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"Multiple Causation, Apportionment, and the Shapley Value"

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Notice introductive

A propos du partage des préjudices en situation de causalité multiple

Qui doit supporter le coût des accidents ? Cette question est aussi ancienne que celle des risques inhérents à la plupart des activités humaines. Pourtant le droit de la responsabilité civile est un droit moderne qui a connu sa consécration dans la rédaction du Code civil français de 1804 à travers l'article 1382 (nouveau 1240). En l'occurrence, le Code reconnaissait la faute comme fondement exclusif de la responsabilité. Pendant l'essentiel du XIX^{ème} siècle la situation va perdurer jusqu'à ce que la question d'une responsabilité objective¹, détachée de la faute, se trouve posée, notamment dans le domaine des accidents du travail. Les considérations économiques, exprimées en termes d'efficacité sociale, ont toujours été présentes dans les débats relatifs à la construction de l'édifice juridique dans le domaine de la responsabilité civile. Par-delà les débats doctrinaux, la question se pose en particulier de savoir dans quelle mesure les évolutions constatées du droit de la responsabilité traduisent ou non des préoccupations en termes d'efficacité économique.

On le comprend, le droit de la responsabilité reste un sujet d'actualité comme il l'a été depuis la rédaction du Code civil. A chaque époque ses priorités et l'histoire est loin d'être terminée, ne serait-ce que si l'on songe aux défis à venir en matière de responsabilité dans le domaine du numérique ou dans le domaine de l'environnement, s'agissant notamment de la prévention et de la réparation des préjudices écologiques. Comment ignorer dans ces conditions les contributions de l'analyse économique du droit sur ces multiples sujets ? L'intérêt d'une démarche interdisciplinaire est désormais bien établi. Loin de vouloir imposer une vision hégémonique de l'économie, elle propose au contraire de mettre le raisonnement et les critères de jugement des économistes au service de l'interprétation des normes² et, plus généralement, de la fabrique du droit. Un plus dans l'analyse des phénomènes juridiques dans la mesure où l'analyse économique peut rendre plusieurs types de services. Elle permet, tout d'abord, de déterminer les principales conséquences d'un changement de règle, s'agissant par exemple d'analyser les effets du passage à une forme de responsabilité objective ou encore de l'introduction de dommages punitifs. Ensuite, l'analyse économique peut mettre au jour la « raison d'être » des institutions juridiques et, par là même, montrer leur unité profonde. Certains auteurs ont ainsi perçu dans bon nombre de domaines du droit³ une logique de gestion prudente des ressources rares (y compris le temps, l'attention et l'information dont les consommateurs et les producteurs doivent disposer dans une économie de marché) et d'incitation à la découverte de façons d'économiser ces ressources.

Tout en reconnaissant que la responsabilité civile peut servir plusieurs objectifs du point de vue de la doctrine juridique, force est de constater que la fonction généralement mise en avant est celle de l'indemnisation de la victime. C'est d'ailleurs le plus souvent au regard de cette fonction réparatrice que la responsabilité pour faute a pu être critiquée par la doctrine⁴. Dans une perspective historique,

¹ H. Mazeaud (1965), *Traité théorique et pratique de la responsabilité civile délictuelle et contractuelle*, 6^{ème} ed. par A. Tunc, Montchrestien, n° 426 et s. Précisons que Tunc voyait dans l'objectivation de la faute un moyen d'en sauvegarder le primat tout en favorisant l'indemnisation des victimes.

² En ce sens B. Deffains et S. Ferey (2010), *Agir et Juger : Comment les économistes pensent le droit*, Presses d'Assas.

³ Ce constat est fait indépendamment des époques que des espaces géographiques et culturels puisque l'on songe aussi bien au droit romain qu'au droit civil ou au droit des contrats contemporains et ce aussi bien dans des systèmes de Common Law que dans des systèmes civilistes.

⁴ A. Tunc, *supra*, n^{os} 170-173.

la prévention des comportements antisociaux (généralement qualifiés de « fautifs ») a également été discutée par la doctrine en tant que fonction de la responsabilité civile⁵. D'un point de vue classique, on serait tenté de résumer les choses en disant que l'indemnisation de la victime a été historiquement vue comme la fonction fondamentale de la responsabilité, mais que la responsabilité ne devait pas pour autant indemniser toutes les victimes. Dès lors que l'objectif était également de veiller à prévenir les comportements antisociaux, ce sont précisément les victimes de tels comportements qui devaient être indemnisées en priorité.

Mais progressivement la doctrine va évoluer. Cela tient surtout à la construction progressive du principe de responsabilité du fait des choses et aux errements qu'a connus la jurisprudence quant à son fondement, dont le choix apparaissait déterminant pour l'avenir. C'est pourquoi le débat entre les partisans de la faute et les pionniers de la théorie du risque fut aussi passionné, les premiers craignant que l'abandon de la faute ne prive la responsabilité civile de toute assise morale, les seconds y voyant plutôt l'amorce d'une révolution qui répondrait parfaitement aux besoins de la société moderne. Bien que divisés sur la question du fondement, ces derniers parlaient généralement d'une même voix lorsqu'ils désignaient le rôle de la responsabilité civile : celle-ci doit avant tout servir l'intérêt des victimes. Ce qui est vrai dans le droit civil français, l'est tout autant dans les autres systèmes juridiques, notamment aux Etats-Unis avec le temps fort que constitue le Restatement of Torts de 1965.

De ce point de vue, il faut savoir gré à Samuel Ferey et Pierre Dehez de s'attaquer à l'un des sujets les plus complexes s'agissant des conditions de l'indemnisation des victimes lorsque le dommage implique plusieurs auteurs. La causalité multiple est en effet une des questions les plus complexes du droit contemporain de la responsabilité civile. La répartition d'une perte subie par une victime entre plusieurs auteurs d'un dommage est en effet difficile, et les tribunaux ne suivent pas toujours des principes clairs et cohérents.

Ainsi par exemple en droit français, l'obligation *in solidum* constitue l'une des doctrines les plus connues en droit de la responsabilité civile. Lorsque plusieurs personnes sont responsables d'un même dommage, elles sont solidairement tenues à réparation envers la victime. Si toutes ou certaines d'entre elles ont commis une faute, elles contribuent entre elles à proportion de la gravité et du rôle causal du fait générateur qui leur est imputable. Si aucune d'elles n'a commis de faute, elles contribuent à proportion du rôle causal du fait générateur qui leur est imputable, ou à défaut par parts égales. Apparue à la fin du XIXe siècle, elle est une exception au principe de division de l'obligation contenu dans le nouvel article 1309 du Code civil. Selon ce dernier, lorsque qu'au minimum deux débiteurs sont tenus d'une même obligation, chacun n'est tenu que pour sa part. Le créancier qui dispose de plusieurs débiteurs ne peut donc réclamer à chacun que sa part dans la dette et non la totalité. Par exemple, un même dommage est le résultat du concours des fautes de plusieurs responsables, le cas échéant à des titres différents, comme, par exemple lorsque le participant à une pratique sportive est blessé en raison, à la fois, d'une faute commise par un des coparticipants et d'un manquement de l'organisateur du jeu à son obligation générale de sécurité. La responsabilité de chacun des intervenants sera engagée à raison de la part que celui-ci a pris à la survenance du dommage, et elle le sera dans les conditions particulières auxquelles elle obéit (responsabilité délictuelle pour le coparticipant et responsabilité contractuelle pour l'organisateur).

Dans l'obligation *in solidum*, c'est l'exact opposé : le créancier peut réclamer la totalité de la dette à chacun des codébiteurs, selon un choix discrétionnaire. La raison d'être d'une telle exception est évidente : avec celle-ci, la victime ne supporte pas le risque d'insolvabilité de l'un des coauteurs. Autrement dit, si la victime subit un dommage causé par plusieurs coauteurs mais que l'un d'eux est insolvable, son dommage sera entièrement réparé avec le mécanisme d'obligation *in solidum* alors que le principe de division l'aurait empêché.

⁵ A. Tunc, *supra*, n° 161

A titre d'illustration on peut mentionner la doctrine de la part de marché aux Etats-Unis qui consiste à répartir la responsabilité contre un ensemble de défendeurs en fonction de leurs parts de marché respectives dans les ventes d'un produit nocif pendant la période où le dommage s'est produit. D'un point de vue économique, on comprend assez bien le fondement de cette doctrine visant à obtenir une internalisation appropriée des coûts du comportement nuisible lorsqu'il n'est pas possible d'identifier lequel des nombreux producteurs de produits nocifs identiques a causé du tort à une seule victime. C'est sans aucun doute la raison pour laquelle cette doctrine a connu un grand succès dans le domaine de la responsabilité du fait des produits défectueux. La responsabilité faisant appel à la « part de marché » peut effectivement fournir une forme d'assurance pour les pertes mais elle est loin de garantir une prévention optimale des risques d'accidents et donc elle peut difficilement être défendue pour des raisons de bien-être social.

L'idée de l'article de Ferey et Dehez est précisément de proposer un cadre d'analyse visant à dépasser la plupart des difficultés rencontrées dans la mise en œuvre de ces différentes approches doctrinales. Pour ce faire, ils proposent une approche axiomatique originale fondée sur la théorie des jeux coopératifs. Ils abordent la question d'un point de vue qui n'est pas purement théorique mais qui permet de comprendre comment mobiliser l'outil dans un cadre juridique opérationnel en montrant comment la valeur de Shapley peut être mobilisée dans un processus séquentiel de répartition d'un dommage. Ils considèrent en particulier les situations les plus complexes de causalité successive auquel ils associent une classe générale de jeux dit de « responsabilité séquentielle ». Ce travail est particulièrement convaincant dans la mesure où il permet de rationaliser économiquement le Restatement (Third) of Torts qui pose certains des principaux fondements du droit américain de la responsabilité civile.

Au-delà de l'exemple américain, cette contribution – et plus généralement le programme qui l'accompagne – peut être qualifiée de fondamentale et elle marque sans conteste une date importante dans la réflexion sur la répartition des dommages en situation de causalité multiple et/ou d'incertitude sur l'identité précise de l'auteur d'un dommage. L'idée défendue par Ferey et Dehez selon laquelle on devrait alors s'éloigner de la fiction au stade de la contribution à la dette devient essentielle. En effet, les juridictions ont souvent à connaître de litiges complexes où plusieurs auteurs – individus ou entreprises par exemple – ont conjointement causé un préjudice à une ou plusieurs victimes tel un automobiliste qui renverse un piéton qui, transporté à l'hôpital, est amputé du fait de la faute du médecin. Elles sont alors confrontées à une tâche ardue : celle de devoir répartir les dommages subis entre plusieurs responsables (ci-après appelés co-auteurs). Loin d'être des cas d'écoles, ces situations sont extrêmement répandues et donnent lieu à contentieux : en responsabilité délictuelle des accidents, lorsque plusieurs individus ont causé l'accident, en responsabilité contractuelle lorsqu'un préjudice a été causé par plusieurs co-contractants (comme dans les chaînes de contrats) ou lorsque plusieurs entreprises ont causé un préjudice à des consommateurs ou à des acheteurs intermédiaires, en droit de l'environnement (pollution par plusieurs agents) ou encore en droit du travail (accident du travail ou maladie professionnelle causés par plusieurs fautes inexcusables d'employeurs successifs) ou en droit de la concurrence (préjudice causé par une entente). Ce rapide survol des litiges donne la mesure des multiples dimensions du problème : nature de la causalité en droit, place de la causalité et de la faute dans la répartition, existence ou absence de critères clairs utilisés par les juges pour résoudre ce type d'affaires, conséquences de la répartition sur les incitations économiques à éviter les dommages, incertitude de la répartition qui peut être source de litiges et de comportements inefficaces...

On comprend notamment que la notion de « causalité alternative » qui accompagne la responsabilité *in solidum* est une situation assez étrange puisque le juge sait bien qu'un seul des auteurs alternatifs poursuivis a causé le dommage et pourtant, tous vont voir leur responsabilité engagée. Nous sommes donc en présence d'une présomption de responsabilité qui interroge. En effet, lorsque le juge se prononce sur la responsabilité sans connaître effectivement l'auteur du dommage, il commet une erreur en répartissant la charge de la dette entre deux auteurs tout en sachant pertinemment que le dommage est imputable à un seul d'entre eux. Il va condamner un défendeur innocent à payer une partie du dommage et exonérer celui qui a effectivement causé le dommage d'une partie de sa responsabilité. Le but de la répartition de la charge de la dette en fonction des parts du marché, au stade de la contribution à la dette, est de limiter l'erreur du juge, de délaisser la fiction pour se rapprocher un peu de la réalité et de garantir l'équité entre les auteurs alternatifs. Faire payer 50 au défendeur qui n'a pas été impliqué causalement dans un préjudice de 100 constitue une erreur, tout comme le fait de ne faire payer que 50 au défendeur qui a causé l'entier préjudice. Partager à égalité amène à se tromper fortement à la hausse sur l'un et à la baisse sur l'autre. Déterminer la part contributive de chacun en fonction de la probabilité qu'il avait de causer le préjudice permet d'assurer une plus grande égalité entre les auteurs alternatifs mais cela pose un problème majeur puisque la répartition du dommage se fait non pas en fonction de l'activité de l'auteur du dommage comme le recommande l'analyse économique du droit mais en fonction du nombre d'auteurs de dommage potentiellement concernés. Une telle solution ne favorise pas la fonction préventive de la responsabilité civile.

D'où l'intérêt de la démarche proposée par Ferey et Dehez d'analyser la causalité juridique comme relevant d'une analyse contrefactuelle. Si cette analyse est bien connue en économie, elle reste méconnue en droit positif alors même qu'elle pourrait convenir à l'analyse du raisonnement du juge. De surcroît, en proposant de dépasser les travaux antérieurs qui ont pris pour habitude de modéliser toujours le dommage multiple par des jeux non-coopératifs, ils proposent une approche en termes de jeux coopératifs qui permet de fonder l'interdisciplinarité droit/économie sur des bases solides en utilisant l'axiomatic comme une manière d'explicitier les principes juridiques utilisés pour répartir un préjudice causé conjointement. Cette approche permet définitivement de développer un traitement juridique du problème de la répartition du dommage dans la plupart des situations rencontrées, et ce quel que soit l'environnement juridique, common law ou système civiliste. En mobilisant les outils de la théorie des jeux coopératifs pour tenter de fournir une quantification du rôle causal joué par chacun des co-auteurs, cette contribution essentielle en économie du droit permet d'analyser les différentes manières possibles de répartir la charge de la dette de réparation entre co-auteurs. En montrant qu'il est possible de construire des ensembles (réduits) de jeux paramétrés correspondant aux principaux cas typiques de ce type de situations (dommages successifs, faute de la victime, dommage conjoint etc.), il devient possible d'étudier les incitations générées par les différentes règles de répartition ainsi rationalisées. Ceci permet de préciser les théories de la causalité utilisée en droit et de montrer que la théorie des jeux coopératifs constitue une manière de quantifier le rôle causal des actions individuelles. Une démarche et un travail exemplaire à diffuser largement auprès des magistrats et des étudiants en droit...

Bruno Deffains, décembre 2022.

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Multiple causation, apportionment and the Shapley value

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Abstract

Multiple causation is one of the most intricate issues in contemporary tort law. Sharing a loss suffered by a victim among multiple tortfeasors is indeed difficult and Courts do not always follow clear and consistent principles. Here, we argue that the axiomatic approach provided by the theory of cooperative games can be used to clarify that issue. We have considered the question from a purely game theoretic point of view in Dehez and Ferey (2013). Here we propose to analyze it in a legal perspective. We consider in particular the difficult case of successive causation to which we associate a general class of games called "*sequential liability games*". We show that our model rationalizes the two-step process proposed by the *Restatement Third of Torts*, apportionment by causation and apportionment by responsibility. More precisely, we show that the weighted Shapley value is the legal counterpart of this two-step process.

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References

1. INTRODUCTION

"Logic has not always the last word in law"
Chief Justice of the New Hampshire
Supreme Court Robert Peaslee¹

Multiple causation is one of the most intricate issues in contemporary tort law. It arises when several tortfeasors cause harm to a victim entitled to recover it and when Courts have to apportion damage among them². Many subfields in private law are concerned with apportionment issues: environmental law (several firms poisoning a river), medical malpractices (surgeon aggravating the consequences of a first accident caused by an initial injurer), health litigation (asbestos exposure by several firms through time), antitrust law (dividing the loss suffered by the consumers due to antitrust practices by several firms) *etc.* Moreover, many models and theories have been proposed in law³, philosophy, economics⁴, and psychology⁵ to capture the features of legal causation and apportionment issues. These legal debates lead the American Law Institute to promulgate a new *Restatement* dedicated to this issue.⁶

The present paper adds to this literature by developing a game theoretic approach in which damages are monetized and modeled as cooperative games where players are the tortfeasors who jointly created an indivisible economic loss to be paid.⁷ Solution concepts are then applied following the axiomatic approach proposed by Shapley. Contrary to law and economics models in the literature, we are more interested in the fairness of the apportionment than in the incentives created by the apportionment rules. Therefore we

¹ Peaslee (1934, p.1131).

² Four multiple causation issues may be distinguished: successive causation, simultaneous causation, alternative causation and victim's contribution.

³ See Hart and Honoré (1985) and Borgo (1979) for a comprehensive analysis of causation related to the theory of law and Coleman (1992) for causation issues related to moral theory. See also Wright (1985a), Keeton (General Editor), Dobbs, Keeton, and Owen (1984) for specific issues of causation in Torts.

⁴ In economics, a constant attention has been devoted to this topic. See Landes, and Posner (1980); Rizzo, and Arnold (1980), (1986); Shavell (1983); Kornhauser, and Revesz (1989); Young, Faure, Fenn, and Willis (2007); Parisi, and Singh (2010).

⁵ For a psychological approach on causation in law, see Rachlinski (1998) and more generally the literature about the hindsight bias in behavioral law and economics.

⁶ See the *Restatement (Third) of Torts: "Apportionment of Liability"* promulgated by the American Law Institute in 1999 and published in 2000, notably "Topic 5: Apportionment of Liability When Damages Can Be Divided By Causation, § 26" (thereinafter, the *Restatement*).

⁷ For a comprehensive view of the economics of causation, see Ben-Shahar (2000). Surprisingly enough, the theory of cooperative game and its solution concepts have never been elaborated in the law and economics literature to analyze multiple causation issues. To our knowledge, and except for an unpublished paper mentioned by Ben-Shahar (2000), no model of multiple causation cases is available in terms of cooperative game. See also the approach proposed by Braham and van Hees (2009) which analyzes the concept of "the degrees of causal contribution for actual events" by using power indices.

consider causation from an *ex post* perspective – once the damage occurred – and not from an *ex ante* perspective.⁸

Contemplating the debates between legal philosophers and law and economics scholars on causation, the *ex ante-ex post* distinction could be said to be a *summa divisio*. On the one side, most legal philosophers interested in corrective justice criticize law and economics findings for its forward-looking oriented theory of causation and prefer developing some *ex post* criteria of causation;⁹ on the other side, law and economics scholars, following Coase, try to show that causation is not the keystone of Torts as soon as the legal system seeks to implement optimal incentives. Dealing with causation in law and economics, Cooter wondered "*how is legal cause imbedded in formal models? Do the formal models clarify the difficult legal issues about causation, as concluded by such writers as Calabresi, Shavell, and Landes and Posner? Is the disappearance of "cause" from the formal models evidence of scientific progress and a reason for celebration, as Russell's views suggest? Or do the formal models obscure legal cause and suppress interesting legal issues, as asserted by critics such as Wright?*" (Cooter 1987, p. 523). One of the findings of our approach is to show economic theory adds also to *ex post* causation theories and apportionment issues. Legal philosophy could learn from economic models of causation in an *ex post* perspective. Such models could then be developed to fill the gap between legal conceptions of causation and law and economic ones. This is one of the findings of the paper.

In the following, we distinguish with Posner and Landes (1980) successive joint tort and simultaneous joint tort, and we focus on a subset of multiple causation cases for the clarity of the exposition: the successive injury. The reason why we focus on successive causation is twofold. Firstly, these cases have specific mathematical properties; secondly, the counterfactuals needed to implement apportionment rules are more easily knowable than in simultaneous cases. Successive injury occurs when, after an injury caused by a first tortfeasor *A* to a victim *V*, the damage is aggravated by tortious acts from a second wrongdoer *B*, then from a third one *C etc. A, B, C...* are said to be the multiple tortfeasors because they cause *together* the final damage suffered by *V*. An example from the *Restatement* may illustrate

⁸ Our approach is more a retrospective causation perspective rather than a prospective causation perspective. According to Ben-Shahar (2000, p. 647) "*Retrospective causation exists if, all else held fixed, but for the action the harmful consequence would not have occurred. Prospective causation exists when an action raises the probability of the harmful consequence. Thus, the distinguishing factor between the two types of causation is the time perspective of the evaluation. Retrospective causation is backward-looking, answering the counterfactual inquiry of whether the action was a necessary condition for the outcome. Prospective causation, in contrast, is forward-looking, answering the ex ante inquiry of whether the action increased the likelihood of injury*".

⁹ As Cooter says, "*Economic models of tort law are based on functional relationships among such variables as the probability of accidents, the harm they cause, and precaution against them. Being mathematical relationships, they are not explicitly causal [...]*" (Cooter 1987, p. 523). For a criticism of economic analysis of law related to *ex post* and *ex ante* perspectives on causation, see Wright (1985b) and Coleman (1992). Our approach shows instead that economics has a lot to say on *ex post* causation.

such a case. Suppose, "A negligently parks his automobile in a dangerous location. B negligently crashes his automobile into A's automobile, damaging it. When B is standing in the road inspecting the damage, B is hit by C, causing personal injury to B. B sues A and C for personal injury and property damage. B's negligent driving and A's negligent parking caused damage to B's automobile. A's negligent parking, B's negligent driving, B's negligent standing in the road, and C's negligent driving caused B's personal injuries." (American Law Institute, 2000, Topic 5, §26, *comment c*). How should judges determine the compensation to be paid by each injurer? Should he consider that the car driver A is liable for the entire damage insofar as, without his action, the damage would have not occurred? Or that each of the tortfeasors is liable for a part of it? Or that one of them is more liable than the other and for which amount? An apportionment rule is needed to correctly share the damage. Such litigations occur as soon as two or more individuals have jointly caused damages. It is easy to think of the different fields of law concerned by this issue: environmental law, nuisance, accidental law, medical malpractices, products liability, insurance law, or even antitrust, *etc.*¹⁰

In our model, an *adjudication* specifies the compensation that each tortfeasor has to pay to the victim. *Adjudications* should be *unobjectionable* (Ferey and Dehez 2013). There is a *minimum* compensation: each tortfeasors should pay at least the damage that he would have caused *alone*. There is also a *maximum* compensation: no tortfeasor should pay more than the *additional* damage that he has caused. The additional damage is measured by the difference between the total damage and the damage that would have resulted without the participation of that tortfeasor. We go further and extend them from individual tortfeasors to *subsets of tortfeasors*, leading to the following two conditions:

- C1 *The contribution of any subset of tortfeasors should be at least equal to the damage they would have caused without the intervention of the others.*
- C2 *The contribution of any subset of tortfeasors should not exceed the additional damage resulting from their participation.*

To apprehend the notion of unobjectionable adjudications, we construct a game with transferable utility – called *liability game* – whose characteristic function precisely measures the *potential* damage caused by any subset of tortfeasors, capturing successive causation if any. We show that the *core* of a liability game defines the set of *all* unobjectionable adjudications, and that the (symmetric) *Shapley value* defines a fair compromise in which tortfeasors differ only in the damage they have caused. A judge may depart from that fair compromise by assigning weights to tortfeasors in order to reflect misconduct or negligence.

¹⁰ Three main approaches are distinguished in law: joint liability, several liability, and joint and several liability. In *joint liability*, each tortfeasor is liable for the full amount of the damages, without any claim against the other tortfeasors. In *several liability*, each tortfeasor is only liable for a given share. In *joint and several liability*, each tortfeasor is liable for the total amount of damages, but has a claim against the other tortfeasors to get their contribution to damage back. Sharing rules are needed in the last two cases.

The resulting *asymmetric* Shapley values define unobjectionable adjudications and, vice versa, weights can be associated to any unobjectionable adjudication.

Both legal practices and economic analysis of law are concerned by our analysis. Firstly, our model provides a characterization of the apportionment rules that could be used by Courts. Secondly, we show that judicial practices, jurisprudence and legal debates underlie the solution concept that we use. For that purpose, we illustrate our model by some Court decisions and by proposals and synthesis provided by the *Restatement*. We show that our approach offers a framework to better understand the two-steps process advocated by the *Restatement* based on apportionment by causation and apportionment by responsibility. Cooperative game theory is relevant for law and we aim to make judges and legal practitioners aware of the implicit logic they use to solve actual cases. Moreover, discussing apportionment issues on the grounds of an axiomatic method may be useful to achieve greater fairness and consistency in adjudication.¹¹

The remainder of the paper is organized as follows. In section 2, we define liability games and show that their core defines the set of unobjectionable adjudications. We show that it coincides with the set of weighted Shapley values in the sequential case. We then compare Shapley value properties with other allocation rules - egalitarian rule, nucleolus or equal surplus. Section 3 deals with legal issues. We show how the rule proposed by our cooperative game model enlightens the main legal principles and practices in tort law. We mainly rely on American common law cases, on the one hand, and on principles and proposals advocated by the *Restatement* on the other. More precisely, we deal with the scope of the Shapley value for the law – normative as well as positive – and we show the two-step process proposed by the *Restatement* follows an apportionment method, which is equivalent to the core and the weighted Shapley value prescriptions. Section 4 concludes.

¹¹ As Coleman (1982, p. 349) asserts "*political authority is necessary and inevitably coercitive [...], exercising it requires a justification [and therefore] any body of the law must be coherent and consistent*". See also Boston (1996, p. 269).

2. LIABILITY GAMES

Litigations about multiple causation are due to the fact that several tortfeasors have jointly caused damage to a victim. We begin by providing a heuristic presentation of our approach. We then define liability games, with a particular attention to sequential liability, and introduce the concepts of core and Shapley value. Throughout this section, we illustrate our approach with 2 and 3 player's cases.

2.1 Heuristic presentation

Let us consider a situation where two persons are involved in damage whose monetary value D is known. A Court must allocate D between the two injurers. This determines the amount each one will be asked to pay. We will consider two cases. The "*simultaneous liability*" where no damage would have resulted if one of the injurer had not been present and the "*sequential liability*" where the damage is successively aggravated.

Equal division is the natural allocation in the simultaneous liability case. A Court may however consider that, because of negligence or fault, one injurer should be asked to pay more than the other. Let's identify the injurers as 1 and 2. An *adjudication* is a pair (x_1, x_2) that specifies an allocation of damage D among the two injurers: $x_1 + x_2 = D$. A system of non-negative weights $w = (w_1, w_2)$ summing to one can be associated to an adjudication $x = (x_1, x_2)$. They are given by $x_i = w_i D$ ($i = 1, 2$). They give a measure of the relative responsibility of each injurer.

Sequential liability is more complicated. The first injurer causes an initial damage d_1 that is aggravated by the second injurer who causes a further damage d_2 . Total damage is then given by $D = d_1 + d_2$. Imposing to each injurer to pay for "his" damage may seem to be, at first sight, a natural solution. It is however not necessarily fair. Indeed, if the first injurer had not been there, no damage would have occurred. The first injurer could be asked to pay, also part of the damage d_2 . A balanced solution is to impose to the second injurer to pay *half* of his damage. In order to allow for a differential treatment of the injurers, a system of non-negative weights (w_1, w_2) summing to one can be associated to an adjudication (x_1, x_2) and, vice-versa, adjudications reveals weights:

$$x_1 = d_1 + w_1 d_2$$

$$x_2 = w_2 d_2 = (1 - w_1) d_2$$

or, equivalently:

$$w_2 = \frac{x_2}{d_2}$$

$$w_1 = 1 - w_2 = \frac{x_1 - d_1}{d_2}$$

(1)

2.2 Liability games

We denote by $N = \{1, 2, \dots, n\}$ the set of tortfeasors involved in the case, $n \geq 2$. All together, they have caused a total damage D .¹² For any subset of players, we need to identify damage that these players would have caused together, without the contribution of the other players. This is the notion of *potential damage* that relies on a counterfactual reasoning. It applies to individual players as well, and the potential damage of the all player set (the "grand coalition") is the total damage D . This defines a function v that associates a real number $v(S)$ to all possible subsets $S \subset N$. The pair (N, v) is a *cooperative game with side payments* where v is the *characteristic function* of the game.¹³ In a general context, $v(S)$ is the "worth" of coalition S that measures the minimum that coalition S can ensure by itself, if it forms. In our context, these games are called "*liability games*".¹⁴

In a simultaneous liability case where each tortious act is a *necessary condition* to damage, the game is easily identified: $v(S) = D$ if $S = N$ and $v(S) = 0$ for all $S \neq N$. This corresponds to the *unanimity game*: no damage occurs once a member of N is missing.¹⁵

In the sequential case, players are identified by their position and the *immediate* damage $d_i \geq 0$ caused by each player is assumed to be known.¹⁶ The corresponding liability game is then entirely defined by the list of immediate damages $d = (d_1, d_2, \dots, d_n)$. In the 2-player and 3-player cases, we then successively have:

$$v(1) = d_1$$

$$v(2) = 0$$

$$v(1, 2) = d_1 + d_2 = D$$

and

$$v(1) = v(1, 3) = d_1$$

$$v(2) = v(3) = v(2, 3) = 0$$

$$v(1, 2) = d_1 + d_2$$

$$v(1, 2, 3) = d_1 + d_2 + d_3 = D$$

The *marginal contributions* of a player to all possible coalitions is a central concept in allocation theory: for any given coalition $S \subset N$, the marginal contribution of a player i to

¹² Players are the injurers and possibly also the victim in which case her indemnity is reduced by the amount she has to pay.

¹³ The characteristic function was first introduced by von Neumann and Morgenstern ([1944] 1953)). See Luce, and Raiffa (1957) for an old yet excellent reading, or Maschler, Solan, and Zamir (2013) for a more recent one.

¹⁴ By convention, the empty set is assigned a zero value: $v(\emptyset) = 0$.

¹⁵ This holds at least for simple simultaneous cases where all the tortfeasors are said to be necessary causes. The findings would be different in more complex cases as the overdetermination cases for example. On causal overdetermination, see for example Hart and Honoré (1985) and Stapleton (2013).

¹⁶ An immediate damage could be zero. If $d_i = 0$ for some player $i < n$, injurer i has caused *indirectly* a damage.

coalition S is defined by $Cm_i(S) = v(S) - v(S \setminus i)$. It is obviously zero if $i \notin S$. In the framework of our model, $Cm_i(S)$ is the marginal damage of injurer i to coalition S . It measures the increase in damage caused by injurer i , with reference to the potential damage of coalition S . For instance, d_2 is the marginal damage of player 2 to coalition $\{1,2\}$ while $D = d_1 + d_2 + d_3$ is the marginal damage of player 1 to coalition $\{1,2,3\}$.

Two players are said to be *equal* if they contribute equally to all coalitions to which they both belong. They are interchangeable. In the simultaneous case, all players are equal. In the sequential case, two players are equal if (and only if) they are consecutive and the first causes no immediate damage. For instance, players 2 and 3 are equal *if and only if* $d_2 = 0$.

2.3 The core

The core of a game (N, v) is a concept introduced by Gillies (1953). It is the set of allocations that give to all coalitions *at least* what they are worth:

$$C(N, v) = \left\{ x \in (x_1, \dots, x_n) \mid \sum_{i \in N} x_i = v(N) \text{ and } \sum_{i \in S} x_i \geq v(S) \text{ for all } S \subset N \right\}$$

No coalition receives less than its worth. In this sense, no coalition can formulate an objection against core allocations. In general, nothing insures that such allocation exists. Applied to a 3-player game, the core is the set of allocations (x_1, x_2, x_3) such that $x_i \geq v(i)$ for all i , and $x_i + x_j \geq v(i, j)$ for all $i \neq j$. Equivalently, it is the set of allocations (x_1, x_2, x_3) such that

$$\begin{aligned} v(1) &\leq x_1 \leq v(N) - v(2, 3) \\ v(2) &\leq x_2 \leq v(N) - v(1, 3) \\ v(3) &\leq x_3 \leq v(N) - v(1, 2) \end{aligned}$$

Applied to a 3-player liability games, the left-hand sides are the potential damage of the individual players and the right-hand sides are their additional damage. Hence, the core is precisely the set of *unobjectionable adjudications* as defined in the introduction: each player pays *at least* his potential damage and *at most* his additional damage. In the simultaneous case, the core imposes no restriction: $0 \leq x_i \leq D$ for all $i \in N$. In the 3-player sequential case, the core is the set of allocations (x_1, x_2, x_3) such that:

$$\begin{aligned} d_1 &\leq x_1 \leq d_1 + d_2 + d_3 \\ 0 &\leq x_2 \leq d_2 + d_3 \\ 0 &\leq x_3 \leq d_3 \end{aligned}$$

Hence, the core of simultaneous and sequential liability games is always nonempty. We observe that unobjectionable adjudications satisfy a basic fairness principle: no one covers a

damage that has occurred "*upstream*" in the sequence. As a result, the first player has always to cover the initial damage.

2.4 The Shapley value

The value is a concept introduced by Shapley (1953). For a given game (N, v) , the Shapley value is an *allocation rule* that specifies for each player his share in $v(N)$, defined as a weighted average of his marginal contributions:

$$SV_i(N, v) = \sum_{S \subset N} \alpha_n(s) (v(S) - v(S \setminus i)) \quad i = 1, \dots, n$$

The weights only depend on coalition size and are given by:¹⁷

$$\alpha_n(s) = \frac{(s-1)!(n-s)!}{n!}$$

As such, it is just a formula, but it can be axiomatized. There exist several characterizations in the literature beyond Shapley's original one.¹⁸ We retain here the alternative axiomatization due to Young (1985) because it is more appropriate within our context. Young proves that it is the *unique* allocation rule that satisfies the following properties:

Efficiency The shares of the players add up to the value of the game.

Symmetry Equal players are entitled to equal shares.

Monotonicity If a game is modified and the marginal contributions of a player do not decrease, then the amount paid by that player cannot decrease.

Efficiency is included in the definition of an allocation rule: the value of the game $v(N)$ is *exactly* distributed. Symmetry is nothing but the axiom of *equal treatment of equals*. Monotonicity is a strong *independence axiom*: what is allocated to a player only depends on his marginal contributions, independently of the other players' contributions.

Applied to a simultaneous liability game, no need for hard computations: by symmetry, the Shapley value imposes every players to pay the same amount. In the sequential 2-players case, we retrieve the rule (1) with equal weights: $w_1 = w_2 = 1/2$. In the 3-players case, we get:

$$\begin{aligned} x_1 &= d_1 + \frac{1}{2}d_2 + \frac{1}{3}d_3 \\ x_2 &= \frac{1}{2}d_2 + \frac{1}{3}d_3 \\ x_3 &= \frac{1}{3}d_3 \end{aligned} \tag{2}$$

This "triangular" formula easily extends to any number of players.

¹⁷ We use lower case letter to identify the size of a set: s is the cardinal of S .

¹⁸ See for instance Moulin (1988).

2.5 The weighted Shapley value

Removing symmetry allows equal players to be treated differently, opening the way to a family of values, called *weighted Shapley values*, obtained by assigning weights to players.¹⁹ In the 3-player case, we obtain the following allocations:

$$\begin{aligned}x_1 &= w_1 d_3 + \frac{w_1}{w_1 + w_2} d_2 + d_1 \\x_2 &= w_2 d_3 + \frac{w_2}{w_1 + w_2} d_2 \\x_3 &= w_3 d_3\end{aligned}\tag{3}$$

This is again a triangular formula, with appropriate weighting. Notice that (4) is valid as long as $w_1 + w_2 > 0$. In the case where $w = (0, 0, 1)$, the last player covers his additional damage d_3 but there is an *indetermination* concerning the division of d_2 . A selection has to be made. Because weights are equal, the natural solution is to apply the symmetric Shapley value to the 2-player game restricted to the coalition $\{1, 2\}$. The corresponding allocation is then given by:

$$\begin{aligned}x_1 &= \frac{1}{2} d_2 + d_1 \\x_2 &= \frac{1}{2} d_2 \\x_3 &= d_3\end{aligned}$$

The allocation that imposes to the first player to cover the entire damage D corresponds to $w = (1, 0, 0)$. The allocation $x = (d_1, d_2 + d_3, 0)$ corresponds to $w = (0, 1, 0)$: the last player is exempted and the second player covers his marginal damage $d_2 + d_3$. If only one player is assigned a zero weight, he is exempted, except of course for the first player who has to pay at least d_1 . Hence, $x_i = 0$ if $w_i = 0$ for all $i \geq 2$ and $x_1 = d_1$ if $w_1 = 0$.

It is easily verified that the allocation corresponding to any set of non-negative weights $w = (w_1, w_2, w_3)$ belongs to the core. On the other hand, Monderer, Samet and Shapley (1992) have shown that core allocations are weighted values: a weighted adjudication is unobjectionable and, vice-versa, unobjectionable adjudications reveal weights.²⁰

The definition of the Shapley value, weighted or not, is easily extended to any number of players: the triangular formulas (2) and (3) indeed extend to any $n \geq 3$. It goes differently for the concept of unobjectionable adjudication. As mentioned in the introduction, it can be extended to accommodate more than three players by going from individual players to

¹⁹ Weighted Shapley values have been axiomatized. See for instance Kalai, and Samet (1987) or Dehez (2011).

²⁰ In general, core allocations are weighted values. The opposite inclusion holds for the class of convex games introduced by Shapley (1971). Simultaneous and sequential liability games are convex. See Dehez, and Ferey (2013) for more details.

coalitions of players. Indeed, consider a core allocation x and a coalition $S \subset N$. By definition of the core, we have:

$$\sum_{i \in S} x_i + \sum_{i \in N \setminus S} x_i = v(N) \quad \text{and} \quad \sum_{i \in N \setminus S} x_i \geq v(N \setminus S)$$

Combining these two conditions, we obtain:

$$\sum_{i \in S} x_i \leq v(S) - v(N \setminus S)$$

Hence, core allocations satisfy the following inequalities:

$$v(S) \leq \sum_{i \in S} x_i \leq v(S) - v(N \setminus S) \quad \text{for all } S \subset N$$

Applied to liability games, they correspond to conditions C1 and C2: no *coalition* pays less than its potential damage, nor more than its additional damage. With this definition of unobjectionable adjudication, all that precedes carries over, in particular the equivalence between unobjectionable adjudications and weighted adjudications. Notice that C1 and C2 are equivalent conditions: an adjudication that verifies one, automatically verifies the other.

2.6. Alternative allocation rules

Before applying the Shapley values to the law, it is useful to consider other well-known allocation rules: equal division, equal surplus and nucleolus. Equal division is the simplest allocation rule:

$$ED(N, v) = \frac{1}{n} v(N) \quad i = 1, \dots, n$$

Applied to liability games, it imposes to each injurer to pay the same amount. This rule applies naturally in the simultaneous case. In the sequential case, it is not appropriate because it does not take into account the relative involvements of the players in the occurrence of damage. Furthermore, it generally does not define an unobjectionable adjudication.

An alternative could be to impose to players to pay for their contribution to the total damage given by $Cm_i(N) = v(N) - v(N \setminus i)$. Given efficiency, the resulting allocation rule, known as "*egalitarian non-separable contribution*", is defined by:

$$ENSC(N, v) = Cm_i(N) + \frac{1}{n} \left(v(N) - \sum_{j \in N} Cm_j(N) \right) \quad i = 1, \dots, n$$

It coincides with the Shapley value in the 2-player case but is much different when more than two players are involved.

Applied to a 3-player sequential game, the ENSC rule gives:

$$\begin{aligned}
 x_1 &= d_1 + \frac{2}{3}d_2 + \frac{1}{3}d_3 \\
 x_2 &= \frac{2}{3}d_2 + \frac{1}{3}d_3 \\
 x_3 &= -\frac{1}{3}d_2 + \frac{1}{3}d_3
 \end{aligned} \tag{4}$$

While it defines unobjectionable adjudications, it obviously leads to counterintuitive results: it fails to satisfy upstream independence and, moreover, a decrease in d_2 leads to an increase in x_3 .

Another well-known allocation rule is the nucleolus introduced by Schmeidler (1969). In the spirit of the leximin criteria proposed by Rawl (1971), it "*minimizes dissatisfaction with priority to the coalitions that are most dissatisfied*", to quote Shubik (1982, p. 339). It has been applied to liability games in Dehez and Ferey (2013). In the 3-player sequential case, the nucleolus has two parts, depending on the relative values of d_2 and d_3 . If $d_3 \leq 2d_2$, it produces the following allocation:

$$\begin{aligned}
 x_1 &= d_1 + \frac{1}{2}d_2 + \frac{1}{4}d_3 \\
 x_2 &= \frac{1}{2}d_2 + \frac{1}{4}d_3 \\
 x_3 &= \frac{1}{2}d_3
 \end{aligned} \tag{5a}$$

If instead, if $d_3 \geq 2d_2$, we have:

$$\begin{aligned}
 x_1 &= d_1 + \frac{1}{3}d_2 + \frac{1}{3}d_3 \\
 x_2 &= \frac{1}{3}d_2 + \frac{1}{3}d_3 \\
 x_3 &= \frac{1}{3}d_2 + \frac{1}{3}d_3
 \end{aligned} \tag{5b}$$

The nucleolus is an element of the core and it therefore defines unobjectionable adjudications. However, *as a rule*, it violates upstream independence: the immediate damage caused by player 2 may affect the amount that player 3 has to pay.

3. APPLYING THE WEIGHTED SHAPLEY VALUE TO THE LAW

Liability games formally defined in the previous section and their solution concepts are relevant to improve our understanding of the apportionment issue. More precisely, we have shown the relevance of the Shapley value and its weighted version, in relation to the core defined as the set of unobjectionable adjudications. As the cooperative games are less used in law and economics literature than the non-cooperative games, we further investigate the scope of our approach for the law from a normative and descriptive point of view.

3.1 The Shapley value as a normative tool

"Normative aspects of game theory may be sub-classified using various dimensions. One is whether we are advising a single player (or group of players) on how to act best in order to maximize payoff to himself, if necessary at the expense of the other players; and the other is advising society as a whole (or group of players) of reasonable ways of dividing payoff among themselves. The axis I'm talking about has the strategist (or the lawyer) at one extreme, the arbitrator (or the judge) at the other." (Aumann 1985, p. 38). In the following, we use the term *normative* in the second sense, the one of the judge.

We have seen that the Shapley value is just one allocation rule among others. Therefore, why should the Shapley value be preferred to any other rule? Should a Court follow apportionment based on the Shapley value? Here we rely on three major arguments to answer this question. Firstly, the properties of the Shapley value are meaningful for the law and need to be carefully examined to assess its normative content and acceptability; secondly, compared to other solutions, the Shapley value seems more relevant to correctly apportion damage among injurers in legal contexts; thirdly, normative statements in terms of game theory has to be compared with traditional law and economics criteria, namely the minimization of social costs.

3.1.1 Axiomatization of the Shapley value

The symmetric Shapley value is a fair compromise between tortfeasors' concurrent claims. To see why, two arguments can be elaborated. Firstly, Shapley's formula is based on marginal damages. In this sense, the Shapley value is an evaluation of the degree of causation of each wrongdoing act²¹ and can be considered as a useful benchmark to evaluate whether an injurer was strongly or weakly causally involved in the damage. Secondly, the axiomatic characterization of the value identifies its foundations as an allocation procedure, in particular efficiency, symmetry and monotonicity.

The law requires efficiency: damage has to be totally recovered by the victim and, at the same time, punitive damages put aside, the victim cannot get more than his damage.

²¹ For a similar statement from a philosophical perspective, see Braham and van Hees (2009).

Symmetry states that two injurers with identical marginal damage to all coalitions of which they are members should pay the same amount. Quoting Young (1994, p. 1215), "*Of all properties that characterize the Shapley value, symmetry seems to be the most innocuous [...] because it calls for a judgment about what should be treated equally. [...] the symmetry axiom is not plausible when the partners [...] differ in some respect other than [benefit] that we feel has a bearing on the allocation*". That is why our approach leaves open the possibility to consider elements beyond causation, as the degrees of fault and responsibility, that are in line with distributive justice and not corrective justice.

Monotonicity states that if the marginal damage of an injurer decreases, what he is asked to pay should not increase, independently of possible changes in other injurers' marginal damage. In other words, the share of an injurer should depend exclusively on his marginal damage.²²

This axiomatic foundation of the value applies to the class of *all* transferable utility games. A natural question is to identify properties that produce the Shapley value when restricted to the class of sequential liability games. Here are two properties particularly appropriate in our context.

Zero immediate damage If an injurer causes no immediate damage, his share and the share of his successor should coincide.

Upstream independence The amount paid by an injurer should not depend on the damage caused by the injurers that precede him.

Zero immediate damage is nothing but symmetry. We have indeed seen that when $d_i = 0$ for some $i < n$, the injurer i and $i + 1$ are equal. Upstream independence says that the share of an injurer is independent of the immediate damages caused by his predecessors. Actually, zero immediate damage and upstream independence suffice to characterize the Shapley value on the set of sequential liability games.

Proposition: Given a sequential liability game, the Shapley value is the unique allocation rule satisfying efficiency, zero immediate damage and upstream independence.

Proof We look for rules φ satisfying the following two properties:

$$\begin{aligned} d_i = 0 \text{ for some } i < n &\Rightarrow \varphi_i(d) = \varphi_{i+1}(d) \\ d_j = d'_j \text{ for all } j = i, \dots, n &\Rightarrow \varphi_i(d) = \varphi_i(d') \end{aligned}$$

Consider the case $n = 3$. If $d = (0, 0, d_3)$, efficiency and zero immediate damage imply:

$$\varphi_1(0, 0, d_3) = \varphi_2(0, 0, d_3) = \varphi_3(0, 0, d_3) = \frac{d_3}{3}$$

²² Actually, that property is enough to characterize the Shapley value. See Young (1985).

Upstream independence then ensures that $\varphi_3(d) = d_3/3$ for all non negative (d_1, d_2, d_3) . If $d = (0, d_2, d_3)$, efficiency and zero immediate damage then imply:

$$\varphi_1(0, d_2, d_3) = \varphi_2(0, d_2, d_3) = \frac{1}{2}(d_2 + \frac{2}{3}d_3) = \frac{1}{2}d_2 + \frac{1}{3}d_3$$

Using upstream independence again, we must have that $\varphi_2(d) = d_2/2 + d_3/3$ for all non negative (d_1, d_2, d_3) . We are then left with d_1 that must be paid by the first player:

$$\varphi_1(d) = d_1 + \frac{1}{2}d_2 + \frac{1}{3}d_3 \text{ for all non negative } (d_1, d_2, d_3)$$

Consequently, zero immediate damage and upstream independence, together with efficiency, define a unique rule that coincides with the Shapley value of the associated liability game. The argument extends to any number of players, starting from the last player and proceeding backward. ♦

Another reason why we insist on the Shapley value as a useful guide for the Court is due to the advantages of the Shapley value (2) compared to other allocation rules, taking into account the context: equal division, egalitarian non-separable contribution (4) and nucleolus (5). All three rules satisfy the weak property of monotonicity used to characterize the Shapley value in the general framework. Within the context of sequential liability games, a stronger monotonicity requirement is the following:

Strong monotonicity An increase in the immediate damage of a player should not reduce the amount paid by any player.

The egalitarian non-separable contribution rule (4) fails to satisfy that property. The following table summarizes the properties of the various allocation rules that we have considered.

	<i>ED</i>	<i>ENSC</i>	<i>Nucleolus</i>	<i>SV</i>
<i>Zero immediate damage</i>	Yes	Yes	Yes	Yes
<i>Upstream independence</i>	No	No	No	Yes
<i>Strong monotonicity</i>	Yes	No	Yes	Yes

Table 1. Properties of different allocation rules

3.1.2. *Incentives and the Shapley value*

Even if our approach does not directly deal with the incentive aspect of apportionment, it leaves room for further development making a bridge between cooperative and non-cooperative approaches on multiple causation issues. As we asserted in the introduction, most

of the literature in law and economics has developed an *ex ante* perspective: the main issue of apportionment is to provide efficient incentives for future injurers. From a normative perspective, it is needed to consider the relationships between the Shapley value solutions and the efficiency criteria used in law and economics literature, i.e. the minimization of the social costs.

One of the results of the non-cooperative literature in law and economics about apportionment is that, under certain circumstances, minimization of social costs requires multiple injurers to pay together more (or in some cases less) than the total amount of damage.²³ In the simplest two player's case, minimization of social costs may require the first tortfeasor to pay $d_1 + d_2$ and the second tortfeasor to pay d_2 . As such, the total amount of the offsettings paid by tortfeasors would be $d_1 + 2d_2$ leading to an overcompensation of the victim. Avoiding overcompensation would be possible by decoupling compensation and damages paid. Even if decoupled liability designs existed, it could be considered as unfair since causation requirements would be violated and tortfeasors will pay more than what they have actually caused. As the Shapley value respects the efficiency axiom and reduces the size of the set of acceptable allocations, choosing a weighted Shapley value for apportioning the damage among tortfeasors does not necessary lead to an optimal (*ex ante*) incentives scheme. We face a trade-off between minimization of the social costs and fairness principles.

However, one step further could be proposed to file this gap²⁴. As the different allocations belonging to the core – which are weighted Shapley-values – lead to different incentives schemes on tortfeasors, the minimization of social cost criteria could be used to choose among them. In other words, it would be acceptable to choose, within the core, the allocation that provides the best *ex ante* incentives in terms of minimization of social costs. This is a second best argument on which it could be possible to elaborate further a bridge between the *ex post* and *ex ante* approaches of causation.

3.2 The Shapley value as a descriptive tool

The solution concepts in cooperative game theory should not only be understood as normative tools to guide a Court. They also provide a framework to better understand existing norms and Courts' decisions.²⁵ An issue that we now address. We first analyze some famous

²³ See Young et al. (2007, p. 123).

²⁴ See also that the structure of sequential liability games, incremental harm caused by a downstream tortfeasors, closely resembles to the case studied in Singh (2007) where a unique tortfeasor takes incremental care levels and causes incremental harm. The difference however lies on the fact that in the Singh scenario any incremental damage caused is not a necessary condition for the following harms.

²⁵ "The distinction between the descriptive and the normative modes is not as sharp as might appear, and often it is difficult to decide which of these two we are talking about. For example, when we use game or economic theory to analyze existing norms (e.g. law), is that descriptive or is it normative? We must also be aware that a given solution concept will often have both descriptive and normative interpretations..." (Aumann 1985, p. 37). In a famous paper, Aumann and Maschler (1985) have contributed to this view by providing a game theoretic analysis of a bankruptcy problem from the Talmud.

cases showing that adjudications may be rationalized in terms of the Shapley value. We then develop further the argument by providing a rationalization of the principles and methods advocated by *the Restatement* to apportion damage among multiple tortfeasors – the "two-steps" method – in terms of the weighted Shapley value.

3.2.1 Example and cases

The most illustrative examples of our model are the successive accident cases where the tortfeasors tortious acts are related.²⁶ To illustrate our approach, we first study in detail a particular case. In *Webb v Barclays Bank Plc & Anor*, the England and Wales Court of Appeal (Civil Division) had to solve a multiple and successive causation cases.²⁷ Here are the facts. Mrs. Webb contracted polio in the second year of her life and stayed with leg and knee vulnerability. In 1994, she was employed by Barclays Bank (thereafter the Bank) and stumbled and fell in their forecourt. She suffered pain and was then cured by the Portsmouth Hospital Trust (thereafter the Trust). After several medical treatments, the Trust advised Mrs. Webb to get an amputation above the knee. She accepted. A few months later, an independent report from others doctors shows that the Trust was negligent about advises provided to Ms. Webb and that such a medical operation was not required. Mrs. Web decided to claim against the Bank and the Trust. In 2000, the Bank settled with Mrs. Web for the entirety of the damage (£. 165,953). The Bank then had a recovery claim against the Trust.

First, the Court wondered whether "when an employee is injured in the service, and by the negligence, of her employer, his liability to her is terminated by the intervening negligence of a doctor brought in to treat the original injury, but who in fact made it worse." (§52). "The answer to this first issue is negative and the negligence in advising amputation did not eclipse the original wrong-doing. The Bank remains responsible for its share of the amputation damages. The negligence of [the Trust] was not an intervening act breaking the chain of causation." (§57). Therefore, the entire damage has to be apportioned between the two injurers.

Second, the Court addressed the issue of apportionment. The logic of apportionment provided by the Court is exactly the same as a weighted Shapley value. The Court begins by dividing the final damage in two part, basis A and basis B: "*First, (Basis A) there was the tripping accident, brought against the claimant's employers, the Bank, for their negligent failure to maintain their forecourt. [...] Second (Basis B), there was the claim for the doctor's negligent advice, as a result of which the leg was amputated.*" (§ 46). Basis A – which is $v(1)$ in terms of our model – is evaluated at £. 53,945 and Basis B – which is $v(12) - v(1)$ in terms

²⁶ We exclude unrelated cases insofar as the second tortious act is not a legal cause of the damages up: apportionment is simple and is proportionate to each harm separately evaluated.

²⁷ *Webb v Barclays Bank Plc & Anor* (2001) EWCA Civ 1141 (16 July 2001).

of our model – at £. 112,008. Then, the Court assessed the degree of responsibility: "*The Bank, by their negligent maintenance of the forecourt, was responsible for getting the vulnerable Mrs. Webb before the doctors employed by the Trust. But it was the latter's negligence that was much more responsible for the amputation and all that went with it. In all the circumstances, we assess the Bank's responsibility at 25% and the Trust's at 75%*". (§ 59) The final apportionment ordered by the Court is therefore for the Bank: Basis A plus 25% of Basis B, and for the Trust 75% of Basis B. In terms of our model, the allocation chosen by the Court is the weighted Shapley value as defined by equation (4) with $w_1 = 0.25$ and $w_2 = 0.75$.

Many others cases and litigations are covered by our model: enhanced injury, background conditions, victim's contribution and also some nuisances or product liability cases. In these issues, a common mathematical structure can be identified once the different tortfeasors (including the victim) follow a temporal chain of causality: in these cases, tortious acts of the tortfeasor i are a physical cause of "direct" damage d_i and a proximate or legal cause of the aggravated damages up in the liability sequence (the enhanced injuries d_j with $j > i$). We provide further examples of these different kinds of litigation. Thereafter, all these cases will be named "successive injury cases".

Successive accident cases. In *Maddux*, the first tortfeasor hits the plaintiff's car and thirty second later, a second driver hits the car and causes other injury. The causal events are so close that the chain of injuries was considered by the Court as a single case.²⁸

Background conditions. In *Steinhauser*, the Court had to adjudicate a case where the tortious act of the defendant had caused a "chronic schizophrenic reaction" from the plaintiff.²⁹ The Court held that the defendants could explore the possibility of the plaintiff having developed schizophrenia regardless of the accident.

Victim's contribution. In *Prospectus Alpha Navigation Co*, the plaintiff's ship was tied up at the defendant's dock.³⁰ Because of a negligent tortious act of the plaintiff's crew, the ship caught fire. However, the defendant was also negligent: he sent the plaintiff's ship away before the fire being completely extinguished. Then, the fire caused further damage. In *Dillon*, a young boy was on a high beam of a bridge trestle. He lost his balance and was falling to the rocks when he grabbed the electric wires, negligently exposed by the defendant, which killed him.³¹

²⁸ *Maddux v. Donaldson*, 108 N.W.2d 33 (Mich. 1961).

²⁹ *Steinhauser v. Hertz Corp.*, 421 F.2d 1169 (2d Cir.1970). See also *Lancaster v. Norfolk and Western railway Co.*, 773 F. 2d 807, 822 (7th Circuit. 1985).

³⁰ *Prospectus Alpha Navigation Co v. North Pacific Grain Growers, Inc.*, 767 F. 2d 1379 (9th Cir. 1986).

³¹ *Dillon v. Twin State Gas & Electric Co.*, 163 Atl. 111 (N.H.1932).

Product liability. In *Hillrichs*, the Court considered that a jury could evaluate the extent of the enhanced injury.³² A corn-harvesting machine was not equipped with an emergency stop device and the plaintiff lost his fingers after his hand had been entangled in. The Court considered that some evidence showed that the injury would have been different with a stop emergency device. In *Reed*, the plaintiff's was involved in a car accident in which the shattering of the fiberglass top of his car hurts his arm.³³ The expert testified that such injury would have been avoided by a metal top. The Court considered that estimation of the enhanced damages was possible.

3.2.2 *The Restatement and the weighted Shapley value*

The usefulness of the Shapley value to better understand apportionment principles in law may be systematized. One of the innovations proposed by the *Third Restatement* compared to the *First* or the *Second* is a "two-step process" to apportion damage among tortfeasors.³⁴ The method provides a unified framework taking account the different issues: causation, degree of responsibility, divisibility, inconsistent verdicts *etc.* The method is stated as follows: "*The factfinder divides divisible damages into their indivisible component parts. The factfinder then apportions liability for each indivisible component part under Topics 1-4. For each indivisible component part, the factfinder assigns a percentage of comparative responsibility to each party or other relevant person [...]. The percentages of comparative responsibility for each component part add to 100 percent [...]. The plaintiff is entitled to judgment in an amount that aggregates the judgments for each component part*".³⁵ This method corresponds to the weighted Shapley value.

First, the *Restatement* states that the damage must be divided by causation when it is possible to assign to one tortfeasor or to subsets of tortfeasors the part of the damage this subset has caused alone.³⁶ The characteristic function of a liability game provides such a division of damage. Reciprocally, the factfinder or the jury instructed by a Court to divide the damage seeking to assign to each subset of tortfeasors the damage they would have caused alone, defines a characteristic function.

Sometimes, the task is easy because the aggravated damages d_i are perfectly observable; sometimes, a counterfactual is needed. The factfinder wonders which amount of damage would have occurred if one or several tortfeasors had not acted tortuously and defines

³² *Hillrichs v. Avco Corp.*, 478 N.W.2d 70 (Iowa 1991).

³³ *Reed v. Chrysler Corp.*, 494 N.W.2d 224 (Iowa 1992).

³⁴ Topic 5 of the *Restatement* is entitled "Apportionment Of Liability When Damages Can Be Divided By Causation". See also the *Restatement* (second) § 879 and Boston (1996).

³⁵ The *Restatement*, Topic 5, §26, *comment c*.

³⁶ Interestingly enough, the *Restatement* mentions explicitly the "set" of tortfeasors: "*Divisible damages may occur when a part of the damages was caused by one set of persons in an initial accident and was then later enhanced by a different set of persons*" (the *Restatement*, Topic 5, *Reporters' note, comment f*).

potential damage. Our model captures these features, given that all the coalitions but the grand coalition are only hypothetical.³⁷ For example, in *Dillon*, the Court used potential damage to drastically reduce the amount paid by the electric company by holding that, even if the company had not been negligent, the boy would have suffered important damage due to his fall.³⁸ The only damage the electric company has caused is, at most, the difference between actual damage and potential one. In other words, the Court has divided the harm by evaluating the potential amount of damage due to the fall *alone*.³⁹ Similar legal reasoning could be found in other issues.⁴⁰ Once defined the characteristic function, the question to know how to divide divisible and indivisible parts among tortfeasors still holds. We now discuss this point.

Once damage is divided, the first step of the methodology provided by the *Restatement* is to apportion damage by causation, namely: each tortfeasor should pay at least for the damage he would have caused alone and at most for the additional damage he has caused.⁴¹ For most legal theorists, it would be unfair for a tortfeasor to pay for more than what he has caused. This basic principle inspired by corrective justice is accepted as the cornerstone of all acceptable apportionment rules. As asserted by Robertson (2009, p.1008), following Carpenter, "*it has long been regarded as a truism that 'a defendant should never be held liable to a plaintiff for a loss where it appears that his wrong did not contribute to it, and no policy or moral consideration can be strong enough to warrant the imposition of liability in such [a] case'.*" As soon as the sum of the payments due by each tortfeasor exactly covers the harm suffered by the plaintiff⁴², the core of a 3-player liability game is the subset of

³⁷ We rely on the classical distinction between prospective causation and retrospective one, see note 8 *supra*.

³⁸ *Dillon v. Twin State Gas & Electric Co.*, 163 Atl. 111 (N.H.1932).

³⁹ Obviously, if the boy had not lost his balance, the tortious act would have not been damageable. On the contrary, if the electric company had not been negligent, a less important damage would have occurred. The key-element the factfinder has to know is whether the boy had already lost his balance before grabbing the electric wires or not. Commenting this case, Chief Justice Peaslee said that "*serious injury, if not death, was certain to ensue, when he was caught upon the defendant's wires and electrocuted*" and therefore, it was fair and logical that the Court allows a compensation for only such a sum "as his prospects for life and health were worth at the time the defendant's fault became causal." (Peaslee 1934, p.1134-1135).

⁴⁰ *Douglas Burt & Buchanan Co. V. Texas & Pacific Railway Co.*, 1922, 150 La 1038, 91 So. 503.

⁴¹ This principle is one of the cornerstones of the *Restatement*: "*no party should be liable for harm it did not cause*" (*Restatement*, Topic 5, §26, comment a, see also comments h and j). That is why a one-step process is unfair: "*a court may decide to use a one-step process of apportionment. The factfinder determines the total recoverable damages and then assigns percentages of responsibility to each person who caused some of the damages [...]. A problem with a one-step process is that it may result in a party being held liable for more damages than the party caused. See comment d. A party's comparative responsibility is distinct from the magnitude of the injury the party caused*" (*Restatement*, Topic 5, §26, comment j).

⁴² A plaintiff's total aggregate recovery from all the contributing tortfeasors can never exceed the amount of his actual damages. See *Miller v. Union Pacific R. Co.*, 290 U.S. 227, 236 (1933). We do not deal with punitive damages and we consider that Courts are able to calculate the full amount of damage to be paid to the victim. *A priori*, our argument does not depend on the methods actually used by Courts to calculate damages except if the calculation leads to non-monotonicity: it could be the case, for example, when a first tortfeasor causes a disease to the victim, following by a second tortfeasor who causes death and compensation for death be less important than compensation for disease. Offsetting benefits are therefore excluded, see Porat, and Posner (2014).

allocations that verify two conditions ("*non-objectionable adjudications*"). The first one is that the contribution of any subset of tortfeasors should be at least equal to the damage they would have caused without the intervention of the others. The second one is that no group of tortfeasors should pay more than what it has caused. Law and legal doctrine acknowledge the importance of these two restrictions to consistently apportion liability. Saying that no tortfeasor should pay more than he has caused is a legal translation of the condition C2 in our game.⁴³ Legal principles and economic conditions converge.⁴⁴

However, most of the times, apportionment by causation is insufficient to define a unique apportionment of the damage (in mathematical terms, the core typically contains many allocations). One remaining issue is precisely to know how to divide the indivisible components and therefore to choose one particular apportionment among the unobjectionable adjudications that correspond to core allocations. The second principle proposed by the two-step method – the apportionment by responsibility – is needed: "*the court should divide damages by causation and then, for each component part, apportion liability by shares of responsibility.*"⁴⁵ Fault degrees of each tortfeasor are introduced and play the role of relative weights. Dividing indivisible damage by responsibility, in the sense of the *Restatement*, consists in assigning weights to each tortfeasor in order to divide the indivisible components. Judge could consider arguments which justify treating unequally the tortfeasors, for example, because their degrees of fault are different. It is easy to show that some examples provided by the *Restatement* follow a weighted Shapley value logic.⁴⁶

⁴³ For example, in *Ravo v. Rogatnick*, the Court states that, in case of successive injuries due to medical malpractice, "*the initial tortfeasor may well be liable to the plaintiff for the entire damage proximately resulting from his own wrongful acts. The successive tortfeasor, however, is liable only for the separate injury or the aggravation his conduct has caused*" *Ravo v. Rogatnick*, 514 N.E.2d 1104 (N.Y. 1987). Or see *Pridham v. Cash & Carry Bldg. Center, Inc.*, 359 A.2d 193 (N.H. 1976), *Prospectus Alpha Navigation Co v. North Pacific Grain Growers, Inc.*, 767 F. 2d 1379 (9th Cir. 1986).

⁴⁴ See the *Restatement (second) of Torts*: "*it should be noted that there are situations in which the earlier wrongdoer may be liable for the entire damage, while the later one will not. Thus an original tortfeasor may be liable not only for the harm which he has himself inflicted, but also for the additional damages resulting from the negligent treatment of the injury by a physician. The physician, on the other hand, has played no part in causing the original injury, and will be liable only for the additional harm caused by his own negligence in treatment.*" (*Restatement (second) of Torts*, 16.1.A., §433A, comment c; see Keaton et al. (1984, p. 352).

⁴⁵ See the *Restatement* Topic 5, §26, *Reporters' note, comment d*. This comment criticizes *Alpha navigation* because the additional damage is partly due by the first tortfeasor insofar as without his tortious act the latter damage would have not occurred: "*The court stopped with causal division by holding that the defendant was liable for all the damage caused by its decision to send the ship away. That is not consistent with the goals of comparative responsibility. The plaintiff's negligence also caused the extra damage; but for the original fire, there would have been no damage*".

⁴⁶ See the *Restatement*, Topic 5, §26, *Reporters' note, comment c*. Let's study one of the examples provided by the *Restatement* to illustrate the two-step process: "*Consider a case in which D, the driver of an automobile, is alleged to have negligently driven an automobile into a highway guardrail. An alleged defect in the automobile's door latch causes the passenger door to open and P, the passenger, to be ejected. P suffers serious neurological injuries and sues D and M, the automobile's manufacture [...]. The court instructs the jury that it must find what damages P would have suffered if the door had not opened (assuming the jurisdiction recognizes that hypothetical injury as a cognizable injury for purposes of causal division) [...]. After making that determination, the jury decides if D and M are legally responsible and assigns percentages of*

Weighted Shapley-value offers possible compromises consistent with conditions C1 and C2 and with the evaluation by the judge of the responsibility of each. In addition, the weighted Shapley value mathematically distinguishes between causation and responsibility apportionments. Reciprocally, each core-allocation is a particular weighted Shapley-value. As such, it is possible to consider that, as soon as a Court chooses an unobjectionable adjudication, it reveals the degree of responsibility of each tortfeasor.

responsibility to them [...]. If the jury found that P would have suffered some damages if the door had remained closed, damages are divisible. The jury determines if D was negligent, M's automobile was defective, the negligence caused the entire damages, and the defect caused the enhanced injury. It finds the amount of damages for the enhanced injury and the damages for the entire injury. The jury then assigns percentages of responsibility to D and M for the enhanced injury". In terms of our model, the court determines the set of tortfeasors $N = \{D, M\}$ and instruct the jury to determine d_1 and d_2 (damage division). This defines the characteristic function v associated to the set of tortfeasors $\{D, M\}$: $v(D) = d_1$, $v(D, M) = d_1 + d_2$ and $v(M) = 0$. The legal issue is to solve the transferable utility game (N, v) : the initial damage is entirely paid by D since he is the only cause of this part of the damage and the enhanced injury is shared between the two tortfeasors by assigning to each of them a degree of responsibility, say w_D et w_M , knowing that the sum equals 100%. Then the payment due by each tortfeasor from this two-step process is exactly the weighted Shapley value associated to the game (N, v) .

4. CONCLUDING REMARKS

To conclude, we develop further comments and propose possible extensions beyond the sequential liability models.

Firstly, the model covers a wide range of cases and provides a better "comprehension" of them. By comprehension, we mean that our model defines as a class of games covering a large variety of cases and identifies the common structure that lies behind them. In other words, it identifies a common mathematical structure unifying all liability litigations. It is therefore interesting to know whether Courts use a common rule to apportion damage among tortfeasors.

Applied to sequential situations, the aim of the *Restatement* is precisely to provide such a general method, and we have shown this method is deeply justified in terms of rationality as soon as it appears as the implementation of a weighted Shapley value scheme.

Secondly, one of the main implications of our findings is the relationships between axiomatic reasoning, rationality and legal adjudication: by using an axiomatic method to determine the shares paid by each tortfeasor and by characterizing different solutions in terms of axioms, the discussion about the best way to apportion damage among tortfeasors is improved in terms of impartiality and rationality. By unicity, acceptance of an apportionment derived from a particular rule is equivalent to acceptance of the underline axioms. One step further would be to determine the incentive effects of the implementation of a Shapley value to make clearer the trade-off between fairness and minimization of social costs.

Thirdly, and more importantly, it is possible to extend our approach to cover other types of multiple causation cases, leading to different liability games. That requires the understanding of the structure of the multiple causation at stake. For instance, one possible extension deals with over determination cases or preemptive causation that lead to paradoxical conclusions in legal theory. Consider the famous example of two fires that jointly destroy a house. A strict "*but for test*" would lead to consider that none of the fire is a cause since the damage would have occurred anyway. Tortfeasors have already argued that they have no obligation to compensate the victim insofar as the causal link is missing. Referring to potential damages, the characteristic function is given by $v(12) = v(1) = v(2) = D$.⁴⁷ This game admits no core allocation i.e. there exists no unobjectionable adjudication. However, the symmetric Shapley value is well-defined: the players being interchangeable, it produces the equal division $(D/2, D/2)$.

Regarding information, as our approach is based on *ex post* causation, coalitions have to be understood as counterfactual states of the world (the state of the world that would have occurred, all things being equal, if one agent had not tortuously acted). In the sequential

⁴⁷ See Stapleton (2013).

liability game, this task is simple and actually requires little information (only n numbers, the d_i , which often are perfectly observable). In other cases, it could be difficult to precisely identify the counterfactual states of the world. Take for instance the asbestos cases. Such a litigation leads to apportionment issues either among several firms that have exposed the victim to asbestos products or among different insurance companies that have covered the risk for a single injurer at different periods of time. Several apportionment principals have been proposed.⁴⁸ As the sequential liability game, asbestos cases have a temporal structure because the disease is due to cumulated past exposure. However, asbestos cases do not share the sequential feature of our model insofar as removing an injurer i from the causality sequence does not prevent the injurers down in the sequence from increasing the expected damage, i.e. the final risk of disease. Therefore, once Courts have considered these injurers are together the cause of the disease, assigning to each coalition its value is more difficult and requires information on the risk level created by each one of the coalitions. One way could be to use the epidemiologic models describing the relationships between probability of disease and length of exposures in order to have an idea of the counterfactual states of the world. The best proxies for the counterfactuals here would be the expected damages of each coalition.

⁴⁸ See *Owens-Illinois, Inc. v. United Insurance Co*, 138 N.J. 437 (1994) “because multiple policies of insurance are triggered under the continuous-trigger theory, it becomes necessary to determine the extent to which each triggered policy shall provide indemnity [...]” (title VII). Court then discusses different rules that could be used to apportion the responsibility between firms and/or insurance companies.

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