta(1+\mu)]} > 0,$$

$$\pi_1^{sc} - \pi_1^{b} = \frac{(1+\varepsilon)\delta_2[\beta_G\psi^{sc}(-\psi^{sc2}g_0^i + \overline{g}_2^c) - \overline{g}_1^c]}{(1+\beta_G\psi^{sc2})[\delta_2\mu + \Theta(1+\mu)]},$$

$$x_1^{sc} - x_1^{b} = \frac{(\mu-\varepsilon)\delta_2(\beta_G\psi^{sc3}g_0^i - \beta_G\psi^{sc}\overline{g}_2^c + \overline{g}_1^c)}{(1+\beta_G\psi^{sc2})[\delta_2\mu + \Theta(1+\mu)]} > 0,$$

$$\tau_2^{sc} - \tau_2^{b} = g_2^{\mathrm{csc}} - g_2^{cb} = \frac{\beta_G\psi^{sc2}\Theta(1+\mu)\overline{g}_2^c + \Theta\psi^{sc}(1+\mu)(\psi^{sc}g_0^i - \overline{g}_1^c)}{(1+\beta_G\psi^{sc2})[\delta_2\mu + \Theta(1+\mu)]} > 0,$$

$$\pi_2^{sc} - \pi_2^{b} = \frac{-(1+\varepsilon)\delta_2[\psi^{sc}(\psi^{sc}g_0^i - \overline{g}_1^c) + \beta_G\psi^{sc2}\overline{g}_2^c]}{(1+\beta_G\psi^{sc2})[\Theta + \mu(\Theta+\delta_2)]} < 0, \text{ since } \psi^{sc}g_0^i - \overline{g}_1^c > 0,$$

$$x_2^{sc} - x_2^{b} = \frac{(\mu-\varepsilon)\delta_2[\psi^{sc}(\psi^{sc}g_0^i - \overline{g}_1^c) + \beta_G\psi^{sc2}\overline{g}_2^c]}{(1+\beta_G\psi^{sc2})[\Theta + \mu(\Theta+\delta_2)]} > 0, \text{ since } \psi^{sc}g_0^i - \overline{g}_1^c > 0.$$

We obtain $\pi_1^{sc} - \pi_1^{b} < 0$ and $x_1^{sc} - x_1^{b} > 0$ if $\psi^{sc2}g_0^i - \psi^{sc}\overline{g}_1^c < \overline{g}_2^c < \frac{\overline{g}_1^c}{\beta_G\psi^{sc}} + \psi^{sc2}g_0^i$, and $\pi_1^{sc} - \pi_1^{b} > 0$ and $x_1^{sc} - x_1^{b} < 0$ if $\overline{g}_2^c > \frac{\overline{g}_1^c}{\beta_G\psi^{sc}} + \psi^{sc2}g_0^i$. ■

Part II: Second case of corner solutions versus the first case

Solutions (40)-(45) and (54)-(61) are indexed according to the aforementioned conventions. Comparing them yields:

$$\tau_1^{sc} - \tau_1^{fc} = \frac{(\psi^{sc} - \psi^{fc})(1+\beta_G\psi^{sc2})(1+\mu)\Theta g_0^i + \mu\delta_2(\psi^{sc}g_0^i - \overline{g}_1^c) + \beta_G\delta_2\psi^{sc}\mu\overline{g}_2^c}{(1+\beta_G\psi^{sc2})[\delta_2\mu + \Theta(1+\mu)]} > 0,$$

$$g_1^{\mathrm{csc}} - g_1^{cfc} = \frac{\Theta(1+\mu)[(\overline{g}_1^c - \psi^{fc}g_0^i) - \beta_G\psi^{sc}\overline{g}_2^c)] + \beta_G\psi^{sc2}\Theta(1+\mu)g_0^i(\psi^{sc} - \psi^{fc})}{(1+\beta_G\psi^{sc2})[\delta_2\mu + \Theta(1+\mu)]},$$

$$\pi_1^{sc}-\pi_1^{fc}=\frac{(1+\varepsilon)\delta_2[(\psi^{fc}g_0^i-\overline{g}_1^c)+\beta_G\psi^{sc2}(\psi^{fc}-\psi^{sc})g_0^i]+(1+\varepsilon)\beta_G\delta_2\psi^{sc}\overline{g}_2^c}{(1+\beta_G\psi^{sc2})[\delta_2\mu+\Theta(1+\mu)]},$$

$$x_1^{sc}-x_1^{fc}=\frac{(\varepsilon-\mu)\delta_2[(\psi^{fc}g_0^i-\overline{g}_1^c)+\beta_G\psi^{sc2}(\psi^{fc}-\psi^{sc})g_0^i]+(\varepsilon-\mu)\beta_G\delta_2\psi^{sc}\overline{g}_2^c}{(1+\beta_G\psi^{sc2})[\delta_2\mu+\Theta(1+\mu)]},$$

$$\tau_2^{sc}-\tau_2^{fc}=g_2^{csc}-g_2^{cfc}=\frac{\beta_G\psi^{sc2}\Theta(1+\mu)\overline{g}_2^c+\Theta\psi^{sc}(1+\mu)(\psi^{sc}g_0^i-\overline{g}_1^c)}{(1+\beta_G\psi^{sc2})[\delta_2\mu+\Theta(1+\mu)]}>0.$$

$$\pi_2^{sc}-\pi_2^{fc}=\frac{-(1+\varepsilon)\delta_2\psi^{sc}(\psi^{sc}g_0^i-\overline{g}_1^c)-(1+\varepsilon)\beta_G\delta_2\psi^{sc2}\overline{g}_2^c}{(1+\beta_G\psi^{sc2})[\Theta+\mu(\Theta+\delta_2)]}<0,$$

$$x_2^{sc}-x_2^{fc}=\frac{-(\varepsilon-\mu)\delta_2\psi^{sc}(\psi^{sc}g_0^i-\overline{g}_1^c)-(\varepsilon-\mu)\delta_2\beta_G\psi^{sc2}\overline{g}_2^c}{(1+\beta_G\psi^{sc2})[\Theta+\mu(\Theta+\delta_2)]}>0.$$

Using $\psi^{fc}g_0^i-\overline{g}_1^c<0$ and $\psi^{sc}-\psi^{fc}>0$, we obtain $g_1^{csc}-g_1^{cfc}>0$, $\pi_1^{sc}-\pi_1^{fc}<0$ and $x_1^{sc}-x_1^{fc}>0$ if $\overline{g}_2^c<\frac{(\overline{g}_1^c-\psi^{fc}g_0^i)+\beta_G\psi^{sc2}g_0^i(\psi^{sc}-\psi^{fc})}{\beta_G\psi^{sc}}$ and *vice versa*. ■

Appendix B. Proof of Proposition 10b.

Using $\Theta\approx\frac{\mu^2+\delta_1}{(1+\mu)^2}+\frac{(1+\delta_1)}{(1+\mu)^2}\sigma_\varepsilon^2$, and deriving $\text{var}(\pi_1)$ and $\text{var}(x_1)$ given by (63), and $\text{var}(\pi_2)$ and $\text{var}(x_2)$ given by (64) with respect to σ_ε^2 yields:

$$\frac{\partial\,\text{var}(\pi_1)}{\partial\sigma_\varepsilon^2}=\frac{\partial\,\text{var}(x_1)}{\partial\sigma_\varepsilon^2}=\frac{[\delta_2\mu(1+\mu)+\mu^2+\delta_1-(1+\delta_1)\sigma_\varepsilon^2](1+\beta_G\psi^2)[\beta_G\delta_2\psi(-\psi^2g_0^i+\psi\overline{g}_1^c+\overline{g}_2^c)]^2}{(1+\mu)\{(1+\beta_G\psi^2)[\delta_2\mu+\Theta(1+\mu)]\}^3},$$

$$\frac{\partial\,\text{var}(\pi_2)}{\partial\sigma_\varepsilon^2}=\frac{\partial\,\text{var}(x_2)}{\partial\sigma_\varepsilon^2}=\frac{[\delta_2\mu(1+\mu)+\mu^2+\delta_1-\sigma_\varepsilon^2(1+\delta_1)](1+\beta_G\psi^2)[\delta_2(-\psi^2g_0^i+\psi\overline{g}_1^c+\overline{g}_2^c)]^2}{(1+\mu)\{(1+\beta_G\psi^2)[\delta_2\mu+\Theta(1+\mu)]\}^3}.$$

We have $\frac{\partial\,\text{var}(\pi_1)}{\partial\sigma_\varepsilon^2}=\frac{\partial\,\text{var}(x_1)}{\partial\sigma_\varepsilon^2}>0$ and $\frac{\partial\,\text{var}(\pi_2)}{\partial\sigma_\varepsilon^2}=\frac{\partial\,\text{var}(x_2)}{\partial\sigma_\varepsilon^2}>0$ if $\sigma_\varepsilon^2<\frac{\delta_2\mu(1+\mu)+\mu^2+\delta_1}{1+\delta_1}$ and *vice versa*. If $\frac{\delta_2\mu(1+\mu)+\mu^2+\delta_1}{1+\delta_1}>\mu$, i.e. $\delta_2>\frac{(1+\delta_1)\mu-(\mu^2+\delta_1)}{\mu(1+\mu)}$, the only case possible is that these derivatives are positive due to the upper bound on the initial degree of opacity, i.e. $\sigma_\varepsilon^2 \le \mu$ (see the proof of Proposition 3).

The variance of inflation and the output gap in period 1, given by (63), is greater (smaller) than the one given by (17) in the benchmark case if $\bar{g}_2^c > \frac{\bar{g}_1^c}{\beta_G \psi} + \psi^2 g_0^i$ (if $\psi^2 g_0^i - \psi \bar{g}_1^c < \bar{g}_2^c < \frac{\bar{g}_1^c}{\beta_G \psi} + \psi^2 g_0^i$, respectively). The variance of inflation and the output gap in period 2, given by (64), is smaller than that given by (17). ■

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Free entry and business cycles under the influence of animal spirits*

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Abstract

We provide a business cycle model in which endogenous markup fluctuations are the main driving force. These fluctuations occur due to some form of 'animal spirits', impelling firms in their entry-exit decisions within each sector. By contrast to existing models of the business cycle emphasizing the role of animal spirits, we do not rely on the sink property of the equilibrium to generate indeterminacy. Hence, while our model does pretty well in accounting for the main features of US business cycles, it avoids several criticisms addressed to these former models, concerning either their dependence upon strongly increasing returns and too high markups, or their implication of countercyclical movements of consumption.

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Keywords: Business cycles, Animal spirits, Imperfect competition.

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1 Introduction

We propose a business cycle model in which the main source of fluctuations consists in "animal spirits", as they influence firms' willingness to enter and compete within existing markets. Although, like the previous literature on endogenous fluctuations, we focus on "animal spirits" as an important factor of the business cycle,[1] we do not rely on the sink property of the dynamic equilibrium to generate indeterminacy. We use instead previous microeconomic theoretical results showing that, under private increasing returns to scale, free entry and perfect market contestability are compatible with an indeterminate number of active firms at equilibrium. To be precise, we mean that different levels of activity, corresponding to different numbers of producing firms within a larger set of potential competitors, are sustainable as Nash equilibria.

The usual approach to free entry is based upon the view that positive profits of incumbent firms are always a sufficient incentive for potential entrants to become active, so that equilibrium in a contestable market is possible only under the zero profit condition. This condition is then used to endogenously determine the number of active firms at equilibrium. However, the standard concept of Nash equilibrium, applying to both busy and idle firms, only requires that there be no admissible way for the latter to attain a positive profit, given the equilibrium strategies of the former. While this is true under different forms of competition, the Cournot equilibrium with free entry (Novshek, 1980)[2] offers a straightforward illustration of this idea. If profits are positive and if the optimal individual scale is negligible with respect to market size, nothing prevents a potential entrant to produce at a near optimal scale and sell at an almost unchanged price. Hence, equilibrium profits are necessarily close to zero. However, if the optimal individual scale is non-negligible, as implied by the existence of internal economies of scale, then not only can active firms' profits be significantly different from zero, but the equilibrium number of these firms may also be largely indeterminate (d'Aspremont, Dos Santos Ferreira and Gérard-Varet, 2000).

As far as we know, no attempts have been made to explore the implica-

[1] A non-exhaustive list of papers on this issue includes Benhabib and Farmer (1994), Farmer and Guo (1994), Galí (1994), Benhabib and Farmer (1996), Schmitt-Grohé (1997), Perli (1998), Wen (1998), Schmitt-Grohé (2000) and Weder (2000). See the very complete survey by Benhabib and Farmer (1999) for other important references.

[2] See also Knieps and Vogelsang (1982), and Brock and Scheinkman (1983).

tions of these microeconomic results at the macroeconomic level, in particular for business cycle studies.[3] The aim of this paper is precisely to explore whether this indeterminacy property may have strong implications for the business cycle. Our main idea is that if the equilibrium number of active firms is indeterminate, the actual number will depend on varying consistent conjecture profiles inducing some firms to produce and at the same time dissuading their potential competitors from doing so. In short, "animal spirits" may influence entry and exit decisions, and so become a driving force of the business cycle. Notice that we are not saying that "animal spirits" are supposed to play the role of exogenous random shocks on the number of active firms, but rather that they may act as a stochastic selection mechanism in the presence of indeterminacy.

A simple empirical implication of our model is that optimistic and pessimistic expectations of firms should be correlated with periods of creation and destruction of business plants, respectively. Figure 1 displays firms expectations about future production together with net business formation in France over the period 1993-2002 (monthly data).[4] Although the match is far from perfect, due principally to very high frequency movements in the process of net business formation, there is a clear correspondence between the two series. At a monthly frequency the correlation is 0.45, and it raises to 0.53 at a quarterly basis. Figure 2 shows in turn that net business formation is strongly correlated with detrended output (HP-filtered). The correlation is around 0.60, and is similar to that found in Portier (1995) for France during the period 1977-1989, and in Chatterjee and Cooper (1993) for the United States. Of course, it is possible to explain such a relationship by considering that output and business formation simultaneously increase in response to real shocks such as technological innovations.[5] In this paper, we simply

[3]In the seminal paper of Chatterjee, Cooper and Ravikumar (1993), strategic complementarities in the entry decisions of consumers-producers may also lead to multiple Pareto-ranked equilibria. However, in that paper, the mechanism generating indeterminacy involves participation costs which are different across agents. This is not in accordance, strictly speaking, with free entry conditions, in the sense given in the microeconomic literature mentioned above.

[4]All the data are provided by INSEE. The expectations series is taken from the monthly survey on manufacturing "expectations about future prodution", and the net business formation series is given by "creations less destructions" as recorded by INSEE. The ouput series is HP-filtered gdp. We thank Guy Laroque and Vladimir Passeron for providing us with these data.

[5]For example, Rotemberg and Woodford (1995) and Portier (1995) show that the num-

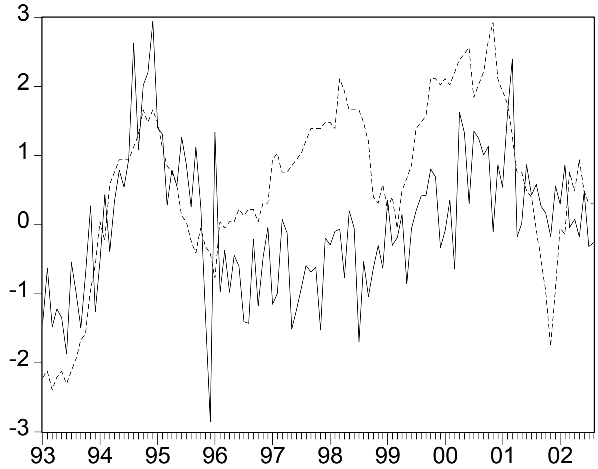

Figure 1: Firms'expectations about future production (dashed line) and net business formation (solid line) in France, 1993-2002.

wish to explore the alternative explanation, which attributes to self-fulfilling expectations of producers the process of entry and exit of new firms within any particular sector.

To illustrate our main idea, we build a simple symmetric model of imperfect competition with a large number of differentiated sectors, within which only a small number of Cournot competitors are active at equilibrium. In line with the microeconomic literature mentioned above, we show that this number may be indeterminate under free entry. Thus, although there is no intrinsic uncertainty, fluctuations in the number of active firms may occur due to endogenous changes in the beliefs of producers with respect to their competitors' behavior. These changes are assumed to be coordinated by reference to some extrinsic stochastic process, with both idiosyncratic and aggregate components. Such a mechanism of entry, taking place in the absence of fundamental uncertainty, allows us to generate endogenous fluctuations in the average markup and, therefore, endogenous fluctuations in the whole economic system. We simulate a dynamic general equilibrium model incorporating such endogenous variations in beliefs, and show that it is able

ber of active firms should vary with output in response to technological and government spending shocks under the zero profit condition.

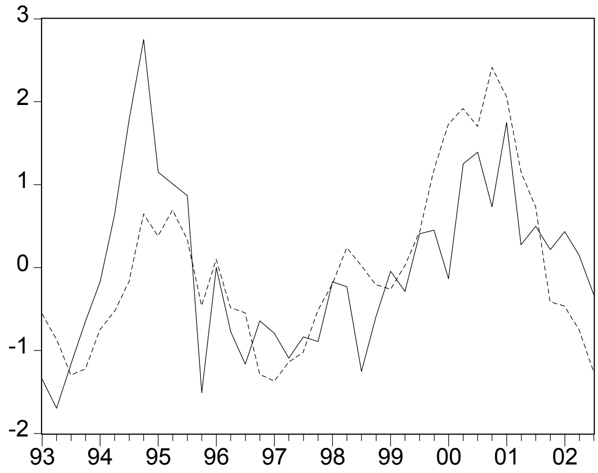

Figure 2: Net business formation (solid line) and HP-filtered output (dashed line), France, 1993-2002.

to generate large fluctuations in real variables. Furthermore, simulation experiments show that the business cycle properties implied by our model are comparable to those measured in the US economy.

The remainder of the paper is organized as follows. Section 2 presents the basic structure of the production sector of our model, and illustrates the indeterminacy result on the equilibrium number of active firms. Section 3 introduces the markup formation process emphasized in section 2 into a dynamic general equilibrium model, and discusses aggregation and coordination issues. Section 4 provides the simulation results and discusses the performance of our model with respect to related models in the literature. Section 5 concludes.

2 Free entry and markup formation

We show in this section that an indeterminate equilibrium number of active firms may prevail in an economy endowed with a quite standard production and market structure. Although the structure we use is chosen so as to make calculations as simple as possible, none of the results here emphasized is specific to that particular structure. In fact, as shown in d'Aspremont, Dos

Santos Ferreira and Gérard-Varet (2000), indeterminacy in the equilibrium number of active firms within perfectly contestable markets may occur under fairly general conditions regarding the market structure and the type of competition which is involved between firms. A necessary condition is obviously that returns to scale be increasing, otherwise a finite number of active firms is incompatible with a free entry equilibrium. However, even a modest degree of increasing returns to scale, resulting from the mere existence of fixed costs, may well be sufficient for indeterminacy.

The economy we consider consists of a large number m of identical sectors, each one producing a homogeneous good, and such that goods stemming from different sectors are imperfectly substitutable from the consumers' viewpoint. In each sector, a virtually large number N of identical firms are involved in Cournot competition within a perfectly contestable output market. Each one of these firms has to decide its output, positive if it chooses to be active, otherwise zero. We begin by examining partial equilibrium conditions in any such sector, leaving their general equilibrium counterpart to the next section.

For simplicity, we assume that individual production takes place under a constant positive real marginal cost c plus a fixed (non sunk) positive cost ϕ, incurred in terms of wasted product. In other words, total real costs to be paid to supply any *positive* amount y of good are given by[6] $c\,(y+\phi)$. Furthermore, we assume that the demand addressed to any sector i depends negatively (with unit elasticity) on the price of this sector relative to the general price level, p_i/P, and positively and linearly on an index of sectoral demand, a_i:

$$y_i = \frac{a_i P}{p_i}, \tag{1}$$

where a_i and P are taken as exogenous from the standpoint of any individual sector i.

Given this set of assumptions, we can study the optimal behavior of any firm $j \in \{1, ..., N\}$ potentially producing in sector i. Any such firm, anticipating aggregate real spending a_i and a vector $y_{i(-j)} = (y_{i1}, ..., y_{ij-1}, y_{ij+1}, ..., y_{iN})$ of outputs supplied by its competitors, has to solve the program:

[6] This is the cost structure corresponding to a production function with constant returns to scale relative to variable inputs, when firms have to pay in addition overhead costs ϕ, expressed in terms of their output.

$$\max_{y_{ij}\in[0,\infty)} \left\{ \left| \begin{array}{cc} \left(\frac{a_i}{y_{ij}+\overline{y}_{i(-j)}} - c\right) y_{ij} - c\phi & \text{if} \quad y_{ij} > 0 \\ 0 & \text{if} \quad y_{ij} = 0 \end{array} \right. \right\}, \tag{2}$$

where $\overline{y}_{i(-j)} = \sum_{j'\neq j} y_{ij'}$ is the total amount of output supplied by firm j's competitors.

According to this program, the optimal behavior of a firm depends on the conjecture it makes on other firms behavior. Naturally, the choices of all other firms are also dependent on the conjectures they make on the former's behavior. These strategic interactions underline the concept of free entry equilibrium, which is nothing else but a qualification of the standard concept of Cournot-Nash equilibrium:

Definition: An *equilibrium* in sector i, given the common expectation of an aggregate sectoral real expenditure a_i, is a vector $(y^*_{i1}, ..., y^*_{iN}) \in [0,\infty)^N$ such that y^*_{ij} is a solution to producer j's program (2) under the conjecture $y^*_{i(-j)}$. It is an *equilibrium with free entry* if some firms are active, while at least one other firm optimally chooses not to produce: $\exists j \in \{1, ..., N\}, y^*_{ij} = 0$, while $\overline{y}^*_{i(-j)} > 0$.

Each one of the N firms within the sector is allowed to rationally choose to be active or inactive, according to its (correct) conjecture of other firms choices. As soon as one firm at least chooses to be inactive (the equilibrium number n of active firms being then smaller than N), we may speak of free entry since there exists a potential entrant which is not hindered by any cost or product differentiation disadvantage relative to its competitors.

We now show how to compute the admissible interval for the equilibrium number of active firms. First, notice that any firm deciding to be active must choose the amount of good it produces according to the first order condition for a positive solution of program (2):

$$\frac{a}{y_j + \overline{y}_{-j}} \left(1 - \frac{y_j}{y_j + \overline{y}_{-j}}\right) = c \tag{3}$$

(where we have dropped for simplicity the subscript referring to the sector). For $\overline{y}_{-j} > 0$, and by concavity of the payoff function in the interval $(0,\infty)$, this condition is sufficient for a global maximum with positive production, provided the corresponding solution is profitable, that is, entails a non-negative profit.

Observe that the left-hand side of (3), with the same equilibrium level of sectoral output $y_j^* + \overline{y}_{-j}^*$ for any j, is decreasing in y_j, so that a positive solution y_n to (2) is unique and identical to all the n active firms, entailing the individual output

$$y_n = \frac{1}{n}\frac{aP}{p_n} \tag{4}$$

and the relative price

$$\frac{p_n}{P} = \frac{n}{n-1}c. \tag{5}$$

Thus, firms set their common price by applying to marginal cost c a markup which, by the assumption of unit demand elasticity, only depends (negatively) upon the number n of active firms. This number is endogenous, and must satisfy, besides the *profitability* condition that any producing firm should make a non-negative profit, a *sustainability* condition for free entry. This condition is weaker than the zero profit condition that is usually introduced in the macroeconomic literature. It merely imposes that any potential entrant is deterred from becoming active because *correctly* realizing that no output would entail a positive profit, given its conjecture of the sectoral demand level a and of the cumulative sectoral output ny_n supplied by its n active competitors.

Within our framework, the *profitability* condition is

$$\left(\frac{a}{ny_n} - c\right) y_n \geqslant c\phi, \tag{i}$$

whereas the *sustainability* condition is

$$\forall y_j \in [0, \infty), \quad \left(\frac{a}{y_j + ny_n} - c\right) y_j \leqslant c\phi. \tag{ii}$$

In particular, the left-hand side of the second inequality expresses the profit of a potential entrant, gross of the fixed cost. This profit is maximized at some level of output $\widehat{y}_n$, which is given by

$$\widehat{y}_n = \frac{a}{c}\sqrt{\frac{n-1}{n}}\left(1 - \sqrt{\frac{n-1}{n}}\right), \tag{6}$$

so that condition (ii) can be given a more tractable, equivalent expression:

$$\left(\frac{a}{\widehat{y}_n + ny_n} - c\right)\widehat{y}_n \leqslant c\phi. \tag{ii'}$$

Condition (ii') simply states that even for the "optimal" level of positive production, a potential entrant can at best exactly cover its production costs, thus being satisfied with inaction.

Conditions (i) and (ii') can now be used to determine the admissible interval for the equilibrium number of active firms. In particular, using (4) and (5) to substitute for y_n and p_n/P, the profitability condition (i) yields

$$n \leqslant \sqrt{\frac{a}{c\phi}} \equiv \overline{n}. \tag{7}$$

Similarly, using (4) to (6) to substitute for y_n, p_n/P and $\widehat{y}_n$, the sustainability condition (ii') can be expressed as

$$1 - \sqrt{\frac{n-1}{n}} \leqslant \sqrt{\frac{c\phi}{a}},$$

or

$$n \geqslant \frac{\overline{n}}{2 - 1/\overline{n}} \equiv \underline{n}. \tag{8}$$

Hence, any integer n in the interval $[\underline{n}, \overline{n}]$ can be the number of active firms at an equilibrium with free entry (there is indeterminacy as soon as the interval contains more than one integer). The zero profit condition, picking up the greatest integer in the interval, thus appears as no more than a particular selection device in presence of indeterminacy.

3 The model

In this section and the following, we study the business cycle properties of a dynamic general equilibrium model which includes markup formation as just characterized. Specifically, we build a standard dynamic general equilibrium model which relies on the market structure described in section 2, and assume that firms enter any sector according to some form of "animal spirits", selecting a number of active firms in the admissible interval. We think of "animal spirits" as the main mechanism governing entries because, as already stressed, producers' equilibrium decisions are strongly dependent on consistent, self-fulfilling *conjectures* about their competitors' behavior. We want further to examine whether such an entry mechanism, associated with extrinsic random shocks on producers' conjectures (and the corresponding markup formation) can provide a model of economic fluctuations whose properties are compatible with those observed in the US economy.

The economy is composed by a huge population (formally, a continuum of unit mass) of identical households maximizing their intertemporal utility, and by a large number m of sectors producing m imperfectly substitutable goods, within which a small number of active firms compete in a Cournot setting. We assume that the utility function of a household is defined, for each period, over the m different goods, with a unit elasticity of intersectoral substitution.[7] As well known, this specification implies that the total demand addressing each sector i at time t is given by

$$y_{i,t} = \frac{Y_t}{m} \frac{P_t}{p_{i,t}}, \tag{9}$$

where $Y_t = m \left[\prod_{i=1}^{m} (y_{i,t})^{1/m} \right]$ and $P_t = \prod_{i=1}^{m} (p_{i,t})^{1/m}$ are the appropriate indices of the aggregate output and the price level. Hence, the demand which addresses each sector depends negatively, with unit elasticity, on the relative price $p_{i,t}/P_t$ prevailing in the sector, and positively and linearly on the level $a_{i,t} \equiv Y_t/m$ of sectoral demand, as assumed in the preceding section.

Because of the presence of fixed costs in the production function, there are increasing returns to scale; otherwise, our model (with Cobb-Douglas preferences and technologies, in particular) is quite standard. But this weak form of increasing returns is enough to entail indeterminacy of the equilibrium number of active firms within each sector, so that different levels of activity are sustainable, even though output markets are all perfectly contestable.

3.1 Households

The representative household is endowed with an initial amount of capital K_0. During each period t, $t = 0, ..., \infty$, it rents its capital stock K_t to the representative firm at the real interest rate r_t, supplies an amount of work H_t at the real wage rate w_t, perceives as a shareholder the real profits made

[7] As mentioned earlier, a unit elasticity of intersectoral substitution is one of the assumptions made only to simplify calculations. None of our results on the indeterminate equilibrium number of firms is dependent on this particular assumption.

by the representative firm Π_t, and consumes a volume C_t of the final goods. The intertemporal budget constraint is

$$K_{t+1} = (1-\delta)K_t + r_t K_t + w_t H_t + \Pi_t - C_t, \tag{10}$$

where δ is the rate of capital depreciation. The program of the representative household is then to maximize the expected utility

$$\underset{0}{E}\left\{\sum_{t=0}^{\infty} \beta^t U\left(C_t, H_t\right)\right\}$$

with respect to $\{C_t, H_t, K_{t+1}\}_{t=0,\ldots,\infty}$, given the budget constraint (10) and the instantaneous utility function (see Hansen, 1985)

$$U\left(C_t, H_t\right) = \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{B}{1+\chi} H_t^{1+\chi},$$

where $\sigma > 0$ is the inverse of the elasticity of intertemporal substitution, and $\chi > 0$ is the inverse of the labor supply elasticity. The optimality conditions can be written as

$$B H_t^{\chi} C_t^{\sigma} = w_t \tag{11}$$

and

$$C_t^{-\sigma} = \beta \underset{t}{E}\left\{\left(1-\delta+r_{t+1}\right) C_{t+1}^{-\sigma}\right\}, \tag{12}$$

which are the traditional consumption-leisure and consumption-saving trade-off conditions for the household.

3.2 Firms

The profit maximization program of the representative firm j in the i-th sector can be conveniently described as a two-stage procedure: In the first stage it chooses, for any given level of production $y_{ij,t}$, the optimal combination of capital $k_{ij,t}$ and labor $h_{ij,t}$ which minimizes its production costs $r_t k_{ij,t} + w_t h_{ijt}$, subject to the production function

$$y_{ij,t} = \left(k_{ij,t}\right)^{\alpha} \left(h_{ij,t}\right)^{1-\alpha} - \phi, \tag{13}$$

where ϕ is a fixed cost (in terms of output). The optimal levels of capital and labor are given by

$$k_{ij,t} = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{w_t}{r_t}\right)^{1-\alpha} (y_{ij,t} + \phi) \tag{14}$$

and

$$h_{ij,t} = \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \left(\frac{w_t}{r_t}\right)^{-\alpha} (y_{ij,t} + \phi)\,. \tag{15}$$

We can deduce from these equations the corresponding level of the real production cost (for $y_{ij,t} > 0$):

$$r_t k_{ij,t} + w_t h_{ij,t} = A r_t^{\alpha} w_t^{1-\alpha} (y_{ij,t} + \phi) \equiv c\,(r_t, w_t)\,(y_{ij,t} + \phi), \tag{16}$$

where $A = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$ is a constant related to technology conditions. Notice that the cost structure which is defined by (16) is the one we have adopted in section 2. The real marginal cost is constant at the firm level (w_t and r_t are taken as given from the individual point of view), but the fixed cost in the production function implies that increasing returns to scale prevail in the (weak) form of a decreasing real average cost.

In the second stage of its maximization program, each firm chooses the price it will charge and the corresponding level of production. It is the description of this second stage that we have undertaken in the preceding section. The optimal pricing decision of an active firm is to apply a markup over the marginal cost, leading in a sectoral symmetric equilibrium, by equation (5), to the relative price

$$\frac{p_{i,t}}{P_t} = \mu_{i,t} c\,(r_t, w_t)\,. \tag{17}$$

The markup depends exclusively, because of constant unit demand elasticity, on the actual number of firms operating within the sector:

$$\mu_{i,t} = \frac{n_{i,t}}{n_{i,t} - 1}, \tag{18}$$

where $n_{i,t}$ takes values in the interval $[\underline{n}, \overline{n}]$.

3.3 Aggregation

We have seen that there are in general multiple Nash equilibria in each sector, with different numbers of producing firms (belonging to the interval $[\underline{n}, \overline{n}]$)

and corresponding different relative prices and levels of activity. Denoting by $\lceil \underline{n} \rceil$ the least integer not smaller than $\underline{n}$ and by $\lfloor \overline{n} \rfloor$ the greatest integer not larger than $\overline{n}$, there actually exists $S \equiv \lfloor \overline{n} \rfloor - \lceil \underline{n} \rceil + 1$ different possible states for any given sector, such that we may associate with any state s ($s = 1, ..., S$) a specific admissible number $n_s \equiv \lceil \underline{n} \rceil + s - 1$ of active firms, and a corresponding markup $\mu_s \equiv n_s/(n_s - 1)$.

Aggregation can be easily achieved as follows. Denoting by $f_{s,t}$ the proportion of sectors in the economy which are in state s at date t, we can determine the average number of active firms, $n_t \equiv \sum_{s=1}^{S} f_{s,t} n_s$, and the average markup μ_t, defined as the weighted geometric mean of the S different admissible markups, using equation (18): $\ln \mu_t = \sum_{s=1}^{S} f_{s,t} \ln \left(n_s / \left(n_s - 1\right)\right)$. Aggregate output can be correspondingly defined, being a weighted geometric mean of the S different admissible sectoral production levels.

Clearly, the evolution both of the average number of active firms and of the average markup, as well as that of all the aggregate variables, *crucially depends on the evolution of the vector* $f_t \equiv (f_{1,t}, ..., f_{S,t})$, itself determined by the process coordinating producers' conjectures.

3.4 Animal spirits

Because of the existence of multiple Nash equilibria within each sector, there exists a coordination problem between firms, which cannot be solved by referring solely to current objective economic conditions or "fundamentals" of the system. Instead, 'animal spirits' of competitors may influence entry and exit decisions, and so become a driving force of the business cycle.

Following a common practice in the literature, we shall consequently assume that firms tackle this coordination problem by referring to some extrinsic stochastic process, which may refer to both idiosyncratic and aggregate components (relevant information is largely sector specific, but the general economic situation significantly influences the way such information is evaluated and interpreted).

Specifically, we assume that at each date t all firms in each sector i receive a signal $s_{i,t} \in \{1, ..., S\}$ suggesting the state which is most likely to be realized at this date in this specific sector. However, this signal $s_{i,t}$ is assumed to be noisy, so that it can only trigger coordination upon the corresponding state if its intensity is above some threshold, say ρ in a $[0, 1]$ scale. Otherwise, if the signal appears too noisy, firms will prefer to coordinate upon the state

$s^*_{i,t-1}$ that has been observed in the previous period.[8]

We assume that $s_{i,t}$ is the outcome of an *independent non-stationary* Markov process, characterized by a transition matrix Π_ω, common to all sectors and itself generated at the economy level by some *stationary* stochastic process with sample space Ω.[9] If signal intensities are distributed over the interval $[0, 1]$ uniformly as well as independently across the different sectors, we obtain by the law of large numbers the following evolution of the column vector $\mathbf{f}_t = (f_{1,t}, ..., f_{S,t})'$:

$$\mathbf{f}_t = [\rho I + (1 - \rho) \Pi_\omega] \mathbf{f}_{t-1}, \tag{19}$$

where Π_ω is an $(S \times S)$ column-stochastic matrix (the s-th element of the s'-th column of Π_ω represents the probability for any firm in a sector of receiving the signal s at period t, conditional on being in state $s' \in \{1, ..., S\}$ at $t-1$, when the economy as a whole is in state $\omega \in \Omega$ at t). By pre-multiplying the two sides of this equation by the row vector $\mathbf{n} = (n_1, ..., n_S)$, we obtain the corresponding evolution of the average number of active firms

$$n_t = \rho n_{t-1} + (1 - \rho) \mathbf{n} \Pi_\omega \mathbf{f}_{t-1}. \tag{20}$$

Clearly, as long as the transition matrix Π_ω stems from a non-degenerate stochastic process, both the average number of active firms and the average markup will be continuous random variables over the supports $[n_1, n_S]$ and $[n_S/(n_S - 1), n_1/(n_1 - 1)]$, respectively.[10] Also, the time properties of n_t and μ_t will crucially depend on the properties of the stochastic process Π_ω.

Examples

To illustrate our argument, we can think of different sensible coordination processes implying distinct stochastic processes for the average number of active firms.

[8] This assumption, which seems *a priori* natural, allows us to account for the persistence in firms'expectations clearly apparent in the data (see Figure 1).

[9] We may for instance refer to the *double chain Markov model*, where two Markov chains are superposed, one controlling the transition process between the transition matrices of the other (see Berchtold, 1999).

[10] With a constant (and regular) transition matrix $\rho I + (1 - \rho) \Pi$, the frequency vector $\mathbf{f}_t$ would converge to a fixed point $\mathbf{f}$, so that aggregate fluctuations would eventually fade away.

As a first example, we can assume that there are two states summing up the overall opinion about the general tendency of the economy: "good times" and "bad times". For instance, "good times" may uniformly encourage the entry of some potential producer, provided profitability is preserved, and "bad times" may prompt some heretofore active firm to cease production, provided sustainability is preserved. In other words, all sectors receive the signal $s_{i,t} = \min \{s_{i,t-1} + 1, S\}$ in "good times", and $s_{i,t} = \max \{s_{i,t-1} - 1, 1\}$ in "bad times". Of course, the degree of reliability of this signal is still evaluated on the interval $[0, 1]$, triggering a state transition only if it is larger than the threshold ρ. Under these assumptions, it is straightforward to show that the number of active firms in each sector follows a random walk with two delaying barriers n_1 and n_S (see for instance Spitzer, 1976). As for the average number of active firms, notice that we obtain in this case for the product $\mathbf{n}\Pi_\omega$ in equation (20): $\mathbf{n}\Pi_g = \mathbf{n} + \begin{bmatrix} 1 & \ldots & 1 & 0 \end{bmatrix}$ if $\omega = g$ ("good times") and $\mathbf{n}\Pi_b = \mathbf{n} - \begin{bmatrix} 0 & 1 & \ldots & 1 \end{bmatrix}$ if $\omega = b$ ("bad times"). As a consequence, the aggregate process described by this equation takes the form:

$$n_t = n_{t-1} + (1-\rho)(1 - f_{S,t-1}) \text{ if } \omega = g \quad (21)$$

$$n_t = n_{t-1} - (1-\rho)(1 - f_{1,t-1}) \text{ if } \omega = b. \quad (22)$$

The evolution of the average number of active firms thus looks like a standard random walk with two shocks. However this is partly misleading since, because of the delaying barriers, the aggregate process depends upon past microeconomic information on the proportions $f_{1,t-1}$ and $f_{S,t-1}$ of sectors that were in the two extreme states 1 and S at date $t-1$.[11]

As a second example, we can refer to a case in which present aggregate information is sufficient to characterize the stochastic process for the average number of active firms. In particular, we can think of a situation in which all firms in the economy receive the same signal $s_{i,t}$ independently of the state prevailing previously in their respective sectors, but with a degree of confidence that remains idiosyncratic. In this case, all the S columns of Π_ω are equal to the same (degenerate) column probability vector $\boldsymbol{\pi}_\omega$ belonging to the canonical basis of $\mathbb{R}^S$, so that equation (20) becomes

$$n_t = \rho n_{t-1} + (1-\rho) n_s, \quad (23)$$

[11] This is therefore a case in which correct aggregation cannot be done without reference to the position of microeconomic units, see for instance Caballero (1992).

where $n_s \equiv \mathbf{n}\boldsymbol{\pi}_\omega \in \{n_1, n_S\}$. We obtain in this case a dynamic stochastic equation in the *aggregate* variable n_t, which is a standard AR(1) process with a finite number of shocks equal to $\#\Omega \leq S$. A simple instance of this case, that we use in our simulations below, is the discrete uniform distribution over the $\#\Omega = S$ possible states.[12]

4 Model properties

In this final section, we proceed to a quantitative evaluation of the model in the spirit of the Real Business Cycle literature. In particular, we ask whether this simple mechanism of entries and exits driven by self-fulfilling changes in firms' expectations can generate business cycles that are relatively close to those observed in the US economy. Testing the business cycle properties of this model is important since, as discussed below, traditional sunspot-driven models based on the sink nature of the steady state have encountered difficulties in explaining some standard features of observed business cycles. Since our model does not rely on that kind of dynamic indeterminacy, one can expect its business cycle properties to be quite different from those in the traditional literature on endogenous fluctuations.

For simplicity, and to allow easier comparison with standard RBC models, we will from now on consider a coordination scheme inducing, as in (23), a dynamic stochastic equation in the *aggregate* variable n_t, of the form:

$$n_t = \rho n_{t-1} + (1-\rho) u_t, \tag{24}$$

where the parameter $\rho \in (0,1)$ controls the degree of persistence, and where u_t is a discrete random variable uniformly distributed over the interval $\{\lceil \underline{n} \rceil, \lfloor \overline{n} \rfloor\}$. Notice that there is a deterministic mean value $\check{n}$ around which the dispersion of the number of active firms remains constant. Indeed, specification (24) implies

[12] In fact, the assumption that all firms in the economy receive the same signal is only auxiliary. The important requirement to obtain a dynamic stochastic equation in the *aggregate* variable n_t (independently of any microeconomic information) is that the probability distribution over the S states be independent of the previous state (or equivalently that all the S columns of Π_ω be equal, for any $\omega \in \Omega$, to the same column probability vector $\boldsymbol{\pi}_\omega$). For instance, if $\pi_\omega = (1-\omega, 0, ..., 0, \omega)'$, where ω is uniformly distributed over $[0,1]$, the variable $n_\omega \equiv \mathbf{n}\boldsymbol{\pi}_\omega$ is a *continuous* uniform random variable.

$$E(n_t) = E(u_t) = \frac{\lceil \underline{n} \rceil + \lfloor \overline{n} \rfloor}{2} \equiv \check{n}$$

and

$$\text{Var}(n_t) \equiv E(n_t - \check{n})^2 = \frac{1-\rho}{1+\rho}\,\text{Var}(u_t).$$

Thus, although our model has, strictly speaking, no stationary state (since u_t is uniformly distributed over $[\lceil \underline{n} \rceil, \lfloor \overline{n} \rfloor]$), the dispersion remains constant around the mean value $\check{n}$. As a consequence, the general equilibrium model described below leads to a dynamic system which is ergodic with a bounded support. This will allow us to log-linearize the dynamic system around the average equilibrium defined by the mean value $\check{n}$, and will therefore make our model comparable to other existing business cycle models which have been solved and evaluated using similar methods.

Log-linearization results in a three-dimensional dynamic system of the following form (the hat on a variable denoting percentage point deviation from the average equilibrium):

$$\begin{bmatrix} \widehat{K}_t \\ \widehat{C}_t \\ \widehat{n}_t \end{bmatrix} = \Phi \begin{bmatrix} \widehat{K}_{t+1} \\ \widehat{C}_{t+1} \\ \widehat{n}_{t+1} \end{bmatrix} + \Gamma \begin{bmatrix} \widehat{e}_{t+1} \\ \widehat{u}_{t+1} \end{bmatrix}, \tag{25}$$

where $\widehat{e}_{t+1} = \widehat{C}_{t+1} - E_t\left(\widehat{C}_{t+1}\right)$ is the one-step-ahead forecast error of consumption, and $\widehat{u}_{t+1}$ is the "animal spirits" innovation. Of course, all the other endogenous variables Y, H, w, r and μ can be uncovered by simple linear functions of the 'state' variables K, C and n.

The analysis of the matrix Φ is crucial for determining the properties of the average equilibrium. The three-dimensional dynamic system (25) includes two predetermined variables, $\widehat{K}$ and $\widehat{n}$, and one non-predetermined variable, $\widehat{C}$. Hence, if the matrix Φ contains as many eigenvalues with modulus strictly above one as there are predetermined variables, the average equilibrium has a saddle-path property. As Benhabib and Farmer (1994) emphasized, this implies that the forecast error $\widehat{e}_t$ is then a deterministic function of the endogenous disturbance $\widehat{u}_t$. If, on the contrary, all the eigenvalues of the matrix Φ have modulus strictly above one, the average equilibrium is a sink, and the forecast error may then enter as an independent shock to the business cycle. The properties of the matrix Φ depend of course crucially on the specific values which are attributed to the structural parameters. In our simulation experiments, and for the reference calibration that we adopt below, we found that only two of the eigenvalues of the matrix Φ have modulus strictly above one, so that the average equilibrium around which we study the local dynamics has a saddle-path property.

4.1 Calibration

Calibration of most structural parameters is done within the range of admissible values which have been used in the related literature. Following Farmer and Guo (1994), we assume a discount factor $\beta = 0.99$, an average value of hours worked equal to 0.2 (20% of time endowment), a quarterly depreciation rate of capital $\delta = 0.025$, an elasticity of production with respect to capital $\alpha = 0.3$, and an infinitely elastic labor supply $\chi = 0$. Typical estimates of the inverse of the intertemporal elasticity of substitution range between 0 and 5. We set $\sigma = 2$, which is an average value not far from the logarithmic case. As for the parameter ρ, the threshold below which the intensity of the idiosyncratic signal is not sufficient to trigger a state transition in producers' conjectures, we assume $\rho = 0.9$ in order to get enough persistence.

Finally, a crucial parameter in our model is ϕ, the real fixed cost in the individual production function. This parameter is important for two reasons. First, it determines the admissible interval $[\underline{n}, \overline{n}]$ within which the number of active firms must remain, and second, it controls the degree of overall increasing returns (the inverse of the elasticity of the cost function) we are willing to introduce in our model:

$$\gamma = \frac{y + \phi}{y}.$$

There has been a great deal of empirical literature in recent years that has attempted to estimate the importance of returns to scale in actual economies. Since the earlier works by Hall (1990) and Caballero and Lyons (1992), who reported large IRS in the US manufacturing industry, recent studies by Basu and Fernald (1995, 1997) and Burnside, Eichenbaum and Rebelo (1995) have arrived at much smaller estimates, not very far from the assumption of constant returns to scale. However, even those recent papers include differences which, from the perspective of our paper, are particularly interesting. Indeed, while Basu and Fernald (1997) end up with nearly constant returns to scale for gross output as a whole, Burnside, Eichenbaum and Rebelo (1995) find evidence of constant returns to scale with respect to *variable* inputs. As emphasized by Basu (1995) in his comment, the test of BER (1995) is actually a test to see whether firms operate under constant marginal costs. But if there are fixed costs in production activities, then BER's estimates are fully consistent with globally increasing returns to scale, provided they appear under the mild form assumed in equation (13). Building on BER's work and adding independent estimates of overhead costs provided by Ramey (1991), Basu (1995) shows that BER's estimates are in fact consistent with 28% of increasing returns to scale.

In our simulations, we have chosen $\gamma = 1.23$ as a benchmark calibration, implying that fixed-cost represents 23% of individual net output, and resulting in the admissible interval $[3, 5]$ for the number of active firms. Clearly, given the recent available evidence, we see this calibration as ranging close to the upper bound of admissible values of increasing returns to scale. For this reason, we have also considered a second calibration with a much more conservative value for γ, and tested the robustness of our results against this alternative setting. Specifically, we assume in this second experiment that $\gamma = 1.05$, the value suggested by Basu (2002) to calibrate a production function similar to ours. For this value, a simple computation shows that the admissible interval for the number of active firms is $[11, 21]$.

4.2 Empirical properties

We now compute the second order moments for the main variables of our model, and compare them to their empirical counterparts.[13] Tables 1 and 2 report the relative volatilities and cross-correlations with output of the main variables as measured in US data, and as implied by our model, under the two different assumptions regarding the size of increasing returns to scale.

The most obvious observation from these tables is that the kind of business cycles implied by the model resemble closely that of the US economy. In particular, Table 1 shows that for the two calibrations considered, the model easily reproduces the weaker volatilities of consumption and real wages relative to output, and the larger volatility of investment. The volatility of hours

[13] To compute the theoretical moments, we generated 100 series of 150 innovations uniformly taken on the support $\{\lceil \underline{n} \rceil, \lfloor \overline{n} \rfloor\}$ and constructed the series for all endogenous variables. The average moments are reported in Table 1, after the series have been filtered by the Hodrick-Prescott filter. Empirical moments are taken from King and Rebelo (1999).

Table 1 - Absolute and relative standard deviations

$\frac{\sigma_Y}{Y}$	Variable (X)	σ_X/σ_Y C	H	I	w
1.81	**US Data**	0.74	0.99	2.93	0.38
1.79	**Model** ($\gamma = 1.23$)	0.62	1.54	2.57	0.08
0.35	**Model** ($\gamma = 1.05$)	0.71	1.76	2.13	0.09

is somewhat larger than in the data, but this is a natural feature of models which do not introduce external effects on labor or exogenous shocks to factor productivity. In addition, Table 2 shows that the model also accounts for the strong procyclical movements of consumption, employment and investment that are typical of real economies.

To understand how fluctuations in the equilibrium number of active firms generate such fluctuations in real variables, it is useful to consider the pseudo impulse-response functions to a positive "animal spirits" shock that we displayed in Figure 3. Figure 3 plots the expected behavior of the main variables (expressed as percentage point deviations from the average state) when it is assumed that the current realization of u_t is 5 and all future values are 4, the mean value of the admissible interval $[\lceil \underline{n} \rceil, \lfloor \overline{n} \rfloor]$. Note that given the calibrated value of $\rho = 0.9$, this realization of u_t implies that the average number of firms increases by only 2.5% (i.e., raises from 4 to 4.1). Following the entry of new firms, there is an immediate and persistent reduction in the average markup, which generates in turn an instantaneous and sustained increase in the demand addressed to each sector. Firms meet this increased demand by raising output, investment and employment. With a labor demand curve shifting along an almost invariant horizontal labor supply curve (remember that the wage-elasticity of hours is infinite), real wages also increase during the boom, albeit only slightly during the first periods. With consumers working more at a higher wage, consumption also rises persistently.

Obtaining simultaneous procyclical movements of output, consumption and investment is particularly important in our model since, as pointed out by Benhabib and Farmer (1999), standard models of indeterminacy via a sink stationary equilibrium have encountered significant difficulties in explaining this feature without relying on large markups or large increasing returns

Table 2 - Contemporaneous correlations with output

Variable (X)	$Corr(Y_t, X_t)$ C	H	I	w
US data	0.88	0.88	0.90	0.12
Model ($\gamma = 1.23$)	0.99	0.99	0.99	0.19
Model ($\gamma = 1.05$)	0.99	0.96	0.99	-0.60

through declining marginal costs.[14] This is because with a constant demand for labor and conventional slopes for the labor demand and labor supply curves, any shift in the labor supply schedule necessarily implies countercyclical movements of the real wage, and thus generally leads to decreased consumption.

In that respect, a key feature of our model is that positive "animal spirits" (leading to the entry of new active firms) induce a decrease in the average markup and thus an outward shift in the demand for labor. This increase in labor demand may be strong enough to offset the depressing effect of labor productivity on real wages, so that real wages eventually rise in response to a positive sunspot. Although this simple mechanism is similar to the countercyclical markup models of Galí (1994) and Schmitt-Grohé (1997), the main obvious difference is that we do not have to require extremely large markups or increasing returns to make them fluctuate endogenously.[15] In fact, as shown in Table 2, the model still generates procyclical movements of consumption with 5% of increasing returns ($\gamma = 1.05$).

Hence, by the standards of the business cycle literature, the model does pretty well at accounting for the main features of US postwar fluctuations. Interestingly, most of these results are robust to the different calibrations on the degree of returns to scale γ. In fact, as shown in Table 1, the only signifi-

[14]See amongst others Benhabib and Farmer (1994), Farmer and Guo (1994), Benhabib and Farmer (1996), and Schmitt-Grohé (2000) for additional details. An interesting exception is the two sector model with home production of Perli (1998), in which an "animal spirits" shock both increases output and market consumption, because of an (unobserved) fall in home production. However, calibrated versions of this model imply that the degree of increasing returns to scale must be set to 20% in order to generate procyclical consumption and sufficient autocorrelation in output.

[15]For example, in Galí (1994), values for the markup above 2.0 are necessary to obtain an indeterminate equilibrium. Galí even uses a markup of 2.8 to generate good results in his simulations.

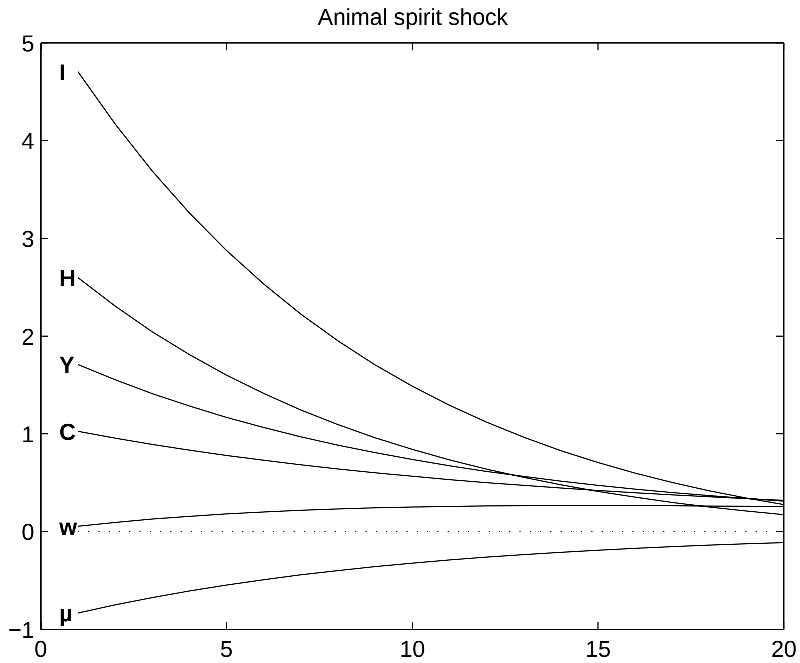

Figure 3: Pseudo impulse response functions for the benchmark model.

cant differences concern the contemporaneous correlation of output with the real wage (with $\gamma = 1.05$, the real wage decreases in the first period before rising up, so that the instantaneous correlation is negative) and the absolute volatility of output. With $\gamma = 1.23$, this volatility is similar to that observed in the US economy, which might be considered as implausibly high for a model with a unique source of disturbances. With $\gamma = 1.05$, this variance is still around 20% that of the US economy. Hence, considering large or small degrees of increasing returns to scale mainly influences the potential of the mechanism we emphasized in this paper to account for a significant part of actual output fluctuations.

5 Conclusion

The idea that endogenous changes in the beliefs of economic players may influence the level of economic activity has been revived by the recent research on business cycles. In particular, dynamic general equilibrium models with a sink stationary equilibrium have shown that endogenous changes in beliefs can be self-fulfilling, and hence consistent with individual optimization and rational expectations.

In this paper, we have explored another way of generating such endogenously driven fluctuations by exploiting the idea, put forward in the microeconomic literature, that different levels of economic activity sustained by different numbers of active firms may be consistent with conditions of free entry and perfect contestability within each sector. We have shown that a simple economy displaying this kind of indeterminacy may be subject to large fluctuations due to endogenous changes in the animal spirits of firms and in their willingness to enter and compete within existing markets. Furthermore, these fluctuations have properties similar to those observed in the US economy.

Finally, because our model does not rely on the sink property of the steady state to generate indeterminacy, it avoids some difficulties that are encountered in recent related models, such as the requirement either to impose large increasing returns through declining marginal costs, or to trade lower increasing returns against countercyclical movements of consumption.

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